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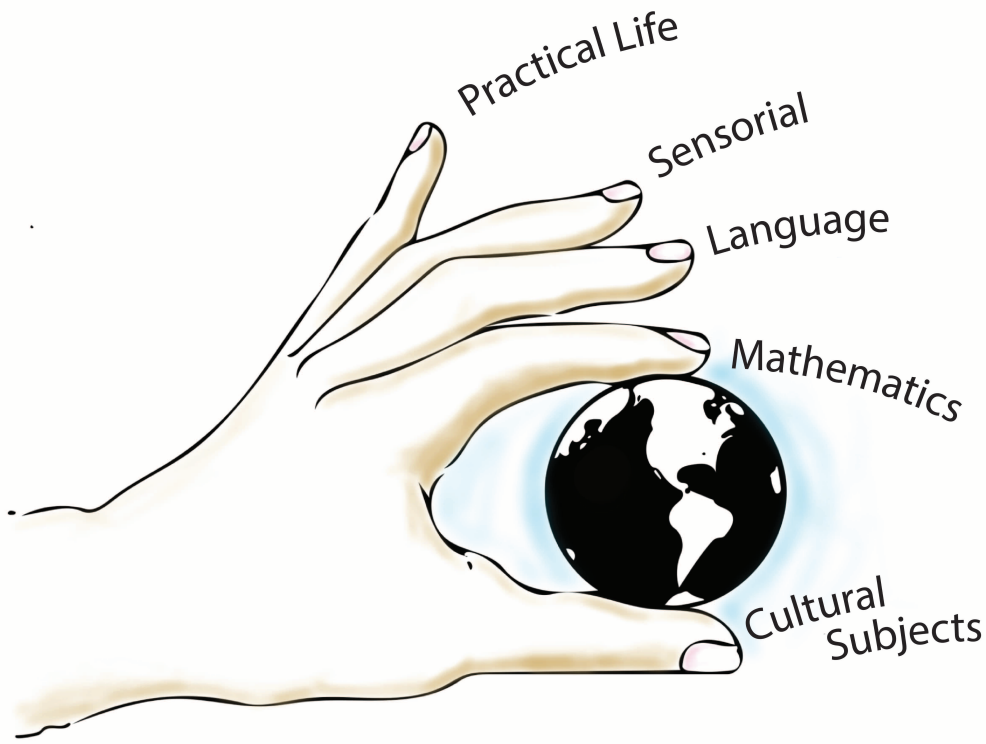

Mathematics

Early Childhood

Teacher Manual

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Introductory Montessori Mathematics

Number Rods

Purposes:

To introduce counting
To associate the names of numerals one through ten with set quantities indicated on the number rods by ten centimeter (one decimeter) red and blue segments
To develop the concept of the sequencing of number
To provide a means for becoming aware of increasing composite quantities
To develop appropriate vocabulary

Preliminary Exercises:

Sequencing the red rods
Floor mat or rug management

Materials:

Ten wooden rods, 2.5 centimeters on each side around the perimeter, varying in length from ten centimeters (one decimeter) to one hundred centimeters (one meter) with each decimeter alternately red and blue
Note: The first rod is red only. All rods are placed with the red segment at the left.
Floor mat or rug long enough to accommodate the longest rod and wide enough for both random arrangement and the rods in sequence

Procedure:

1. Invite a child to the lesson once the red rods can be sequenced.
2. Ask the child to place a floor mat or rug near the shelf where the number rods are stored and to bring the number rods to the mat, placing them at random.
3. Tell the child to sequence these rods in the same manner as the red rods.
4. If interest is shown in learning to count the rods, you and the child return all of the rods to the shelf except the first three. If no interest is shown, have the child return all of the rods and terminate the lesson.
   Note: Rods always are placed horizontally.
5. If the lesson continues, have the child sit on your right side. Keeping the two and three rods to the left of the mat, place the one rod in front of the child and say, "One", at the same time placing the fingers of the dominant hand on the rod. Invite the child to touch the rod and to say, "One."
6. Remove the one rod, place it at the left of the mat and move the two rod in front of the child. Say, "Two," then count "One, "at the same time placing the fingers of the dominant hand on the first segment of the rod. Count "Two," placing the fingers of the dominant hand on the second segment. Invite the child to do the same.
7. Remove the two rod, place it at the left of the mat and move the three rod in front of the child. Say, "Three," then count "One, " at the same time placing the fingers of the dominant hand on the first segment of the rod. Count "Two," placing the fingers of the dominant hand on the second segment. Count "Three," placing the fingers of the dominant hand on the third segment. Invite the child to do the same.

8. Repeat this first period of the three period lesson until the child appears to have assimilated the terms or terminate the lesson if a lack of interest is indicated.
9. When the second period of the lesson is given, place all three rods, not in sequence, in front of the child. Say, "Count the two rod." If the child is able to do so, continue to ask the child to count any of the three rods. If unable to count the rod requested, terminate the lesson by saying, "Thank you. We will have another lesson later."
10. If the third period of the lesson is given, remove the three rods and place at the left of the mat. Place any one of the three rods in front of the child, ask the child to count the rod and tell what it is. Present the rods in random order for the child to count and identify.
    Note: Terminate the lesson when the child loses interest or is unable to follow the procedure as directed by the teacher.
11. At the conclusion of the lesson, thank the child and say that lessons may be requested. Have the child help return the rods to the shelf. Additional quantities will be taught over a period of time until the concept of counting from one to ten is established.

Control of Error:

Alternating red and blue sections of the rods Successive increase in length of rods
The teacher

Observations:

Handling of materials
Placement of the rods
Technique of touching rods
Association of quantity with spoken numeral
Child's counting of sections of the rods
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to place the red rods on one floor mat or rug and the number rods on another. Ask the child to look at a red rod, remember its length and bring the corresponding number rod to the mat to verify that they are the same length.

Vocabulary:

one two three four five six seven eight nine ten number rods count sequence

Sandpaper Numerals

Purposes:

To develop muscular impression for correct formation of numerals To develop ease in writing
To learn recognition of numerals and their names

Preliminary Exercises:

Practice with number rods
Work with tactile exercises

Materials:

Container with cards having fine sandpaper or textured numerals one through nine mounted on green Masonite or mat board, ten on blue, with the baseline drawn under each numeral and a dot to indicate the starting point.
Note: Directions for preparation and patterns for numerals follow.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Both teacher and child wash and dry hands thoroughly.
3. Take the container of numerals to a table and seat the child on your dominant side. Remove the one and two, keeping the container of other numerals on your non-dominant side.
4. Using the first two fingers of the dominant hand, show the child how to feel the numeral exactly as it is to be written and say the name of the numeral. Repeat several times, then offer the child the numeral to feel. Offer the other numeral in the same manner and alternate the numerals as the procedure is repeated as many times as judged necessary for that particular child.
   Note: It may be necessary to guide the child's fingers for more complex numerals.
5. To give the second period of the lesson, place both numerals before the child and say, "Feel the.....," giving the name of a numeral just presented in the firstperiod of the lesson. Randomly say the names of a numeral just presented in the first period of the lesson. Randomly say the names of either of the numerals presented in the first period, always having the child feel the numeral and say its name. If the child is unable to identify any one of the numerals, terminate the lesson by saying, "Thank you. We will have another lesson tomorrow." Do not correct the child by pointing to the correct numeral or by making any comment.
6. If successful with the second period of the lesson, go on to the third period by presenting one numeral at a time. Have the Have the child feel each and say its name. If unable to do so, terminate the lesson as before and start over with the second period upon the next presentation at a later time. Do not correct the child by telling the name of the numeral.
7. Upon termination of the lesson, whether successful or not, thank the child and say that lessons with numerals may be requested. Stand, replace chair and return materials to the shelf.

Note: Many lessons are required before the child is able to demonstrate that the numerals can be felt correctly. Only after the teacher is positive that correct feeling has been established is the child invited to feel the numerals independently. NEVER allow writing of numerals until child can demonstrate the muscular memory for the shapes of numerals without looking at them.

Control of Error:

Tactile awareness of the difference between the rough surface of the numeral and the smooth board on which it is mounted
Visual awareness of the difference in shapes of the numerals
Auditory awareness of the name associated with each numeral
The teacher

Observations:

Preparation of the hand before the lesson
Fingers used for feeling the numerals
Position of fingers
Point of beginning the feeling of numerals
Direction of movement when feeling the numerals
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Number Rods and Cards

Purposes:

To prepare for future work in mathematics
To associate the numerals 1 through 10 with corresponding quantities To develop appropriate vocabulary

Preliminary Exercises:

Use of number rods
Practice with sandpaper numerals to develop the ability to recognize the numerals one through ten

Materials:

Number rods
Numeral cards (not textured) with 1 through 9 in green (unit's color), 1O in blue (ten's color)
Floor mat or rug

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been practiced. Ask that a floor mat or rug be placed near the shelves where the number rods are stored.
2. Have the child place the number rods on the mat in random order with the length parallel to the long edge of the mat which the child is facing.
3. Take the numeral cards to the mat and place in a stack near the front. Have the child sit on your right side.
4. Give the child any numeral card and ask that it be placed in front of the last section on the right of its corresponding rod. The child counts the rods and finds the appropriate one which matches the numeral card. The rods are not placed in order for this exercise.
5. After continuing this with seve,ral rods and cards, reverse the procedure. Indicate a rod and ask the child to find the corresponding numeral card. For this exercise, the numeral cards should be placed in random order horizontally at the bottom of the mat.
6. At the conclusion of the lesson, return materials to the shelf. Thank the child and say that this exercise may be chosen.

Control of Error:

The child's ability to associate the numeral with the counted quantity on the rod

Observations:

Placement of rods and cards
Handling of materials
Child's reaction to error
Length of work time and number of repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

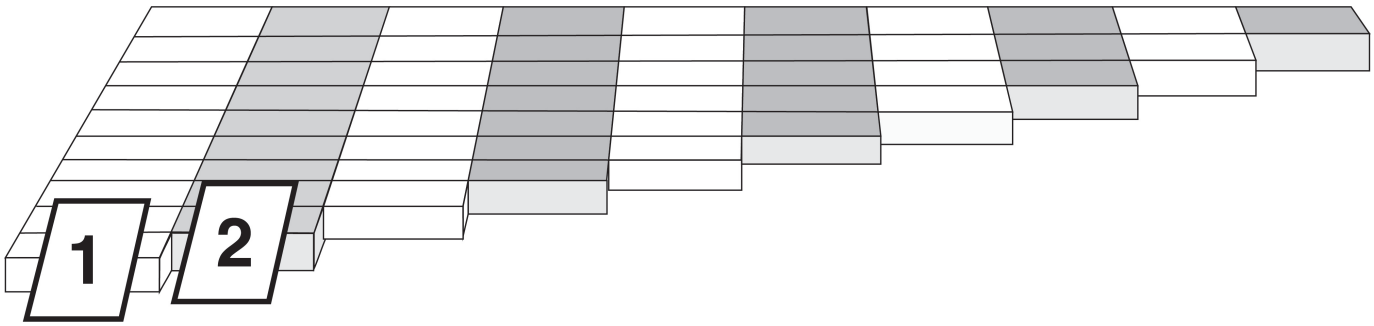
Invite the child to find the one rod and to place the appropriate numeral card in front of it as before. Tell the child to find the next rod, to place it in sequence above the one rod, then to find the appropriate numeral. Indicate that rods and cards are sequenced as previously.

Invite several children who have worked with the rods and cards to participate in a group game. Have one child get a floor mat and others get the number rods to be placed at random on the mat with the long edges parallel to the long side of the rug. Place the numeral cards randomly sequenced in a row on a table nearby. Let each child take a turn counting any rod, then finding the appropriate numeral card to be placed in front of the rod as before.

Invite children to look at the number rods displayed in random order on the mat, then to bring the red rods of the same length for confirmation. The child returns the red rod once the match has been verified.

Vocabulary:

sequence random associate parallel



Spindle Box

Purposes:

To introduce the concept of zero and its written symbol.
To present the sequence of numerals zero through nine To reinforce number-numeral concepts
To develop appropriate vocabulary

Preliminary Exercises:

Work with number rods
Use of sandpaper numerals to establish recognition of one to ten
Note: Lessons with sandpaper numerals continue until the child can write numerals spontaneously.
Exercises with number rods and cards combined

Materials:

Wooden box with ten compartments on the backs of which are written the numerals O through 9 in sequential order Forty-five spindles in a container

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Carry the spindle box and invite the child to take the box of spindles to a table.
3. Place the box in front of the child and the container of spindles to its left.
4. Point to the zero and say, "This is zero. Zero means not any. We will put zero spindles here."
5. Point to the numeral 1. Ask, "What is this?"
6. Tell the child to count that number of spindles from their container into the compartment under the numeral.

Note: Do not say the numeral when telling child to place spindles into compartments.

7. Continue in like manner for sequential numerals until it is observed that the child can work independently. Leave and observe unobtrusively.
8. When it is observed that the exercise has been completed, instruct the child to return the spindles to their container, one at a time before returning the materials to the shelf.

Control of Error:

Exact number of spindles for compartments

Observations:

Placement of the materials on the table
Handling of materials
Placement of spindles
Child's reaction to error
Length of work time and number of repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Each quantity of spindles may be counted into the hand, then placed all together into the appropriate

compartment Vocabulary:

zero compartment spindle

Number Games

Purposes:

To further reinforce number- numeral concepts
To give practice in application of number- numeral concepts
To develop the appropriate vocabulary

Preliminary Exercises:

Use of number rods and numeral cards
Work with spindle box

Materials:

Paper 2 3/4" square with one numeral written in the center, folded so that the numeral is not seen

Note: Color of paper may match the objects to be counted or may be white.

Container labeled set 1 with several papers for each numeral O through 10 and with small objects to be counted such as pebbles, small shells, dried beans or nuts in a shell

Procedure for Verbal Game:

1. Invite several children who know 0 to 10 to participate and have them sit in chairs either near the teacher or around a table.
2. Say, "Pat your head two times." The teacher and children carry out the action and count. Various commands may be given as long as interest remains high.

Procedure for Game with Numerals and Objects:

1. Invite several children who know 0 to 10 to participate. Have them sit on chairs around a table.
2. Take the container of numerals and objects to the table.
3. Have the first child on your left select a paper on which a numeral is written.
4. Ask the child unfold the paper, read the numeral silently and turn it over on the table so that the numeral cannot be seen.
5. Tell the child to count out that quantity of objects onto the table. Counting may be done silently.
6. The other children count the objects and tell the quantity.
7. The paper is turned over and the child says the numeral. If the number of objects and the numeral do not agree, the child may make the correction, but the teacher neither comments on the error nor makes the correction. Objects are returned to their container.
8. The other children take numerals and count objects in the same manner, in turn. Note: The child who draws zero should know to take no objects.
9. Terminate the lesson before interest wanes. Stand, replace chair and thank the children. Say that the game may be chosen.

Control of Error:

Verbal Game: the teacher
Objects Game: the numeral paper and the children

Observations:

Handling of materials
Correspondence of quantities with numerals
Identification of numeral
Child's reaction to error
Degree of interest and concentration

Variations:

Take every opportunity to use number-numeral concepts in the environment, for example, number of napkins and place mats for lunch, number of chairs at a table, number of colored pencils for using insets.
Invite the children to use the materials without the teacher present.
Frequently vary objects used with the numerals.
Add numerals eleven through nineteen after work with the appropriate Seguin Board.

Vocabulary:

terms used in verbal game such as clap, nod, hop,
etc. names of objects used with numerals

Numeral Cards and Counters

Purposes:

To develop the ability to sequence numerals 1-1O
To associate concrete quantity with abstract numerals To provide a visual impression of even and odd numbers To provide a means for learning skip counting
To develop the appropriate vocabulary

Preliminary Exercises:

Work with number rods, sandpaper numerals and numeral cards
Use of spindle box
Number games

Materials:

White mat, divided into ten sections, of a size to accommodate numeral cards and counters in your environment (Sizes are not uniform among suppliers.)
Container of numeral cards with 1 through 9 in green and 1O in blue
Fifty-five green counters in a container

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Place the cards, counters and mat on a table in front of the child who is seated on your dominant side.
3. Keeping the container in front of the child, have the child spread the mat behind the container and place the cards randomly in a row above the mat, then move the container to the non-dominant side of the mat.
4. Ask the child what comes first when counting and to find that numeral. Indicate placement in far left section of mat.
5. Have the child put one counter under the numeral one, pointing to show placement.
6. Continue by asking the child to place the next numeral to the right of the one. Indicate that counters are placed side by side. After the numeral three is placed, indicate that the third counter is centered under the first two.

Note: The teacher neither says the name of the numeral nor counts with or *tor* the child.

7. As soon as it is apparent that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Exact number of counters
Mat to ensure correct placement of numerals and counters in columns

Observations:

Handling of materials
Placement of mat and cards
Placement of counters
Child's reaction to error
Length of work time and number of repetitions
Degree of interest and concentration
Length of period of contemplation

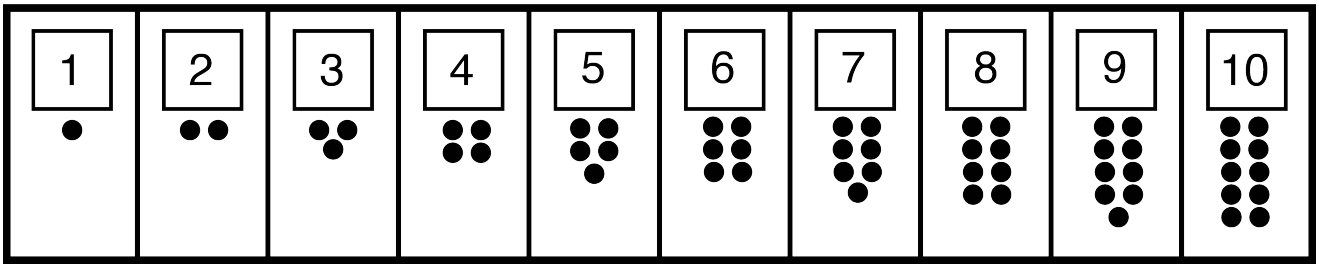
Variations:

Once it is observed that the child understands the number- numeral concepts, a three period lesson on "odd" and "even" may be taught. There is no isolation for 1 st and 3rd periods.

The teacher moves the cards for the even numbers above the mat and invites the child to read the even numbers in sequential order, then to read the odd numbers remaining on the mat in sequential order.

Vocabulary:

next next to odd even sequence counters



Addition with Small Red and Blue Number Rods

Purposes:

To develop an understanding of the process of addition
To give practice in addition
To develop appropriate vocabulary

Preliminary Exercises:

Work with the large number rods
Exercises with spindle box and cards and counters

Materials:

Small red and blue number rods for table work
Container for small number rods

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child take the rods to a table and sit on your dominant side.
2. Ask the child to arrange the rods near the center of the table in the same manner as the large number rods, leaving space at the bottom for horizontal placement of two rows of rods.
3. Move the ten rod below the other rods. The child may recognize it as ten without counting, or there may be a need to count it.
4. Place the nine rod below the ten rod. Have the child count it if not recognized as the nine rod. Ask the child to place a rod at the end of the nine rod to make it equal to the ten rod. Once the one rod is in place, say, "Nine plus one equals ten. When one is added to nine, the sum is ten." Replace the nine and one rods.
5. Follow the same procedure with eight and two, seven and three, six and four.
6. Explain there is not another five to add to equal ten, but show that other rods such as the two and three or the four and the one may be added to the five rod to equal ten.
7. Stand, replace chair and thank child. Say that addition with the table rods may be chosen. Materials are returned to the shelf by the child when exercise is complete.

Control of Error:

Length of the ten rod
The teacher

Observations:

Placement of rods
Counting the rods
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

The child may place the rods together in any combinations, provided eleven through fifty-five are known.

Vocabulary:

small rods added equal plus sum

Subtraction with Small Red and Blue Number Rods

Purposes:

To develop an understanding of the process of subtraction
To give practice in subtraction
To develop appropriate vocabulary

Preliminary Exercises:

Work with the large number rods
Addition with small red and blue rods
Materials:
Small red and blue number rods for table work

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child take the rods to a table and sit on your dominant side.
2. Ask the child to arrange the rods near the center of the table in the same manner as the large number rods, leaving space at the bottom for horizontal placement of rods.
3. Move any two rods below the other rods and place end to end horizontally, for example, the nine rod and the one rod.
4. Have the child count both of the rods. Using the above example, ten would be counted.
5. Take away the one rod and ask the child to count the remaining rod, in this example, nine. State the problem. In this example, say, "Ten minus one equals nine. When one is subtracted from ten, the difference is nine."
6. Continue to combine rods for the child to count, remove the rod on the right and have the child count again. State the problem with the result as before.
7. Stand, replace chair and thank the child. Say that a lesson with subtraction using the small rods may be requested. The materials are returned to the shelf by the child when the exercise is complete.

Control of Error:

Fixed quantities on rods
Child's ability to count

Observations:

Placement of rods
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

The child may form larger subtrahends provided the symbols and quantities for eleven through nineteen are known.

Vocabulary:

minus subtracted difference

Introduction to the Decimal System

Presentation of Golden Beads

Purposes:

To identify by sight, weight and touch one unit, one ten, one hundred and one thousand
To associate names with the quantities
To establish a basis for understanding the decimal system
To develop appropriate vocabulary

Preliminary Exercises:

Practice with number game
Use of cards and counters

Materials:

Presentation tray containing one golden unit bead, ten bar, hundred square and thousand cube (Note: The unit is on the right followed by the ten to its left, then the hundred and finally the thousand on the left.)
Felt table mat

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Place the presentation tray toward the back of the table in front of the child who is seated on your dominant side.
3. Place mat between the tray and the front of the table.
4. Remove the unit from the tray, place it on the mat in its far right position and say, "Unit." Invite the child to handle it and to replace it on the tray.
5. Remove the ten bead bar, place it on the mat in tens position and say, "Ten." Invite the child to handle it and to replace it on the tray.
6. Repeat the procedure with the hundred square and thousand cube. Move the tray to your non-dominant side.
7. By the three period lesson, teach the terms unit, ten, hundred and thousand, always keeping the quantities in their relative places on the felt mat in front of the child.

Note: It may take several lessons to teach the terms to some children.

8. Return the beads to the tray. Stand, replace chair, and thank the child. Return material to the shelf.

Control of Error:

The teacher
Size, weight and configuration of each quantity

Observations:

Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation and degree of interest and concentration

Variations:

Invite the child to count units on the ten bar, tens in the hundred square and hundreds in the thousand cube.

Vocabulary:

unit ten hundred thousand golden beads decimal system place value

Counting Through the Beads

Purposes:

To develop understanding of place value
To provide concrete materials for comprehension of the decimal system
To develop appropriate vocabulary

Preliminary Exercise:

Work with the presentation tray

Materials:

Felt-lined tray with golden bead material as follows: nine units, nine ten bars, nine hundred squares, and nine thousand cubes, all in proper place value positions
Note: Use only materials composed of separate golden beads.
Felt mat in dark color
Two unit bead holders for nine beads each

Procedure:

1. Invite a child to the lesson once the preliminary exercise has been done. Take the tray containing nine of each place value and place at the left front of a table. Have the child sit on your dominant side.
2. Place the mat on the table in front of the child and one of the unit bead holders at the far right of the mat.
   Note: The teacher handles the bead material as the child counts.
3. Place one unit bead at the top of the bead holder and ask the child to count it, saying unit after the number. Continue placing unit beads on the holder as the child counts. (One unit, two units, etc.)
4. When nine units are reached, say, "If we had one more unit, we would have ten units or one ten." Remove the unit beads on their holder with your right hand as you place a ten vertically in tens place on the mat with your left hand. Replace the holder of unit beads on the tray in units place.
5. Continue placing tens vertically from right to left as the child counts. (One ten, two tens, etc.) When nine tens are reached, say, "If we had one more ten, we would have ten tens or one hundred." Remove the tens with your right hand as you place a hundred in hundreds place with your left hand. Replace the tens on the tray to the left of the units.
6. Continue placing hundreds, stacking them on top of each other, one at a time, as the child counts. (One hundred, two hundreds, etc.) When nine hundreds are reached, say, "If we had one more hundred, we would have ten hundreds or one thousand." Remove the stack of hundreds with your right hand as you place a thousand in thousands place with your left hand. Replace the hundreds on the tray to the left of tens.
7. Continue placing thousands from right to left in rows of three each, starting at the top right, as the child counts. (One thousand, two thousands, etc.) When nine thousands are reached, say to the child, "This is as far as we are going to count with this lesson."

8. Return the thousands to the tray at the left. Stand, replace chair, thank the child and say that this lesson may be requested at any time. Return the materials to the shelf.

Control of Error:

Placement and removal of beads by teacher
Limit of nine in each place value

Observations:

Handling of materials
Child's counting of beads
Child's reaction to error
Degree of interest and concentration

Variations:

None

Vocabulary:

one more 2 tens through 9 tens 200 through 900 2000 through 9000

Banking Game with Golden Beads

Purposes:

To develop an understanding of the decimal system
To learn place value
To prepare for operations with golden beads and cards To develop auditory and visual memory
To provide a means for learning the sequence of 1 to 9999 To develop appropriate vocabulary

Preliminary Exercises:

Counting through the place values with golden beads

Materials:

Unlimited unit beads, ten bars, hundred squares and thousand cubes, arranged according to place value on a shelf.
Felt lined tray for each child participating
Unit bead holder for ten beads on each tray

Procedure:

1. Two or three children who have done the preliminary exercises may be invited to participate.
2. Show the children the beads on the shelf and tell them that this will be the bank. Note: Be sure there is a space near the bank on which to place one tray at a time so that beads may be collected without having to place the tray on the floor.
3. Give each participating child a tray with a unit bead holder. Sit at a table on which the children can place their trays of beads and where you can see the bank.
4. Begin by asking each child in turn to go the the bank and to bring you the beads requested on the tray, for example, "Please bring me four tens." Note: The beads are always placed in their relative place value position on the tray. Units will always be at the far right, thousands at far left with tens and hundreds placed appropriately.
5. When the child presents the tray to you, say, "What have you brought?" In this way, the child can have practice in reading the quantities of beads. Move the beads to their place value position if necessary. Thank the child and request that the beads be returned to their proper place in the bank. Note: If the quantity requested is not presented on the tray, it does not matter as long as the child can read the amount. Auditory memory should improve as the exercise is carried out over a period of weeks.
6. After each child has had an opportunity to bring beads from one place value position, ask for two quantities, each from a different place value, for example, "Please bring me six tens, two units." Move the beads to their place value positions if the child has not done so. Request that the beads be replaced if the child does not remember to do so.

7. Allow any child to terminate the exercise as soon as it is observed that interest has waned, but continue as long as some are interested. Ask for three quantities, each from a different place value position, then eventually ask for four. The latter exercises may take a while for the child to remember what has been requested. Some children will be able to remember four quantities, but there may be those participating who can remember only two. Give the child the quantity which can be remembered and gradually build up auditory memory for four quantities.
8. When the exercise is terminated, the trays are replaced on the shelf by the children. Say that the exercise with the beads may be requested. Note: The teacher must conduct the exercise to prevent confusion of place value position in requesting bead quantities.

Control of Error:

The teacher
Configuration and weight of beads

Observations:

Degree of development of auditory memory
Handling of materials
Placement of beads on tray and in bead bank
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Starting with beads from just one place value position, place beads on the tray to be read aloud by the child, then returned to the bank. Gradually increase the number of place value positions until all four can be remembered.

Vocabulary:

bank

Presentation of Large Numeral Cards

Purposes:

To identify the numerals 100 and 1000
Note: The numerals 1 and 10 were learned with the sandpaper numerals.)
To provide a means for identifying place value through color coding: units green, tens blue, hundreds red and units of thousands green
To associate names of quantities with numerals
To establish a basis for understanding the decimal system
To develop appropriate vocabulary

Preliminary Exercises:

Practice with golden bead presentation tray and counting through the beads
Exercises with the quantities of golden beads

Materials:

Tray or container for large numeral cards with 1 unit (green), 1 ten (blue), 1 hundred (red) and 1 thousand (green)
Note: The size of the cards is proportionate to the value, so that the units cards are the smallest and the thousands cards are the largest. The cards in the container are always arranged so that 1111 is showing on the top cards.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take the large numeral cards to a table and place on your non-dominant side. Seat the child on your dominant side.
3. Show the child the green unit card and say, "What is this?" Remove it and repeat the procedure with the blue ten card.
4. Place the red hundred card in front of the child and say, "Hundred." Remove it and place the green thousand card before the child, saying, "Thousand." Repeat this first period of the lesson as many times as judged necessary.
5. Place the hundred and thousand cards in front of the child and say, "Show me thousand," "Show me hundred." Repeat this second period of the lesson several times unless the child was unable to identify the numerals, in which case, terminate the lesson by saying, "Thank you." Later present the first period of the lesson again before proceeding to the second period.
6. Once the second period of the lesson can be done, move to the third period by showing either numeral, one at a time, and asking, "What is this?"
7. Lay out all four cards with unit on the right, ten to its left, hundred to its left, thousand to its left and ask the child to identify them.
8. Replace cards in the container, thank the child and return the materials to the shelf.

Control of Error:

The teacher
Color code of numerals

Observations:

Child's reaction to error

Length of period of contemplation and degree of interest and concentration

Vocabulary:

numeral cards

Counting Through the Cards 1 through 9000

Purposes:

To develop an understanding of place value

To provide a means for associating the abstract symbol of the numeral with the concrete bead material

Preliminary Exercises:

Work with golden bead units, tens, hundreds and thousands

Three period lessons for learning to recognize the unit, ten, hundred and thousand numeral cards

Materials:

Container of large numeral cards with green unit cards 1 through 9, blue ten cards 10 through 90, red hundred cards 100 through 900 and green thousand cards 1000 through 9000

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Take the container of large numeral cards to a table and place to your non-dominant side. Have the child sit on your dominant side.
2. Remove the cards from the container and place the stacks on the table before you, not in front of the child, so that you have easy access to them.
3. With your non-dominant hand, pick up the stack of unit cards which are in sequence with the one card on top.
4. Say, "Let's count the cards as I place them before you." Lay the one card on the table in front of the child and say, "One unit," to give the child the idea that the place will be said as well as the numeral's name.
5. Stack the two card on top of the one and encourage the child to continue counting as the subsequent numeral cards are stacked.
6. After the nine units card is placed, say, "If we had one more, we would have ten units or one ten," at the same time removing the stack of unit cards with the non-dominant hand as you place the ten card with the dominant hand.
7. Count through the tens, stacking the cards as before. After the nine tens card is placed, say, "If we had one more ten, we would have ten tens or one hundred." Remove the stack of tens cards as you place the one hundred card before the child.
8. Count through the hundreds, stacking the cards as before. After the nine hundreds card is placed, say, "If we had one more hundred, we would have ten hundreds or one thousand." Remove the stack of hundreds cards as you place the one thousand card before the child.
9. Count through the thousands, stacking the cards as before. After the nine thousands card is placed, say, "That is as far as we will count with these cards." Rearrange the cards in the original sequence with the ones showing in every place.
10. Thank the child, stand, replace chair and return the materials. Say that card counting may be chosen.

Control of Error:

The teacher

Size and color of cards

Observations:

Child's reaction to error

Degree of interest and concentration

Variations:

None

Vocabulary:

No new vocabulary

Banking Game with the Numeral Cards

Purposes:

To develop an understanding of the decimal system
To learn place value
To prepare for operations with golden beads and cards To develop auditory and visual memory
To provide a means for learning the sequence of 1 to 9999
To establish fluency in reading numerals
To develop appropriate vocabulary

Preliminary Exercises:

Counting through the cards
Exercises with golden beads

Materials:

Container of small numeral cards with green unit cards 1 through 9, blue ten cards 10 through 90, red hundred cards 100 through 900 and green thousand cards 1000 through 9000
Felt mat (optional}
Tray for each child participating, ideally long enough to accommodate cards from each place value position when placed end to end

Procedure:

1. Two or three children who have done the preliminary exercises may be invited to participate.
2. Take the box of cards to a table of appropriate size.
3. Ask one child to lay out the unit cards in a column at the far right, starting at the top right with one. Have another child lay out the tens cards in a column to the left of the units cards, placing the ten card at the top next to the one card. Ask another child to repeat the procedure with the hundreds cards placed to the left of tens, then have the thousands cards placed to the left of hundreds. This is the **card bank.**
4. Give each child a tray on which to place the requested numeral cards and sit where you can see the cards, but not at the table containing them.
5. Begin by asking each child in turn to go the the card bank and to bring you the card requested on the tray, for example, "Please bring me the card for four tens."
   Note: The cards are always placed in their relative place value position on the tray, but are not overlapped at this point. Units will always be at the far right, thousands at far left with tens and hundreds placed appropriately.
6. When the child presents the tray to you, say, "What have you brought?" In this way, the child can have practice in reading the quantities on the cards. Move the card to its place value position if necessary. Thank the child and request that the card be returned to its proper place in the card bank. Note: If the card requested is not presented on the tray, it does not matter as long as the child can read the numeral. Auditory memory should improve as the exercise is carried out over a period of weeks.

7. After each child has had an opportunity to bring single cards, ask for two, each in a different place value, for example, "Please bring me the cards for six tens, two units." Move the cards to their place value positions if the child has not done so.
8. Allow any child to terminate the exercise as soon as it is observed that interest has waned, but continue as long as some are interested. Ask for three cards, each in a different place value, then build up to four. The latter exercises may take a while for the child to remember what has been requested. Some children will be able to remember four numerals, but there may be those participating who can remember only two. Give the child the quantity which can be remembered and gradually build up auditory memory for four cards.
9. When the exercise is terminated, have the children collect the cards sequentially in stacks with the numeral one showing at the top, replace them in the container, then return them to the shelf. Say that the exercise with the cards may be requested. Note: The teacher must conduct the exercise to prevent having two cards from the same place value position on the tray.

Control of Error:

The teacher
Size and color of cards

Observations:

Degree of development of auditory memory
Handling of materials
Placement of cards on tray and in card bank
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Place cards on the tray to be read and returned to the card bank, starting with just one card and building up to four.

Vocabulary:

card bank vertical

Combining Golden Beads and Large Numeral Cards (Grand Array)

Purposes:

To associate the concrete quantity with its abstract symbol
To prepare for the banking game with both cards and beads
To reinforce vocabulary associated with the decimal system

Preliminary Exercises:

Practice with the card banking game
Exercises with the bead banking game

Materials:

Container of large numeral cards with green unit cards 1 through 9, blue ten cards 1O through 90, red hundred cards 100 through 900 and green thousand cards 1000 through 9000

Forty-five unit beads, forty-five ten bead bars, forty-five hundred squares and as many as possible golden bead thousand cubes with imitation thousand cubes to make a total of forty-five

Note: Nine holders for nine unit beads each may be used.

Four floor mats, each 36 inch long to accommodate all the beads and cards in symbolic colors: green felt for units, 6" wide; blue felt for tens, 8 1/2" wide; red felt for hundreds, 1O" wide; green felt for thousands, 13" wide

Note: These measurements are approximate. Mats should be of a size to accommodate the dimensions of the numeral cards in your environment.

Procedure:

1. Invite several children to participate once they have done the preliminary exercises.
2. Have one child lay out the green mat and place green unit cards in a column at the right edge of the mat while another child places the appropriate number of golden unit beads vertically to the left of the numeral .
3. If more than two children are involved, ask another to lay out the blue mat to the left of the units mat and place blue tens cards in a column so that the units and tens are exactly opposite. Another child places the appropriate number of ten bead bars vertically to the left of the tens cards, starting at the edge of the card and building right to left.
4. Have the red mat placed to the left of the blue mat with the red hundreds cards laid out in a column and the appropriate number of hundred squares stacked to the left of the numerals.
5. Have the widest green mat placed to the left of the red mat with the green thousand cards laid out in a column and the thousand cubes stacked very carefully to the left of the numerals.
6. Once the array of cards and beads is complete, most of the children will want to inspect it. This exercise attracts many observers who are not yet ready to participate but who will be inspired to work with the beads.

7. Those who participated in the exercise will return the materials to the shelf once all who choose to examine the array have done so. Any child who has had lessons involving the handling of beads and cards may also assist in returning materials.

Control of Error:

Exact number of beads for each place value
Color and size of cards
Physical increase in size of each quantity
Dimensions and color of felt mat sections

Observations:

Placement of cards and beads
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

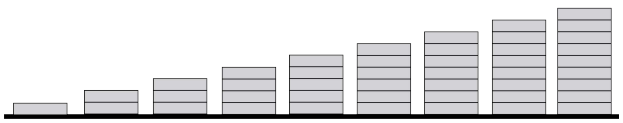
None

Vocabulary:

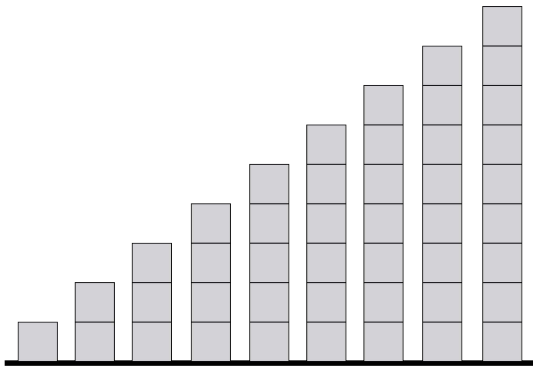
grand array

	1000		100	10	1
	2000		200	20	2
	3000		300	30	3
	4000		400	40	4
	5000		500	50	5
	6000		600	60	6
	7000		700	70	7
	8000		800	80	8
	9000		900	90	9

The Grand Array



Side View of Hundreds



Side View of Thousands

Banking Game with Golden Beads and Numeral Cards

Purposes:

To offer further practice in associating numerals and quantities
To develop an understanding of the decimal system
To give practice in reading numerals
To learn place value
To prepare for group operations in addition, multiplication, division and subtraction To develop appropriate vocabulary

Preliminary Exercise:

Laying out grand array

Materials:

Container of small numeral cards with green unit cards 1 through 9, blue ten cards 1O through 90, red hundred cards 100 through 900 and green thousand cards 1000 through 9000
Shelf with unlimited golden bead material, minimum of 45 units, tens, hundreds and thousands
Felt-lined trays, each with a unit bead holder for ten beads

Procedure:

1. Invite several children to participate once the preliminary exercises have been done.
2. The small numeral cards are placed on a table in columns as in previous exercises by children who have laid out the grand array.
3. Be sure that the beads are in their correct place value positions on the shelf and that there is space near the shelf to place the tray for collection of beads.
4. Give a tray to each child who chooses to participate, provided that the preliminary exercises have been done. Sit beside the card bank and place numeral cards at the lower edge of each tray, starting with two places and progressing to all four. The cards are laid on the tray separated, not overlapped.
5. Tell the child to get as many beads as are indicated by the numerals, then to bring them to you.
6. Say, "What have you brought me?" so that the child has practice in reading numerals. This also gives an opportunity for self-correction. If the wrong amount of beads is presented, do not correct. Have the child return the beads and the cards.

Note: The child refers to the tens by the number presented rather than by the usual terms, for example, three tens instead of thirty.

7. Once cards and beads in all four places are presented, stack the cards in their correct sequence tapping them on the units end to align the cards and read the quantity, using the terms twenty, thirty, etc., as appropriate. For example, the child says, "I have one thousand, three hundred, six tens, two units." After stacking the cards, the teacher says, "Yes, you have one thousand, three hundred, sixty-two." Do not say "and" between hundreds and tens. "And" signifies a decimal point.

Note: At this point, lessons with the Seguin boards may be introduced to teach the numerals and quantities eleven through nineteen and twenty through ninety.

8. Continue as long as interest is observed, then have the children return the materials to the shelf. Say that the exercise may be requested.

Control of Error:

Color and size of cards
Physical increase in size of each
quantity The teacher

Observations:

Reading of quantities
Placement of cards and beads
Handling of materials
Child's reaction to error
Length of work time and number of
repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Place beads on the child's tray to be counted. The child then gets the appropriate cards and places them below the beads at the bottom of the tray.

After much practice with the banking game, children may act as bead banker or card banker, but the teacher places the cards on the trays, or the beads as in the variation above. These "bankers" may give out the materials as requested by the children with trays according to their cards or, in the reverse exercise, their beads.

Vocabulary:

numerals read from cards

Counting Unlimited Golden Beads

Purposes:

To further reinforce understanding of the decimal system and place value
To introduce the changing of quantities *over* nine in units, tens and hundreds places
To prepare for group operations with changing in addition, multiplication, division and subtraction
To develop appropriate vocabulary

Preliminary Exercises:

Banking game with beads and cards

Materials:

Small numeral cards
Unlimited units, tens, hundreds and thousands golden bead material
Felt-lined tray
Unit bead holder for ten beads
Felt mat

Procedure:

1. Invite a child to the lesson. Request that the small numeral cards be laid out in columns on a table as in previous exercises. This is the card bank.
2. Place the felt mat on a table of appropriate size.
3. Have the child take any quantity of beads from units, tens and hundreds but no more than three thousand and place them at random all *over* the tray.
4. Ask the child to take the tray to the table with the felt mat and sort the beads onto the mat according to place value, with units on the right, tens to their left, hundreds to the left of tens and thousands at the far left.
5. Tell the child to count the units onto the unit bead holder. When ten have been placed on the unit bead holder, these are taken back to the bank to be exchanged for a ten bead bar which is placed on the table with the other ten bars. This process is continued until there are nine or fewer units. After counting the units, the child places the appropriate small numeral card in front of the units.
6. The tens are counted and changed for a hundred after *every* ten tens. The tens are carried to the bank on a small tray and the hundred is placed with the other hundreds on the felt mat. After counting the tens, the child places the appropriate small numeral card in front of the tens.
7. The process is repeated with hundreds and thousands with all the cards superimposed.
8. Once it is observed that the procedure is understood, tell the child that this exercise may be chosen at any time.

Control of Error:

Child's ability to count and exchange beads Association
of bead quantities with appropriate numerals

Observations:

- Counting and exchanging bead quantities
- Selection of numeral cards
- Placement of cards and beads
- Handling of materials
- Child's reaction to error
- Length of work time and number of repetitions
- Length of period of contemplation
- Degree of interest and concentration

Variations:

None

Vocabulary:

exchange

Group Operations with Golden Beads

Addition without Changing

Purposes:

To develop understanding of the concept of addition
To provide a basis for comprehending addition of numbers in four decimal places To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Practice with golden bead operations through banking game with beads and cards

Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Three sets of small numeral cards 1 through 3000
Note: Small cards are color coded the same as the large cards.
Unlimited golden beads arranged according to place value on a tray and /or on the shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used.
Unit bead holders to accommodate ten unit beads, one holder for each tray and one for the felt-covered table
Three felt-lined trays
Card with addition problems without changing for the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which do not involve changing.

Procedure:

Note: Prior to presenting group operations, the teacher must lay out all of the beads and cards required for the exercise to determine which tables are suitable to accommodate the materials. This is done when the children are not present and the materials are put away once it has been determined which tables to use.

1. Invite three children to the lesson once they have done the preliminary exercises. Place the large felt mat on the appropriate table previously determined by the teacher. Indicate the bead bank on the shelf or bring the tray of golden beads to the table to be the bead bank.
2. Have one child arrange the large numeral cards on the appropriate table as previously determined by the teacher. Ask the other two children to arrange the three sets of small numeral cards on the appropriate table.
3. Ask each of the three children to take a tray. Give one an addend composed of small numeral cards, for example, 1346. Give another child an addend with small numeral cards, for example, 3211. Give the third child an addend with small numeral cards, for example, 4422. Tell each child to get golden beads in the amount shown with the small numeral cards on each tray.
4. Indicate that each addend is placed on the table covered by the felt mat with beads on the left, cards to the right of the beads. The addends are placed in a column as are the small numeral cards, making sure that proper place value is maintained.

5. One of the children is invited to add the units by counting unit beads from each addend into the unit bead holder placed at the bottom of units column, then is instructed to get a large numeral card to be placed below the small unit cards, indicating the sum of units.
6. Another child is invited to add the ten bars by counting and moving them down to the left of the units, placing them right to left, as if forming a hundred square. The child is instructed to place the large numeral card indicating the sum of tens below the small tens cards, superimposing the unit card on the ten card.
7. The third child is invited to add the hundreds by counting and moving them down to the left of the tens, stacking them as if to form a thousand cube. The child is instructed to place the large numeral card indicating the sum of hundreds below the small hundreds cards, superimposing the ten and unit cards on the hundred card.
8. Any one of the three children may add the thousands by counting and moving them down to the left of hundreds, placing them in pairs from back to front and stacking if the space dictates. The child is instructed to place the large numeral card indicating the sum of thousands below the small thousands cards with the other large numeral cards superimposed on it.
9. The teacher restates the problem by reading the cards on the felt mat "1346 plus 3211 plus 4422 equals 8979. When we add an addend of 1346, an addend of 3211 and an addend of 4422, the sum is 8979." pointing to each addend as it is stated and to the sum.
10. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher
Written addition problems to ensure that the teacher gives out appropriate addends, the sum of which does not exceed nine in any place value
Small numeral cards not exceeding 3000
Color of place values on numeral cards

Observations:

Handling and placement of cards
Handling and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker. Others may add the beads.

Vocabulary:

addend sum superimpose

Multiplication without Changing

Purposes:

To develop understanding of the concept of multiplication
To provide a basis for comprehending multiplication of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Practice with golden bead operations through banking game with beads and cards
Group exercises with addition without changing

Materials:

Large dark colored felt table mat
Large numeral cards up 1 through 9000
Three sets of small numeral cards 1 through 3000
Note: Small cards are color coded the same as large cards.
Unlimited golden beads arranged in proper place value position on a tray and/or shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used.
Four unit bead holders to accommodate ten unit beads each, one holder for each tray and one for the felt-covered table
Three felt-lined trays
Card with multiplication problems without changing for the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which do not involve changing.

Procedure:

1. Invite three children to the lesson once they have done the preliminary exercises. Place the large felt mat on an appropriate table. Indicate the bead bank on the shelf or bring the tray of golden beads to the table to be the bead bank.
2. Have one child arrange the large numeral cards on the appropriate table. Ask the other two children to arrange the three sets of small numeral cards on the appropriate table.
3. Ask each of the three children to take a tray. Give each the same multiplicand with the small numeral cards. Tell them to obtain golden beads from the bead bank according to the cards.
4. Indicate that each multiplicand is placed on the felt-covered table with beads on the left, cards to the right of the beads. The multiplicands are placed in a column as are the small numeral cards, making sure that proper place value is maintained.
5. One of the children is invited to count unit beads from each multiplicand into the unit bead holder placed at the bottom of units column, then is instructed to get a large numeral card to be placed below the small unit cards, indicating the product of units.
6. Another child is invited to count and move the ten bead bars to the left of the units, placing them right to left, as if forming a hundred square. The child is instructed to place the large numeral card indicating the product of tens below the small tens cards, superimposing the unit card on the ten card.

7. The third child is invited to count the hundreds by moving them down to the left of the tens, stacking them as if to form a thousand cube. The child is instructed to place the large numeral card indicating the product of hundreds below the small hundreds cards, superimposing the ten and unit cards on the hundred card.
8. Any one of the three children may count the thousands, moving them down to the left of hundreds, stacking if the space dictates. The child is instructed to place the large numeral card indicating the product of thousands below the small thousands cards with the other large numeral cards superimposed on it.
9. The teacher restates the problem by reading the cards on the felt mat, for example,"1213 times 3 equals 3639. When the multiplicand is 1213 and the multiplier is 3, the product is 3639," pointing to the multiplicands and to the product.
10. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher
Written multiplication problems to ensure that the teacher gives appropriate multiplicands, the product of which does not exceed nine in any place value
Small numeral cards not exceeding 3000
Color of place values on numeral cards

Observations:

Handling and placement of cards
Handling and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker. Others may count the beads.

Vocabulary:

multiplicand multiplier product multiplication

Division without Changing

Purposes:

To develop understanding of the concept of division
To provide a basis for comprehending division of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Practice with golden bead operations through banking game with beads and cards
Group exercises with addition and multiplication without changing

Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Three sets of small numeral cards 1 through 3000
Unlimited golden beads arranged in proper place value position on tray and/or shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used.
Four unit bead holders to accommodate ten unit beads each, one holder for each tray and one for the felt-covered table
Three felt-lined trays
Card with division problems without changing for the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which do not involve changing.

Procedure:

1. Invite three children to the lesson once they have done the preliminary exercises. Place the large felt mat on an appropriate table. Indicate the bead bank on the shelf or bring the tray of golden beads to the table to be the bead bank.
2. Have one child arrange the large numeral cards on an appropriate table. Ask the other two children to arrange the three sets of small numeral cards on the appropriate table.
3. The teacher places the dividend in large numeral cards and golden beads on the felt-covered table, for example, 3996.
4. Each of the three children is asked to take a tray. Say that the divisor is three which means that the beads will be divided equally among the three children.
5. The teacher begins by dividing the thousands, putting one thousand on each tray until all the thousands have been divided. The process is repeated with hundreds, tens and units.
6. Each child is invited to count the beads representing the quotient on the tray, then to place small numeral cards on the tray under the beads.
7. The teacher restates the problem by reading the large cards on the felt mat. "3996 divided by three equals 1332. When the dividend is 3996 and the divisor is 3, the quotient is 1332," pointing to the dividend and to the quotient on each tray. Point out that each tray has the same amount of beads, so the answer is what one receives.
8. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher

Written division problems to ensure that the teacher uses an appropriate dividend for the number of children acting as divisors

Small numeral cards not exceeding 3000

Color of place values on numeral cards

Observations:

Handling and placement of cards

Handling and placement of beads

Counting of beads

Child's reaction to error

Length of work time and number of repetitions

Length of period of contemplation

Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker.

Vocabulary:

divisor dividend quotient division

Subtraction without Changing

Purposes:

To develop understanding of the concept of subtraction
To provide a basis for comprehending subtraction of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Practice with golden bead operations through banking game with beads and cards
Group exercises with addition, multiplication, division without changing Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Two sets of small numeral cards 1 through 9000
Unlimited golden beads arranged in proper place value position on tray and/or shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used. Unit bead holders to accommodate ten unit beads each on the tray and one for the felt-covered table
One felt-lined tray
Card with subtraction problems without changing for the teacher's use ONLY Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which do not involve changing.

Procedure:

1. Invite three children to the lesson once they have done the preliminary exercises. Place the large felt mat on an appropriate table. Indicate the bead bank on the shelf or bring the tray of golden beads to the table to be the bead ba_nk.
2. Have one child arrange the large numeral cards on a table. Ask another child to arrange two sets of small numeral cards appropriately.
3. The teacher places large numeral cards representing minuend on a felt covered table and small numeral cards representing subtrahend on a tray. Ask a child to place beads indicated by the large numeral cards on the table to the left of the cards. Tell another child to remove from the minuend to the tray the quantity of beads represented by the subtrahend, starting with units. The beads and cards of the subtrahend remain on the tray and are "taken away" by having the child stand next to the felt-covered table, holding the tray.
4. The other child is invited to count the beads remaining on the felt-covered table by moving them downward, starting with units, and placing small numeral cards to their right, superimposed.
5. The teacher restates the problem by reading the large numeral cards on the felt-covered table, the small numeral cards on the tray and the small numeral cards on the felt-covered table, for example, "4389 (large cards) minus 2176 (small cards on tray) equals 2213 (small cards on table)", or we can say, With a minuend of 4389 and a subtrahend of 2176 the difference is 2213." The teacher points to the quantities and/or cards as the problem is stated.
6. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher

Written subtraction problems to ensure that the teacher uses appropriate subtrahends and minuends

Color of place values on numeral cards

Observations:

Handling and placement of cards

Handling and placement of beads

Counting of beads

Child's reaction to error

Length of work time and number of repetitions

Length of period of contemplation

Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker. Others may count the beads.

Vocabulary:

minuend subtrahend difference subtraction

Addition with Changing

Purposes:

To develop understanding of the concept of addition with changing
To provide a basis for comprehending addition of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in addition without changing
Counting unlimited golden beads

Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Three sets of small numeral cards 1 through 9000
Unlimited golden beads arranged in proper place value position on the shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used.
Unit bead holder to accommodate ten unit beads, one holder for each tray and one for the felt-covered table.
Three felt-lined trays
Card with addition problems involving changing for the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems.

Procedure:

1. Invite three children to the lesson once they have done addition without changing. Place the large felt mat and the tray of golden beads on appropriate tables or use the bead bank from the shelf.
2. Have one child arrange the large numeral cards. Ask the other two children to arrange the three sets of small numeral cards.
3. Ask each of the three children to take a tray. Place a different addend with the small numeral cards on each tray, then tell the children to put that quantity of golden beads from the bead bank on the tray, for example, 2659, 4222 and 2641.
4. Indicate that each addend is placed on the table covered by the felt mat with beads on the left, cards to the right of the beads. The addends are placed in a column as are the small numeral cards, making sure that proper place value is maintained.
5. One child is invited to add the units by counting unit beads from each addend into the unit bead holder placed at the bottom of units column. When ten beads have been counted, the bead holder containing the ten beads is taken to the bead bank and exchanged for a ten bead bar which is placed above the column of tens. The child continues counting units, then is instructed to get a large numeral card to be placed below the small unit cards, indicating the sum of units.

6. Another child is invited to add the ten bars by counting and moving them down to the left of the units, placing them right to left. When ten tens have been counted, the child returns them to the bank on a small tray to be exchanged for one hundred square which is placed above the hundreds column. The child continues counting tens, then places the large numeral card indicating the sum of tens below the small tens cards, superimposing the unit card on the ten card.
7. The third child is invited to add the hundreds by counting and moving them down to the left of the tens, stacking them as if to form a thousand cube. When ten hundreds have been counted, these are returned to the bank on a small tray to be exchanged for a thousand cube which is placed above the thousands column. The child continues to count hundreds and places the large numeral card indicating the sum of hundreds below the small hundreds cards, superimposing the ten and unit card on the hundred card.
8. Any one of the three children may add the thousands by counting and moving them down to the left of hundreds, placing them in pairs from back to front and stacking if the space dictates. The child is instructed to place the large numeral card indicating the sum of thousands below the small thousands cards with the other large numeral cards superimposed on it.
   Note: There must never be more than nine in thousands place since the cards do not go beyond nine thousands.
9. The teacher restates the problem by reading the cards on the felt mat. "2659 plus 4222 plus 2641 equals 9522. When we add an addend of 2659, an addend of 4222 and an addend of 2641, the sum is 9522," pointing to each addend when stated and to the sum.
10. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher
Written addition problems with changing to ensure that the teacher gives appropriate addends
Color of place values on numeral cards
Unit bead holder

Observations:

Handling and placement of cards
Handling and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker. Others may add the beads.

Vocabulary: addition with changing

Multiplication with Changing

Purposes:

To develop understanding of the concept of multiplication with changing
To provide a basis for comprehending multiplication of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in multiplication without changing
Group exercises in addition with changing

Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Three sets of small numeral cards 1 through 3000
Unlimited golden beads arranged in proper place value position on the shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used. Unit bead holder to accommodate ten unit beads, one holder for each tray and one for
the felt-covered table.
Three felt-lined trays
Card with multiplication problems with changing or the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which involve changing.

Procedure:

1. Invite children to the lesson once they have done the preliminary exercises. Place the large felt mat and the tray of golden beads appropriately or use the bead bank from the shelf.
2. Have one child arrange the large numeral cards. Ask the other two children to arrange the three sets of small numeral cards appropriately.
3. Ask each of the three children to take a tray. Place the same multiplicand in small numeral cards on each tray, for example, 2456. Tell the children to place golden beads from the bead bank on their trays according to the small numeral cards.
4. Indicate that each multiplicand is transferred to the table covered by the felt mat with beads on the left, cards to the right of the beads. The multiplicands are placed in a column as are the small numeral cards, making sure that proper place value is maintained.
5. One of the children is invited to count unit beads from each multiplicand into the unit bead holder placed at the bottom of units column. When ten units have been counted, the child takes them to the bead bank on the unit bead holder and exchanges them for a ten bead bar which is placed above the column of tens. The child continues to count the units, then is instructed to get a large numeral card to be placed below the small unit cards, indicating the product of units.
6. Another child is invited to count and move the ten bead bars to the left of the units, placing them right to left, as if forming a hundred square. When ten tens have been counted, these are taken to the bead bank on a small tray and exchanged for a

hundred square which is placed above the column of hundreds. The child is instructed to continue counting the ten bars and to place the large numeral card indicating the product of tens below the small tens cards, superimposing the unit card on the ten card.

7. The third child is invited to count the hundreds by moving them down to the left of the tens, stacking them as if to form a thousand cube. When ten hundreds have been counted, these are taken to the bead bank on a small tray and exchanged for a thousand cube which is placed above the column of thousands. The child is instructed to continue counting the hundred squares and to place the large numeral card indicating the product of hundreds below the small hundreds cards, superimposing the ten and unit cards on the hundred card.
8. Any one of the three children may count the thousands, moving them down to the left of hundreds, placing them in pairs from back to front and stacking if the space dictates. The child is instructed to place the large numeral card indicating the product of thousands below the small thousands cards with the other large numeral cards superimposed on it.
9. The teacher restates the problem by reading the cards on the felt mat. "2456 times 3 equals 7368. When the multiplicand 2456 is multiplied by the multiplier, 3, the product is 7368," pointing to the multiplicands and to the product.
10. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher
Written multiplication problems to ensure that the teacher gives out appropriate multiplicands
Small numeral cards not exceeding 3000
Color of place values on numeral cards

Observations:

Handling and placement of cards
Handling and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker. Others may count the beads.

Vocabulary:

multiplication with changing

Division with Changing

Purposes:

To develop understanding of the concept of division with changing
To provide a basis for comprehending division of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics.
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in division without changing
Group exercises with addition and multiplication without changing

Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Three sets of small numeral cards 1 through 9000
Unlimited golden beads arranged in proper place value position on the shelf
Unit bead holder to accommodate ten unit beads, one holder for each tray and one for the felt-covered table
Three felt-lined trays
Card with division problems with changing for the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which involve changing.

Procedure:

1. Invite three children to the lesson once they have done the preliminary exercises. Place the large felt mat and the tray of golden beads appropriately or use the bead bank on the shelf.
2. Have one child arrange the large numeral cards. Ask the other two children to arrange the three sets of small numeral cards.
3. The teacher places the dividend in large numeral cards on the felt-covered table and asks a child to place that quantity of golden beads to the left of the cards, for example, 1725.
4. Ask each of the three children to take a tray and say that the divisor is three which means that the beads **will** be divided equally among the three children.
5. The teacher says, "Each for each is what we teach. Fair and square when you share." The teacher places the thousand on the tray of one child and says, "Is this fair? We will change the thousand for ten hundreds." After the change is made, the teacher divides the hundreds among the three children, but since there are hundreds which cannot be divided equally, these are changed for tens. The teacher divides the tens. Those which cannot be divided equally are changed for units. The teacher divides the units.
6. Each child is invited to count the beads representing the quotient on the tray, then to place small numeral cards on the tray under the beads.
7. The teacher restates the problem by reading the large cards on the felt mat. "1725 divided by three equals 575. When a dividend of 1725 is divided by a divisor of 3, the quotient is 575," pointing to the dividend cards and to the quotient in beads on each tray.

8. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher

Written division problems with changing to ensure that the teacher uses appropriate dividends for the number of children acting as divisors

Small numeral cards not exceeding 3000

Color of place values on numeral cards

Observations:

Handling and placement of cards

Handling and placement of beads

Counting of beads

Child's reaction to error

Length of work time and number of repetitions

Length of period of contemplation

Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker, and by having more children as divisors.

Provide division problems with changing which have remainders, for example, 1723 divided by 3 equals 574, remainder 1. The remainder is indicated by a small numeral card placed beside the unit beads which remain on the felt-covered table.

Vary the number of children who act as "divisors."

Vocabulary:

division with changing

Subtraction with Changing

Purposes:

To develop understanding of the concept of subtraction with changing
To provide a basis for comprehending subtraction of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in subtraction without changing
Group exercises in addition, multiplication and division with changing

Materials:

Large dark colored felt table mat
Large numeral cards 1 through 9000
Two sets of small numeral cards 1 through 9000
Unlimited golden beads arranged in proper place value position on the shelf
Note: If thousand cubes made of beads are not available, imitation cubes may be used. Unit bead holder to accommodate ten unit beads on tray and one for felt-covered table One felt-lined tray
Card with subtraction problems with changing for the teacher's use ONLY
Note: Children are not shown the card. It is to ensure that the teacher provides appropriate problems which involve changing.

Procedure:

1. Invite two children to the lesson once they have done the preliminary exercises. Place the large felt mat and the tray of golden beads on an appropriate table, or use the bead bank on the shelf.
2. Have one child arrange the large numeral cards on a table. Ask the other child to arrange two sets of small numeral cards appropriately.
3. Place the minuend with superimposed large numeral cards on a table and the subtrahend with the superimposed small numeral cards on a tray. Tell one child to remove to the tray the quantity of beads represented by the subtrahend from the minuend on the felt-covered table, starting with units. The beads and cards of the subtrahend remain on the tray and are "taken away" by having the child stand next to the felt-covered table, holding the tray.
4. The other child is invited to count the beads remaining on the felt-covered table by moving them downward, starting with units, and placing small numeral cards to their right, superimposed.
5. The teacher restates the problem by reading the large numeral cards on the felt-covered table, the small numeral cards on the tray and the small numeral cards on the felt-covered table, for example, "4829 (large cards) minus 2384 (small cards on tray) equals 2445 (small cards on table)", or we can say, "With a minuend of 4829 and a subtrahend of of 2384 the difference is 2445." The teacher points to the quantities and/or cards as the problem is stated.
6. Continue with more problems as long as interest is shown. The children put away the materials when the group operation is terminated.

Control of Error:

The teacher
Written subtraction problems with changing to ensure that the teacher uses appropriate subtrahends and minuends
Color of place values on numeral cards

Observations:

Handling and placement of cards
Handling and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

More children may be involved in the group operation by having a bead banker and a card banker. Others may count the beads.

Vocabulary:

subtraction with changing

Individual Operations with Golden Beads

Introduction to Recording

Purposes:

To introduce the use of color-coded paper for recording
To establish skills needed for recording problems and answers
To establish a foundation for future abstract operations in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Practice with sandpaper numerals to establish the ability to write them
Group exercises in addition, multiplication, division and subtraction without and with changing

Materials:

Red, green and blue colored pencils
Ruler for drawing lines
Lined paper 2 1/2" wide and 11" long with th, h, t and u printed at the top
Small numeral cards for unit, ten, hundred, thousand
Tray with nine unit beads, nine ten bars, nine hundred squares and nine thousand cubes Two unit bead holders for nine beads, one on the tray, one to place on the felt mat Pencil
Small felt mat

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Take the materials to a table and place the tray of beads toward the back of the table in front of the child.
2. With the child seated on your dominant side, demonstrate the drawing of vertical lines on the paper about 1/2" apart, starting on the far right under the u with green to represent units, then blue under t to represent tens, red under the h to represent hundreds and another green line under th to represent units of thousands. Explain the significance of the colors of the lines and the abbreviations at the top of the lines by placing a unit bead at the top of the unit line, a ten bar at the top of the ten line, a hundred square at the top of the hundred line and a thousand cube at the top of the thousand line. Above the beads place appropriate small numeral cards for unit, ten, hundred and thousand. Return the bead material to the tray and put away the cards.
3. Place the lined paper on the child's dominant side, the felt mat in front of the tray and the unit bead holder at the far right of the felt mat in units place. Ask the child to move a unit bead to the unit bead holder on the mat, starting at the top, and to write the numeral 1 directly on the green line at the far right. Another unit bead is moved to the holder on the mat and the numeral 2 is written on the green line directly under the 1. Continue the procedure until all nine units have been placed and recorded. Remove the unit bead holder containing the unit beads.

4. Have the child move a ten bar from the tray to the felt mat in tens place and write the numeral ten on the lines paper with the 1 directly on the blue line, the O on the green line. Continue until all nine tens have been placed on the mat and recorded. Remove the tens.
5. Follow the same procedure for placing and recording hundreds, stacking the hundreds in the proper place. The 1 is written on the red hundred line, one O is written on the blue ten line, the other O on the green unit line. Remove the hundreds.
6. Have the child place the thousand cubes on the mat, one at a time, writing the numeral 1 on the green line at far left, 0 on the red line, O on the blue line and O on the green line at far right. Continue until all nine thousands have been placed and their numerals writen. Remove the thousands and return all the materials to the shelf.

Control of Error:

Abbreviations **u, t, h,** and **th** Color coding of lines
Nine of each bead quantity

Observations:

Handling of beads
Recording of numerals on colored lines Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Write numerals on the symbolically colored lines and ask the child to set out that quantity of golden beads.
Set out a quantity of golden beads and ask the child to write the numerals representing the quantity on the symbolically colored lines.

Vocabulary:

recording symbolic color

Individual Addition Exercises

Purposes:

To develop understanding of the concept of addition
Tc provide a basis for comprehending addition of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To provide practice in recording arithmetic operations
To develop appropriate vocabular

Preliminary Exercises:

Practice in group operations with golden beads, without and with changing
Introduction to recording

Materials:

Dark colored felt table mat
Tray of golden beads: nine thousand cubes, thirty hundred squares, thirty ten bars, thirty units, arranged in proper place value position
Note: These beads become the "personal" bank for the child doing the exercise. If more are needed, the child may get them from the shelf.
Five unit bead holders to accommodate ten unit beads each, one holder for the tray and four for the felt-covered table
Cards or booklets with addition problems without changing, written on symbolically colored lines with answers on the back of the card or in the back of the book
Note: Problem cards are 3" by 5" unless otherwise noted. Booklets may be 2 1/2" by 3" or 3" by 4 1/2", but the size must be consistent within the classroom.
Lined paper with symbolically colored lines
Note: Paper with the symbolically colored lines printed on it can be purchased, or green, blue and red lines may be drawn with a ruler.
Red ribbon

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child place the felt mat at the front of a table with the tray of golden beads at its left, then get pencil, paper and the problem card or booklet for addition without changing.
2. Ask the child to copy a problem from the card or booklet onto the paper, placing the numerals directly on the symbolically colored lines as before. The paper and problem card or booklet are on the child's dominant side.
3. Tell the child to place the quantity of beads indicated for each addend in separate rows, starting at the top of the felt, keeping them in their place value positions so that all the unit beads on their bead holders are in a column on the far right, the tens are in another column to the left of the units, the hundreds are in a column to the left of tens and the thousands are in a column at the far left. Place ribbon in front below the last row of beads.
4. Have the child add the units of each addend by counting into an empty bead holder placed below the ribbon in the units column, then write the sum of all the units on the paper, placing the numeral on the green unit line.

5. Tell the child to count the tens of each addend, moving them down to the left of the units just counted and placing the tens vertically from right to left. The sum of the tens is written on the paper with the numeral on the blue line.
6. Tell the child to count the hundreds of each addend, moving them down to the left of tens just counted, stacking the hundreds as if to build a cube. The sum is written on the paper with the numeral on the red line.
7. Tell the child to count the thousands of each addend, moving them down to the left of hundreds just counted. The sum is written on the paper with the numeral on the green line. Remind Hie child that the answer may be checked from the back of the card or booklet.
8. The child may continue with more problems as long as interest is shown. The child puts away the materials when the work is terminated.

Control of Error:

Written addition problems without changing to ensure that the sum does not exceed nine in any place value
Color coding of lines and abbreviations u, t, h, and th
Answers on back of card or booklet

Observations:

Recording of problems and answers Handling
and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of
repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Provide addition problems with changing on cards or in booklets for individual use.

Vocabulary:

no new vocabulary

Individual Multiplication Exercises

Purposes:

To develop understanding of the concept of multiplication
To provide a basis for comprehending multiplication of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To provide practice in recording arithmetic operations
To develop appropriate vocabulary

Preliminary Exercises:

Practice in group operations with golden beads
Exercises in individual golden bead addition

Materials:

Large dark colored felt table mat
Tray of golden beads: nine thousand cubes, thirty hundred squares, thirty ten bars, thirty units, arranged in proper place value position
Note: It more beads are needed, the child may get them from the shelf.
Five unit bead holders to accommodate ten unit beads each, one holder for the tray and four for the felt-covered table
Cards or booklets with multiplication problems without changing
Lined paper with symbolically colored lines
Note: Paper with the symbolically colored lines printed on it can be purchased, or green, blue and red lines may be drawn with a ruler.
Yellow ribbon

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the felt mat at the front of a table with the tray of golden beads at its left, then get pencil, paper and the problem card or booklet for multiplication without changing.
2. Ask the child to copy a problem from the card or booklet onto the paper, placing the numerals directly on the symbolically colored lines as before. The paper and problem card or booklet are on the child's dominant side.
3. Tell the child to place the quantity of beads for the multiplicand in separate rows according to the number of times indicated by the multiplier. Start at the top of the felt, keeping the beads in their place value positions so that all the unit beads on their bead holders are in a column on the far right, the tens are in another column to the left of the units, the hundreds are in a column to the left of tens and the thousands are in a column at the far left. Place ribbon in front of beads.
4. Have the child count the units of each multiplicand into an empty bead holder placed below the ribbon in the units column units column, then write the product of all the units on the paper, placing the numeral on the green unit line.
5. Tell the child to count the tens of each multiplicand, moving them down to the left of the units just counted and placing the tens vertically from right to left. The product of the tens is written on the paper with the numeral on the blue line.

6. Tell the child to count the hundreds of each multiplicand, moving them down to the left of tens just counted, stacking the hundreds as if to build a cube. The product is written on the paper with the numeral on the red line.
7. Tell the child to count the thousands of each multiplicand, moving them down to the left of hundreds just counted. The product is written on the paper with the numeral on the green line. Remind the child that the answer may be checked from the back of the card or booklet.
8. Continue with more problems as long as interest is shown. The child puts away the materials when the work is terminated.

Control of Error:

Written multiplication problems without changing to ensure that the product does not exceed nine in any place value
Color coding of lines and abbreviations **u, t, h,** and **th**
Answers on back of card or booklet

Observations:

Recording of problems and answers Handling
and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Provide multiplication problems with changing on cards or booklets for individual use.

Vocabulary:

No new vocabulary

Individual Division Exercises

Purposes:

To develop understanding of the concept of division
To provide a basis for comprehending division of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To provide practice in recording arithmetic operations
To develop appropriate vocabulary

Preliminary Exercises:

Practice in group operations with golden beads
Exercises in individual operations for addition and multiplication

Materials:

Dark colored felt table mat
Tray of golden beads: nine thousand cubes, thirty hundred squares, thirty ten bars, and thirty units, arranged according to place value on a tray
Tray for dividend
Nine unit bead holders to accommodate ten unit beads each, and one for the tray Cards or booklets with division problems without changing
Nine large green skittles
Lined paper with symbolically colored lines
Note: Paper with the symbolically colored lines printed on it can be purchased, or green, blue and red lines may be drawn with a ruler.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the felt mat at the front of a table with the tray of golden beads at upper left and the empty tray in front of it. Have the child get pencil, paper and the problem card or booklet for division without changing.
2. Explain that the bracket symbol is another way to write division. Ask the child to copy a problem from the card or booklet onto the paper, placing the numerals directly on the symbolically colored lines as before and leaving space at the top for the quotient to be written. Have the child draw a bracket as shown on problem card or booklet. The paper and problem card or booklet are on the child's dominant side.
3. Tell the child to place the quantity of beads for the dividend on the empty tray, keeping the beads in their place value positions. Have the child place large green skittles for the divisor in a column at the left of the felt mat, allowing enough space between to place the beads to the right of the skittles. For example, if the problem is 8488 divided by 4, four large green skittles are set out at the left side of the mat. Tell the child that the skittles are used in place of the children among whom the beads were divided in group division.

4. Have the child begin dividing the beads among the skittles, starting with thousands. Each skittle is given a thousand until all have been divided, then the number that one skittle received is recorded on the appropriate green line above the bracket. Next the hundreds are divided, placing them to the right of the thousands. The child counts and records on the red line above the bracket The tens and finally the units are divided and recorded.
5. Continue with more problems as long as interest is shown. The child puts away the materials when the work is terminated.

Control of Error:

Written division problems without changing to ensure appropriate dividends for divisors
Color coding of lines and abbreviations **u, t, h,** and **th**
Answers on back of card or booklet

Observations:

Recording of problems and answers Handling
and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Provide division problems with changing on cards or in booklets for individual use.
Provide division problems with remainders.

Vocabulary:

skittle

Individual Subtraction Exercises

Purposes:

To develop understanding of the concept of subtraction
To provide a basis for comprehending subtraction of numbers in four decimal places
To establish a foundation for future abstract operations in mathematics
To provide practice in recording arithmetic operations
To develop appropriate vocabulary

Preliminary Exercises:

Practice in group operations with golden beads
Exercises in individual addition, multiplication and division with golden beads

Materials:

Dark colored felt table mat
Tray of golden beads: nine thousand cubes, thirty hundred squares, thirty ten bars, and thirty units, arranged according to place value on a tray
Unit bead holder to accommodate ten unit beads on the tray and two for the felt mat
Cards or booklets with subtraction problems without changing
Lined paper with symbolically colored lines
Note: Paper with the symbolically colored lines printed on it can be purchased, or green, blue and red lines may be drawn with a ruler.
Green ribbon.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the felt mat at the front of a table with the tray of golden beads at its left, then get pencil, paper and the problem card or booklet for subtraction without changing.
2. Ask the child to copy a problem from the card or booklet onto the paper, placing the numerals directly on the symbolically colored lines as before. The paper and problem card or booklet are on the child's dominant side.
3. Have the child place the beads for the minuend at the top of the felt mat. Beginning with units, the quantity indicated by the subtrahend is subtracted by moving those beads in front of the minuend. Place ribbon in front of the beads.
4. The beads remaining at the top of the mat are moved below the ribbon and counted. The answer is recorded as the difference as each place value is counted.
5. Continue with more problems as long as interest is shown. The child puts away the materials when the work is terminated.

Control of Error:

Written subtraction problems without changing to ensure that the subtrahend is less than the minuend
Color coding of lines and abbreviations **u, t, h,** and **th**
Answers on back of card or booklet

Observations:

Recording of problems and answers Handling
and placement of beads
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Provide subtraction problems with changing on cards or booklets for individual use.

Vocabulary:

no new vocabulary

Short Bead Stair Introduction

Purposes:

To reinforce number concepts
To prepare for addition, multiplication and subtraction operations with recording
To offer a means for learning tables other than by rote memory
To provide the basis for learning 11 through 19 with the Seguin board
To develop appropriate vocabulary

Preliminary Exercises:

Work with cards and counters
Practice with "sandpaper" numerals for development of the ability to write numerals
Exercises with combined golden beads and numeral cards

Materials:

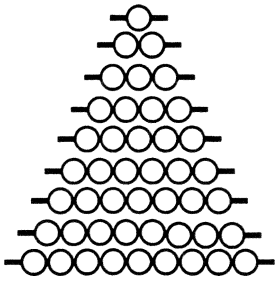
Container with one set of short bead stair, each quantity represented by a different color bead strung on stiff wire: one represented by a single red bead; two, by two green beads; three, pink; four, yellow; five, light blue; six, gray; seven, white; eight, lavender; nine, dark blue

Note: Different manufacturers provide beads in other colors, but the above are the colors specified by Montessori in **Montessori Elementary Material.** The gray beads are no longer available.

Small felt mat in a light, neutral color

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take a container with one set of short bead stair and a felt mat to a table. Place the container in front of the child who is sitting on your dominant side.
3. Ask the child to place the felt mat to the right of the container, then find the bead bar representing one. Indicate that it is placed at the top center of the mat.
4. Say, "What comes next after one?" Have the child count the bead bars to find the next quantity and indicate that it is placed below the one bead. The two bead bar is placed so that there is an equal amount of bead showing on either side of the one bead.
5. Ask the child to continue counting and placing bead bars so that an equilateral triangle is formed.
6. Thank the child and say that the short bead stair may be chosen for arranging.



Control of Error:

- Different color of beads for each quantity
- Formation of an equilateral triangle
- Child's ability to count to nine

Observations:

- Handling of materials
- Placement of beads on mat
- Counting the beads
- Child's reaction to error
- Length of work time and number of repetitions
- Length of period of contemplation
- Degree of interest and concentration

Variations:

- None

Vocabulary:

- short bead stair

Eleven Through Nineteen with Bead Material

Purposes:

To learn names of numbers eleven through nineteen
To call attention to composition of the numbers as ten plus quantities one through nine
To expand counting skills through nineteen
To develop appropriate vocabulary

Preliminary Exercises:

Work with golden beads and large numeral cards combined
Introduction to short bead stair

Materials:

Nine ten bead bars and one set of short bead stairs in a labeled container
Small felt mat

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take the container of beads and the felt mat to a table. Have the child sit on your dominant side. Place the container of beads on your non-dominant side and the felt mat in front of the child.
3. Place a ten bead bar horizontally on the mat and add the one bead to its right. Say, "Ten and one, eleven." Invite the child to count, then repeat the word eleven.
4. Replace the beads in the container, then remove a ten bead bar and a two bead bar and place horizontally with the two bar at right of the ten. Say, "Ten and two, twelve." Invite the child to count, then repeat the word twelve.
5. Randomly present eleven and twelve several times. This is the first period of the three period lesson.
6. For the second period of the lesson, place both bead quantities horizontally before the child and say, "Show me eleven." "Show me twelve." Move the beads to different places on the mat and randomly request that either quantity be identified. Continue the process as long as needed or until interest wanes.
7. If the child is still interested, go to the third period of the lesson after removing the beads from the mat. Place either quantity horizontally before the child and say, "What is this?" If the child can identify the quantity, remove those beads and place the other quantity before the child for identification. Randomly present either quantity with only one displayed at a time.
8. Thank the child at any point of lesson termination and say that other lessons will be given. Have the child replace the materials.
9. By the same procedure, teach the quantities through nineteen in subsequent lessons

Control of Error:

The teacher
The color of the beads added to the ten bead bars to call attention to the quantities

Observations:

Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation and degree of interest and concentration

Vocabulary:

eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen

Written Symbols For Eleven Through Nineteen

Purposes:

To learn the symbols eleven through nineteen
To call attention to composition of numerals as ten plus the quantities one through nine
To develop appropriate vocabulary

Preliminary Exercises:

Work with bead quantities for eleven through nineteen

Materials:

Box of Seguin boards with column of nine tens, preferably blue, numeral cards in green
Floor mat or rug

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Ask the child to unroll a floor mat or rug. Take the box of Seguin boards and numerals to the mat or rug, open the box and place it in the lid near the top of the mat or rug. The long side of the box is parallel to the long side of the mat or rug.
3. Remove the two boards, placing the one with five tens in the center of the mat or rug and the other touching it so that there is a column of nine tens. Leave space at the bottom for the numeral cards.
4. Have the child remove the one through nine numeral cards and arrange sequentially in a row at the bottom of the boards.
5. Show the child how to slide the numeral one into the slot on the right side of the top ten. Say, "Ten and one, eleven," as the one is moved over the zero. Invite the child to slide the numeral two over the zero of the next ten as you say, "Ten and two, twelve."
6. Randomly say eleven and twelve several times as you point to the numerals. This is the first period of the three period lesson. (Numeral not being named may be covered with a small piece of beige felt.)
7. For the second period of the lesson, say, "Show me eleven," "Show me twelve." Continue the process as long as needed or until interest wanes.
8. It the child is still interested, go to the third period of the lesson. Say, "What is this?" as you point to a numeral. Randomly ask for either quantity.
9. Thank the child at any point of lesson termination and say that other lessons will be given. Have the child replace the materials.
10. By the same procedure in subsequent lessons, teach the numerals through nineteen once the previous ones are learned.

Control of Error:

Placement of numerals in sequence over the zeros
Recognition of quantities on short bead stairs

Observations:

Handling of materials
Child's reaction to error

Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Vocabulary:

eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen

Combination of Beads and Numerals for Eleven Through Nineteen

Purposes:

To associate concrete material with abstract numerals
To facilitate understanding of number concepts
To develop appropriate vocabulary

Preliminary Exercises:

Work with beads to learn the quantities eleven through nineteen
Exercises with the Seguin boards to learn the numerals eleven through nineteen

Materials:

Nine ten bead bars and one set of short bead stairs in a labeled container
Seguin boards in a box with nine tens in a column, preferably in blue
Numeral cards one through nine, preferably in green
Floor mat or rug

Procedure:

1. Invite a child to a lesson once the preliminary exercises have been done.
2. Have the child unroll a floor mat or rug and place the Seguin board on it as before with the numerals one through nine across the bottom of the mat.
3. Ask the child to get the container of ten bars and short bead stair and arrange the short bead stair in an equilateral triangle as in previous work, building it at the left of the board above the numerals. The ten bars are placed horizontally at the left of the short bead stair.
4. Pointing to the top of the board, have the child place that quantity, a ten bar, to the left of the board in a horizontal position, leaving about a hand's width between the bead bar and the board.
5. Ask what number comes next after ten, then have the child slide the numeral one over the zero of the top ten from the right side. Tell the child to form eleven with the bead stair one placed to the right of the ten bar.
6. Have the child continue forming the numerals and composing the appropriate quantities as learned in the previous exercises. Leave and observe unobtrusively.

Control of Error:

The color of the beads added to the ten bead bars to call attention to the quantities
Placement of numerals in sequence over the zeros on the Seguin boards

Observations:

Handling of materials
Placement of beads and unit cards
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

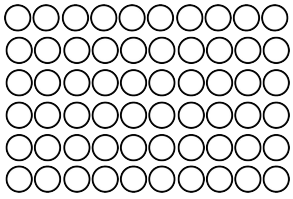
Variations:

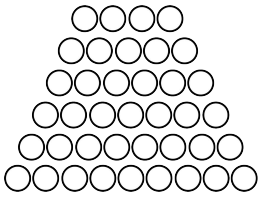
Invite the child to write the numerals eleven through nineteen.

Vocabulary:

No new vocabulary

11
12
13
10
10
10
10
10
10





4	5	6	7	8	9

Ten Through Ninety with Bead Material

Purposes:

To learn names of numbers twenty through ninety
To call attention to the composition of the numbers as tens
To expand counting skills through ninety
To develop appropriate vocabulary

Preliminary Exercises:

Banking game with golden beads and numeral cards
Work with Seguin boards and bead materials for eleven through nineteen

Materials:

Forty-five ten bead bars in a labeled container
Small felt mat

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take the materials to a table and spread the felt mat in front of the child who is seated on your dominant side. The container of beads is on your non-dominant side with the lid under it, if there is a lid.
3. Place one ten bead bar vertically on the mat and ask the child to identify it. Lay a second ten bead bar vertically so that it touches the left side of first bar and say, "Two tens are twenty." Point to the bars while counting, "Ten, twenty."
4. Add a third ten bead bar placed vertically at the left of the first two bars so that the bars are touching and say, "Three tens are thirty." Point to the bars while counting, "Ten, twenty, thirty." Replace beads in the container. Repeat this step several times.
5. For the second period of the lesson, place two tens vertically near the top of the mat and three tens vertically below them. Say, "Show me thirty." If the child is able to do so, say, "Show me twenty." Randomly ask that these quantities be identified several times. Replace the beads in the container.
6. For the third period of the lesson, place two ten bead bars vertically on the mat and and ask, "What is this?" If the child is able to identify twenty, repeat the procedure for thirty. Continue the identification process several times.
7. Thank the child at any point of lesson termination and say that other lessons will be given. Have the child replace the materials.
8. By the same procedure, teach the quantities through ninety in subsequent lessons.

Control of Error:

The teacher
Exact number of ten bars to form quantities ten through ninety

Observations:

Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Vocabulary:

twenty thirty forty fifty sixty seventy eighty ninety

Written Symbols For Twenty Through Ninety

Purposes:

To learn the numerals twenty through ninety
To call attention to the composition of the numerals as tens
To develop appropriate vocabulary

Preliminary Exercises:

Work with bead quantities and numerals for eleven through nineteen
Exercises with ten bead bars for learning twenty through ninety

Materials:

Seguin boards in a box with numerals ten through ninety in a column, preferably in blue
Floor mat or rug

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Ask the child to unroll a floor mat or rug. Take the box of Seguin boards to the mat or rug, open the box and place it in the lid near the top of the mat or rug. The long side of the box is parallel to the long side of the mat or rug.
3. Remove the two boards, placing the board with ten through fifty in the center of the mat or rug and the other touching it so that there is a column of numerals ten through ninety.
4. Point to the ten and say, "What is this?" Point to the next numeral and say, "Two tens, twenty." Point to the next numeral and say, "Three tens, thirty." Randomly say twenty and thirty several times as you point to the numerals. (Numeral not being named may be covered with a small piece of beige felt.) This is the first period of the three period lesson.
5. For the second period of the lesson, say, "Show me twenty," "Show me thirty." Continue the process as long as needed or until interest wanes.
6. If the child is still interested, go to the third period of the lesson. Say, "What is this?" as you point to a numeral. Randomly ask for either numeral.
7. Thank the child at any point of lesson termination and say that other lessons will be given. Have the child replace the materials.
8. By the same procedure in subsequent lessons, teach the numerals through ninety once the previous ones are learned.

Control of Error:

Sequence of numerals on the Seguin boards
The teacher

Observations:

- Handling of materials
- Child's reaction to error
- Length of work time and number of repetitions
- Length of period of contemplation
- Degree of interest and concentration

Vocabulary:

- No new vocabulary

Combination of Beads and Numerals for Twenty Through Ninety

Purposes:

To associate concrete material with abstract numerals
To facilitate understanding of number concepts
To develop appropriate vocabulary

Preliminary Exercises:

Work with beads to learn the quantities twenty through ninety
Exercises with the Seguin boards to learn the numerals twenty through ninety

Materials:

Forty-five ten bead bars in a labeled container
Seguin boards in a box with numerals ten through ninety in a column, preferably in blue
Floor mat or rug

Procedure:

1. Invite a child to a lesson once the preliminary exercises have been done.
2. Have the child unroll a floor mat or rug and place the Seguin boards on it as before.
3. Ask the child to get the container of ten bars and place it at the left of the boards near the bottom of the mat.
4. Pointing to the top of the board, have the child place that quantity, a ten bar, to the left of the board in a vertical position, close to the board.
5. Ask the child to read the next numeral and to place the appropriate number of ten bars at the left, with the second ten placed to the left of the first to form twenty.
6. Have the child continue reading the numerals and composing the appropriate quantities as learned in the previous exercises. Leave and observe unobtrusively.

Control of Error:

Fixed numerals on the Seguin boards
Fixed ten bead bars for the counting of tens only, rather than units
Exact number of ten bars to form ten through ninety

Observations:

Handling of materials
Placement of ten bars
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to write the numerals twenty through ninety.

Vocabulary:

No new vocabulary

Counting Ten Through Ninety-nine with Golden Beads and Seguin Boards

Purposes:

To arouse awareness of the sequence of numerals and quantities through ninety-nine
To develop counting skills
To associate quantities with their numerals
To prepare for future work in mathematics
To develop appropriate vocabulary

Preliminary Exercises:

Work with Seguin boards and beads for eleven through nineteen
Exercises with Seguin boards and beads for twenty through ninety

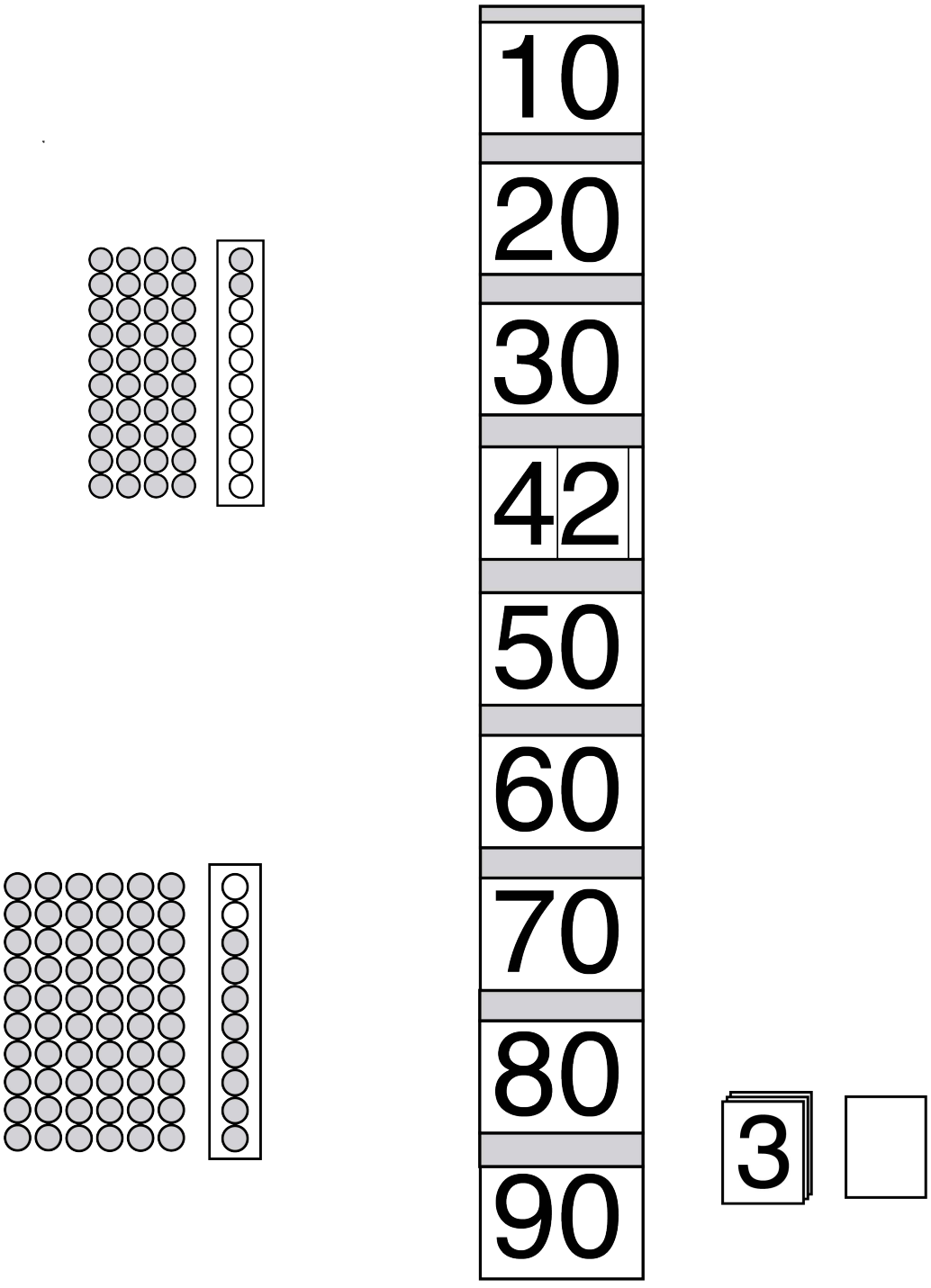
Materials:

Seguin boards with numerals ten through ninety in blue and unit cards one through nine in green
Container with nine unit beads and nine ten bars
Two unit bead holders for nine beads
Floor mat or rug

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Have the child unroll the floor mat or rug and place the Seguin boards with ten through ninety as in previous exercises. The ten bars are placed near the bottom left of the boards with units on a unit bead holder to their left.
3. Ask the child to place the unit cards in order in a stack at the lower right corner of the board. The unit bead holder is placed beside the ten at the top left of the Seguin boards.
4. Point to the numeral ten at the top of the Seguin boards and tell the child to place that quantity (one ten bar) to the right of the unit bead holder.
5. Say, "What is the next numeral after this?" The child should realize that it is eleven. Indicate that the child is to place the one card over the zero on the ten and a unit bead at the top of the unit bead holder.
6. The one card is removed and turned face down to the right of the numeral cards. The two card is placed over the zero. Another unit bead is placed on the unit bead holder below the first bead.
7. The child continues to remove and replace numerals and to form the appropriate quantity with unit beads until nineteen is reached. Say, "If we had one more, we would have two tens or twenty." Remove the nine units on their holder to the lower position and place a ten bar beside the twenty as you move first ten bead bar down next to it. An empty unit bead holder is placed directly to left of the two ten bars.
8. Tell the child to continue adding numerals and beads in sequence until twenty-nine is reached. Say, "If we had one more, we would have three tens or thirty." Remove the nine units on their holder to the lower position and place a ten bar beside the thirty as you move the two ten bead bars down next to it. An empty unit bead holder is placed directly to the left of the three tens.

9. The child should be able to continue the procedure up to ninety-nine, taking away the nine units and moving a ten bar beside the succeeding numeral along with the ten bars from the previous position.
   Note: Because this exercise takes a long time, it can be left out at the end of the day or at lunchtime to be completed later. It would be extremely frustrating if the work had to be put away without completion, only to be started over again. Children would not be interested in choosing the activity.



Control of Error:

Unit bead holders
Number of unit beads and ten bars limited to nine each
Fixed numerals on the Seguin boards

Observations:

Handling of materials
Placement of ten bars and unit beads
Placement of numeral cards over the zero on the tens
Child's reaction to error
Length of work time and number of repetitions Length
of period of contemplation
Degree of interest and concentration

Variations:

Two children may do this exercise, one handling the beads and the other, the numerals.

Vocabulary:

numeral names twenty-one through ninety-nine

Short Bead Stair Addition

Purposes:

To develop an understanding of the process of addition
To give practice in addition
To learn how to record addition problems
To develop appropriate vocabulary

Preliminary Exercises:

Practice in addition with table-sized red and blue rods
Introductory exercise with the short bead stair
Work with Seguin boards to learn 11 through 19
Practice with sandpaper numerals to ensure effortless writing of numerals

Materials:

Two sets of bead bars in a red container
Small felt mat
Addition tables for ones through nines
Paper with ten rows of five squares each, the squares measuring 3/4"
Red pencil and ruler
Pencil
Addition summary tables
A small card on which is written the addition sign
A small card on which is written the equal sign
Date stamp

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take cards with the addition and equal signs to a table and tell the child that this is the plus sign, used to signify addition. By the three period lesson, teach the names of the signs.
3. Choose any addition table and the container of bead bars. Have the child help take all of the materials to a table. Place the box of beads before the child with the felt mat in front of it. The recording paper, pencils and ruler are on the child's dominant side, next to the felt mat. The chosen addition table is placed on the dominant side of the felt mat. The recording paper is placed on the right of the addition table.
4. Using a ruler and red pencil, show the child how to draw red answer squares on the recording paper, then move the red pencil and ruler out of the way on the dominant side once the red squares have been drawn. The child's name is written on the prepared paper.
5. Ask the child to open the container of beads and to place the lid beneath the container, if there is a lid. Tell the child to form a stair at the top of the mat with one set of bead bars and to place near the bottom left of the mat the bead bar from the other set which represents the table chosen. For example, if the five table was chosen, the five bead bar will be placed horizontally near the bottom left of the mat.
6. Have the child copy the first problem from the addition table on the recording paper, placing a numeral or sign in each box except the red one.

7. Ask the child to read the problem and to place the one bead bar from the short bead stair triangle to the right of the bead bar at the bottom of the mat, in the example given above, the five bead bar for the five table. Have the child count the beads at the bottom of the mat and write the answer in the red square. Tell the child to restate the problem with the answer. Return the one bead bar to the short bead stair triangle.
8. Tell the child to copy and read the next problem in the table, then to place the two bead bar from the short bead stair triangle to the right of the bar representing the table chosen. Ask the child to count the bead bars and record the answer as before, then return the two bead bar to short bead stair.
9. Tell the child to continue with the exercise once it is apparent that the procedure is understood. Say that you will show the child how to check the answers after the table is completed, then leave and observe unobtrusively.
10. After the table is completed, have the child get the addition summary table and demonstrate how to find the answers on it by placing the completed table to the left of the corresponding column on the summary table.
12. Show the child how to use the date stamp and say that the date is to be stamped on all work.
13. The child returns all materials to the shelf and places the completed table in the file or container for individual work, depending upon the method used to store each child's work.

Control of Error:

Printed addition tables for copying
A separate square on the paper for each numeral and each symbol
The red answer square
Colors of bead bars
Summary tables for answers

Observations:

Drawing of red squares
Copying of problems
Arrangement and counting of beads
Use of summary table
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Teach the terms **addend** and **sum** by the three period lesson, indicating each on a written addition problem.
Provide random addition problems, not in table form, containing various combinations.
Introduce addition problems using ten through nineteen. Quantities past ten are formed by placing ten bars horizontally with beads from the short bead stair to the right of the ten bar. Wider paper will be needed for these problems.

Vocabulary:

addition table addition summary table plus equals addend sum
recording paper date stamp ruler random

Snake Game

Purposes:

To provide further reinforcement for learning addition combinations to ten
To develop counting skills
To develop appropriate vocabulary

Preliminary Exercises:

Work with short bead stair addition
Exercise with golden beads and numeral cards combined

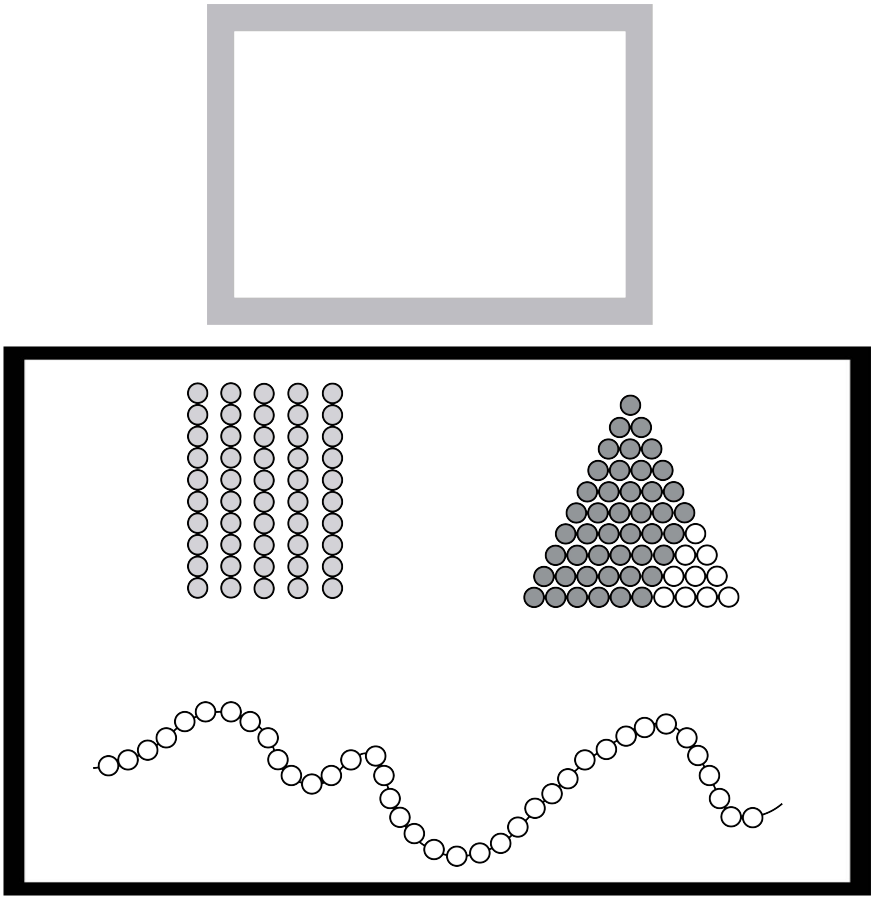
Materials:

Container with one set of short bead stair plus a five bead bar, five ten bead bars and one set of black and white bead bars
Light-colored felt mat
Place holder

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take the materials to a table and have the child sit on your dominant side.
3. Place the felt mat in front of the child with the container of beads behind it.
4. Ask the child to make an equilateral triangle at the upper right of the mat with the black and white bead bars, starting with the one and placing the others in sequence below it. The bead bars are placed so that the white beads are on the right.
5. Have the child arrange the ten bars vertically at the upper left of the mat.
6. Show the child how to place the short bead stair bars on the mat end to end in random quantities in a "snake" arrangement.
7. Beginning at the left end of the "snake", have the child count beads in sequence to ten with the first finger of the right hand, then to hold the tenth bead with the thumb and first finger of the left hand. Using the right hand, the child counts the beads remaining on the bar being held by the left hand, then places the appropriate black or black and white bead bar above those beads and a ten bar to its left. Tell the child to remove all of the short bead stair bars just counted and place horizontally at the top of the mat between the black and white beads and the ten bars.
8. Have the child begin counting again with the right hand, starting with the black or black and white bead bar which is at the right of the ten bar. When ten is reached, the child holds the tenth bead with the left hand as before, continues to count the beads remaining on the short bead stair bar, then places the appropriate black or black and white bead bar above those beads and a ten bar to its left. Tell the child to remove all of the beads below this ten bar. which would be the short bead stair bars and the black and white bead bar to the left. These are returned to the top of the mat with the black and white bead bar put into the proper order.
9. Have the child continue to count and replace the short bead stair bars with tens so that there will be five ten bars forming the "snake" originally made with the short bead stair bars.

10. Tell the child to place the ten bars horizontally, one below the other, on the left of the mat with space between for the short bead stair bars. Ask the child to find combinations of short bead stair bars which make ten and place them below the ten bars. Because of the controlled amount of beads, it will come out even.
11. Stand, replace chair and thank the child. Say that the "snake" game may be chosen. Have the child return the materials to their container and replace them on the shelf.



Control of Error:

Number of short bead stair bars whose combinations will come out even
Exact number of ten bars to match the short bead stair combinations

Observations:

Placement of bead bars
Counting of beads
Substitution of ten bars and black and white bead bars Return
of black and white bead bars to their triangle formation
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use a random number of shor bead stair bars which are provided in another container with a set of black and white bead bars. It will be necessary for the child to obtain a box of ten bars from the golden bead bank on the shelf for use in this exercise. If the number of short bead stair bars does not come out even to make combinations of ten, the appropriate black and white bead bar is placed above remaining short bead stair bar.

Note: In some cases, it may be necessary to exchange short bead stair bars for equivalent amounts to be able to obtain suitable combinations to form ten. For example, if there is a five bar which is too great an amount to combine with the other bead bars in the random quantity set, it must be exchanged for a one and a four or three and two so that the proper combinations to make ten are possible.

Vocabulary:

"snake" game black and white bead bars combinations come out even substitute

Addition Strip Board

Purposes:

To further reinforce the learning of addition tables
To provide a systematic way in which to analyze addition tables
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in addition without changing using golden beads
Addition with small red and blue rods
Addition with short bead stair
Experience with recording addition problems
Work with Seguin boards for learning eleven through nineteen

Materials:

Addition strip board with nine to twelve rows of squares, depending upon the manufacturer, each row having eighteen squares divided by a vertical red line after the tenth square, with numerals one through ten in red, eleven through eighteen in blue, above the top row of squares

Note: The colors of the numerals and strips may be reversed by some suppliers.

Container with nine blue strips graduated in size with red numerals one through nine written at the far right and nine red strips graduated in size and divided into squares like those on the board with blue numerals one through nine written in the last square on the right

Note: The one strip is exactly the size of one square on the board, the two strip is exactly the size of two squares, etc.

Red container of addition tables for combinations of one through nine, squared paper for recording problems and answers, pencil

Red container of addition problem tickets, each of which has a separate addition problem for combinations of one through nine

Addition summary tables

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done and take the materials to a table. Ask the child to choose any addition table. Place the addition board in front of the child with addition table, paper and pencil on the child's dominant side.
2. Ask the child to arrange the blue strips beside the addition board, starting at the top with one and having the left sides of the strips even. The red strips are arranged in front of the blue strips in the same manner. The strips are arranged on the child's non-dominant side.
3. After drawing red answer boxes, have the child copy the first equation on the paper. The teacher places the strips on the first row, restating each addend as it is placed, then points to the numeral above it. For example, for the three's table, 3 + 1 = is the first equation. The blue strip labeled three is placed on the first row of the board and the red one strip is placed to its right on the same row.

4. Tell the child to look at the numeral above the one strip and to record that numeral on the paper in the answer box which, in the example given above, is four. Have the child restate the problem and the answer and record the answer.
5. The child replaces the red strip, writes the next equation and places the appropriate red strip next to the blue strip which remains constant for its table. Following the example given above, the red two strip would be placed at right of the blue. The answer is read above the numeral at the end of the red strip, in the example given above, five, and recorded.
6. As soon as it is observed that the procedure is understood, tell the child to check the recorded answers against those on the addition summary table after all of the table is completed.
7. Thank the child and say that any addition table may be chosen. Leave and observe unobtrusively. The child returns the materials to the shelf when finished. Child's name and date are put on completed work which is filed by the child in the proper storage place.

Note: It may be necessary to demonstrate how to return the red and blue strips to the container if it is the type requiring that the blue nine and red one, blue eight and red two, blue seven and red three, etc., be placed exactly for all to fit.

Control of Error:

Numbered strips and numerals above the squares on the board
Addition summary tables

Observations:

Arrangement of strips beside the board
Placement of strips on the board
Recording of answers
Use of addition summary tables
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to determine all of the combinations in order which form ten by arranging the strips on the addition strip board.

Call attention to the fact that $9 + 1 = 10$ and $1 + 9 = 10$, then remove $9 + 1$. Continue with the other combinations so that only $1 + 9$, $2 + 8$, $3 + 7$, $4 + 6$ and $5 + 5$ remain. This work may be checked against the second addition summary table.

Invitechild to use random addition problem tickets for additional exercises with the addition strip board.

Vocabulary:

addition strip board

Short Bead Stair Multiplication

Purposes:

To develop an understanding of the process of multiplication
To give practice in multiplication
To learn how to record multiplication problems
To develop appropriate vocabulary

Preliminary Exercises:

Introductory exercise with the short bead stair
Work with Seguin boards to learn 11 through 99
Exercises in addition with recording using the short bead stair

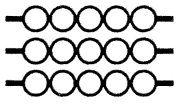
Materials:

Ten short bead stair bars of each quantity, each in a separate yellow container, for example, ten one bead bars in a yellow container labeled one, ten two bead bars in a yellow container labeled two, etc.
Small felt mat
Multiplication tables for multiplication combinations ones through tens
Paper for recording with ten rows of five squares each, the squares measuring 3/4"; paper with ten rows of nine squares each for longer addends
Red pencil and ruler
Pencil
Multiplication summary tables
A small card on which is written the multiplication sign
Date stamp

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take the card with the multiplication sign to a table and tell the child that this is the times sign, used to signify multiplication.
3. Choose any multiplication table and the corresponding container of bead bars. Have the child help take all of the materials to a table. Place the box of beads toward the center of the table before the child with the felt mat in front of it. The recording paper, pencils and ruler are on the child's dominant side, next to the felt mat. The chosen multiplication table is placed beside the recording paper.
4. Have the child draw red answer squares on the recording paper as before, then move the red pencil and ruler out of the way on the dominant side once the red squares have been drawn. The child's name and date are placed on all papers.
5. Ask the child to open the container of beads and to place the lid beneath the container, if there is a lid.
6. Have the child copy the first problem from the multiplication table on the recording paper, placing a numeral or sign in each square except the red one.
7. Ask the child to read the problem and to place one of the bead bars from the container near the top of the mat. Have the child count the beads and write the answer in the red square. Tell the child to restate the problem with the answer.
8. Tell the child to copy and read the next problem in the table, then to remove another bead bar from the container. Place the second bead bar under the first. Ask the child to count the bead bars and record the answer as before.

9. Tell the child to continue with the exercise once it is apparent that the procedure is understood. Say that you will show the child how to check the answers after the table is completed, then leave and observe unobtrusively.
10. After the table is completed, demonstrate how to find the answers on the multiplication summary table by placing the completed table to the left of the corresponding column on the summary table.
11. The child returns all materials to the shelf and places the completed table in the file or container for individual work, depending upon the method used to store each child's work.



(5 x 3 = 15)

5 x 1 =
5 x 2 =
5 x 3 =
5 x 4 =
5 x 5 =
5 x 6 =
5 x 7 =
5 x 8 =
5 x 9 =
5 x10 =

Control of Error:

Prepared multiplication tables for copying
A separate square on the paper for each numeral and each symbol
The red answer square
Colors of bead bars
Summary tables for answers

Observations:

Drawing of red squares
Copying of problems
Arrangement and counting of beads
Use of summary table
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Teach the terms multiplicand, multiplier and product by the three period lesson, using a written problem on which to indicate each.
Provide multiplication problems, not in table form, containing random combinations.
Introduce multiplication problems using ten through nineteen if enough short bead stairs are available. Wider paper will be needed.

Vocabulary:

multiplication table multiplication summary table times multiplicand
multiplier product

Multiplication Board

Purposes:

To further reinforce the learning of multiplication tables
To provide a systematic way in which to analyze multiplication tables
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in multiplication without changing using golden beads
Multiplication with short bead stair
Experience with recording multiplication problems
Work with Seguin boards for learning eleven through nineteen and twenty through ninety-nine

Materials:

Multiplication board with ten rows of ten cavities per row and numerals one through ten above the top row of cavities
Container of one hundred green beads to fit the cavities and a green counter Container of small numeral cards with one through nine in green, ten in blue Multiplication tables one through ten, paper, pencil
Multiplication summary tables
Note: The colors of the numerals and beads may be provided in red by some suppliers, but the color scheme should conform to the conventional green for units, blue for tens.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done and take the materials to a table. Ask the child to choose any multiplication table. Place the multiplication board in front of the child with multiplication table, paper and pencil on the child's dominant side. The container of beads is placed on the left side of the board. The numeral cards are placed above the board.
2. After drawing the red answer boxes, have the child copy the first equation on the paper and place the numeral card to the top left of the board which indicates the table. For example, if it is the six table, the numeral six is placed on the board.
3. Tell the child to place the green counter at the top of the first column above the numeral one and count six beads into the cavities in a column under it.
4. Have the child count the beads and record the answer, then restate the problem and answer.
5. Ask the child to write the next equation, move the counter above the two and continue counting beads into the cavities in that column until it is as long as the first column. Have the child record the amount counted.
6. As soon as it is observed that the procedure is understood, tell the child to check the answers on the paper against those on the multiplication summary table after all of the equations are completed.
7. Thank the child and say that any multiplication table may be chosen. Leave and observe unobtrusively. The child's name and date are put on the completed work and the child returns the materials to the shelf when finished.

Control of Error:

Numeral to indicate table being worked and numerals at the top of the board
Multiplication summary tables

Observations:

Placement of beads, counter and numeral card on the board
Counting of beads
Recording of answers
Use of multiplication summary tables
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use printed tables on which the answers are to be written.
Invite the child to use printed problems in random order on which answers are to be written.

Vocabulary:

multiplication board cavities

Counting Short Bead Chains For Squares of Numbers One through Ten

Purposes:

To prepare for formation of squares of numbers
To illustrate linear and square configuration of the same quantity as an aid to conservation skills
To give practice in counting
To reinforce multiplication skills
To develop appropriate vocabulary

Preliminary Exercises:

Work with both Seguin boards
Multiplication with short bead stairs

Materials:

Short bead chains representing the squares of one through nine, color coded as for the short bead stair, and fixed squares of beads for each chain
Chain representing the square of ten, composed of ten golden bead ten bars, and fixed square of ten golden bead ten bars
Rack for hanging chains in a vertical position with shelf at top to hold squares and arrows Note: The one bead bar is placed at the far right position on the rack and the ten chain is at the far left. The arrangement is incorrectly pictured in many books and catalogs. Container for each quantity of arrows colored the same as the bead chains, each arrow having the numeral to indicate the number of beads counted cumulatively with the arrow for the square slightly wider
Note: Some suppliers place the numerals on the arrow vertically while others place them horizontally.
A strip of felt, ribbon or fabric-backed vinyl on which to place each separate bead chain, rolled for storage and marked with the numeral to indicate the bead chain to be placed on it
Note: The strip must be long enough to accommodate its chain and square, wide enough for placement of the square at the end of the chain. Small floor mat on which to place the container of arrows

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Have the child decide which chain to count, then get the appropriate strip which is unrolled on the floor. Show the child how to carry the chain with one hand at the top and one near the bottom of a long chain so that it does not swing, then how to place it to one side of the strip.
3. Ask the child to bring the small floor mat and the appropriate container of arrows which is placed on the mat. If the numerals are on the arrows in a vertical position, the mat and container are placed at the bottom left under the strip and the child is positioned at the bottom. If the numerals are in a horizontal position, the mat and container are placed at the top left of the strip and the child is positioned at the left side of the strip.

4. Tell the child to begin counting the chain from the left, if arrows are vertical, or from the top if horizontal. When the connection between the first two bars is reached, have the child find the arrow with that numeral and place it with the point touching the appropriate bead.
5. Ask the child to continue counting and placing arrows and say that any of the short bead chains for squaring may be counted. Leave and observe unobtrusively.
   Note: On longer chains, the child may move the small mat and container of arrows to be accessible rather than having to go back to the end of the strip each time.

Control of Error:

Matching color of chains and arrows
Numerals on arrows limited to appropriate ones for each particular chain
Length of strip on which bead chain is placed

Observations:

Choice of strip and arrows for chain to be counted
Handling of material
Placement of chain on strip
Placement of arrows on strip
Counting of beads
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Show the child how to fold the chains to form a square which corresponds to the fixed square. After this demonstration, the child may place the appropriate square at the end of the chain once counting is completed.

Invite the child to make a small booklet in which to record the numerals from the arrows The cover of the booklet is the color of the bead chain and arrows from which the numerals are recorded.

Vocabulary:

square short bead chain arrow linear configuration

Counting Bead Chains For Cubes of Numbers One through Ten

Purposes:

To prepare for formation of cubes of numbers
To illustrate cubic and square configuration of the same quantity as an aid to conservation skills
To give practice in counting
To reinforce multiplication skills
To develop appropriate vocabulary

Preliminary Exercises:

Work with counting short bead chains

Materials:

Bead chains representing the cubes of one through ten, color coded as for the short bead stair
Cabinet for hanging chains in a vertical position with shelf at top to hold cubes and arrows and shelves under each set of chains to hold squares
Fixed squares of beads for each chain with which to form the cube
Fixed cube for each chain
Container for each quantity of arrows colored the same as the bead chains, each arrow having the numeral to indicate the number of beads counted cumulatively
Note: Some suppliers place the numerals on the arrow vertically while others place them horizontally.
A strip of felt, ribbon or fabric-backed vinyl on which to place each separate bead chain, rolled for storage and marked to indicate the bead chain to be placed on it
Note: The strip must be long enough to accommodate its chain and cube and wide enough for placement of squares beside the chain.
Small floor mat on which to place the container of arrows

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Have the child decide which chain to count, then get the appropriate strip which is unrolled on the floor.
3. Have the child stand in front of the bead cabinet with the palm of the non-dominant hand upward. Ask the child to grasp the chosen chain's ring on the right hook to remove the first section of chain and place the chain over the child's thumb. Continue removing rings from the hooks and placing chain over child's fingers.
4. Ask the child to carry the chain to the strip and carefully lower the end of the chain to the end of the strip. Show the child how to remove the first ring with the dominant hand and carefully lay the next sections of the chain on the strip. The child continues to lift the rings and place the chain until it is laid on the strip in its entirety.
5. Ask the child to bring the small floor mat and the appropriate container of arrows which is placed on the mat. It the arrows are used in a vertical position, the mat and container are placed at the bottom left under the strip and the child is positioned at the bottom. If horizontal, the mat and container are placed at the top left of the strip and the child is positioned at the left side of the strip.

6. Tell the child to begin counting the chain from the left, if numerals on arrows are vertical, or from the top if numerals are horizontal.(See diagram below.) When the connection between the first two bars is reached, have the child find the arrow with that numeral and place it with the point touching the appropriate bead.
7. Ask the child to continue counting and placing arrows until the larger arrow is reached. The square representing that which has just been counted is placed above or to the left of that section of chain.
8. Tell the child to continue counting and placing arrows and bead squares to the end of the chain.

Note: On longer chains, the child may move the mat and container of arrows to be accessible rather than having to go back to the end of the strip each time.

9. After all the numeral arrows have been placed, have the child return the arrows to the container, then replace the small mat and container of arrows to the cabinet.
10. Before removing the chain from the strip, remind the child of the chain's number Have the child count that number of bead sections to find the first connecting ring. Ask the child to grasp this ring with the dominant hand and to place it over the child's first finger. Count the number of sections to the next connecting ring and indicate that this ring hangs down. Count to the next ring and have the child place it over the second finger. Continue in this manner until the entire chain is hanging in sections from the child's fingers.
11. Have the child carry the chain to the cabinet, then grasp the first ring and hang it on the first hook. Ask the child to continue placing the rings on the hooks of the cabinet until the entire chain is replaced in the cabinet.

Control of Error:

Matching color of chains and arrows
Numerals on arrows limited to appropriate ones for each particular chain Length of strip on which chain is placed
Large ring connector between chain sections

Observations:

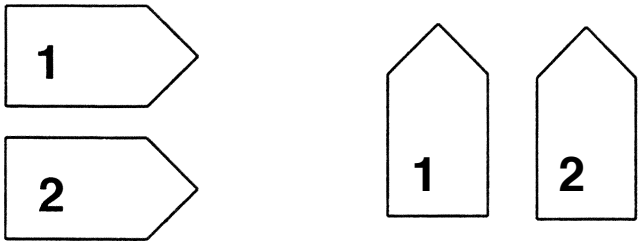
Choice of strip and arrows for chain to be counted Handling of materials
Placement of chain on strip
Placement of squares
Counting of beads
Placement of arrows on strip
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Demonstrate the stacking of squares to form the cube of the chain just counted. After this, the appropriate cube is placed at the end of the chain once counting is completed.

Vocabulary:

cube (as related to other than the golden bead cube)



Division Board

Purposes:

To reinforce the learning of division tables
To provide a systematic way in which to analyze division tables
To develop appropriate vocabulary

Preliminary Exercises:

Group exercises in division without changing using golden beads
Experience with recording answers to problems
Work with Seguin boards for learning twenty through ninety-nine

Materials:

Division board with nine rows of nine cavities, nine spaces for skittles in the green band across the top, with numerals one through nine above the spaces and down the left side
Container of eighty-one green beads or pegs to fit the cavities and nine small green skittles
a small card on which is written the division sign
Division tables with table and answer written on the back, pencil for writing answers, red pencil for underlining answers without remainders
Paper for recording with ten rows of five squares each, the squares measuring 3/4". Note: The divisor can be no greater than nine and the dividend no greater than eighty-one on this board nor can the quotient be greater than nine.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done and take the materials to a table. Ask the child to choose any division table. Place the division board in front of the child with division table, paper and pencil on the child's dominant side. The container of beads or pegs and skittles is placed at the top of the board. Show the child the card with the division sign and say that it is the "divided by" sign used to signify division.
2. Have the child write the first equation on the paper and count the number of beads or pegs in the dividend as indicated. For example, if it is twenty-four, that is the quantity counted into the lid of the container or into a small bowl which is then moved to the left of the board.
3. Tell the child to read the first equation again and place the number of skittles across the top as indicated by the divisor, for example, nine.
4. Have the child give each skittle a bead or peg in turn, starting at the left of each row. Just as in golden bead division, each skittle must get the same number of beads, so those beads which cannot be divided evenly remain in the lid or bowl and are called the remainder. Tell the child to count the number of beads or pegs which one skittle received and record the answer on the paper, in this example, $24 \div 9 = 2$ r.6.
5. Ask the child to replace the beads or pegs into the lid or bowl, to write next equation, and remove the ninth skittle since the next problem has eight as the divisor.
   Note: The number of beads or pegs will remain constant for the table being used.

6. Have the child give each skittle a bead or peg as before and record the number which one skittle received. Tell the child to underline in red each answer not having a remainder.
7. As soon as it is observed that the procedure is understood, tell the child to check the answers on the back of the table after all of the equations are completed.
8. Thank the child and say that any division table may be chosen. Leave and observe unobtrusively. The child returns the materials to the shelf when finished.

Control of Error:

Control with equations and answers

Observations:

Placement of beads or pegs and counter on the board
Counting of beads or pegs
Recording of answers
Use of control
Handling of materials
Child's reaction to error
Length of work time and number of repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use tables on which to write quotients.
Invite the child to solve random problems.

Vocabulary:

division board skittle

Short Bead Stair Subtraction

Purposes:

To develop an understanding of the process of subtraction
To give practice in subtraction
To learn how to record subtraction problems
To develop appropriate vocabulary

Preliminary Exercises:

Introductory exercise with the short bead stair
Subtraction or decomposition with red and blue table rods
Exercises in addition with recording using the short bead stair.

Materials:

One set of short bead stairs and one ten bar in a green container
Small felt mat
Subtraction tables for subtraction combinations one through ten
Paper for recording with ten rows of five squares each, the squares measuring 3/4"
Red pencil and ruler
Pencil
Subtraction summary tables
A small card on which is written the subtraction sign
Date stamp

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Take the card with the subtraction sign to a table and tell the child that this is the minus sign, used to signify subtraction.
3. Choose any subtraction table and the container of bead bars. Have the child help take all of the materials to a table and place the box of beads with the felt mat in front of it. The recording paper, pencils and ruler are on the child's dominant side, next to the felt mat. The chosen subtraction table is placed beside the recording paper.
4. Have the child draw red answer squares on the recording paper as before, then move the red pencil and ruler out of the way on the dominant side once the red squares have been drawn.
   Note: Each subtraction table will be a different length. The child may choose to cut the paper where the table ends, or to skip a line and continue on the same paper with another table that will fit the space. This will require that the child count the lines needed for another table.
5. Ask the child to open the container of beads and to place the lid beneath the container, if there is a lid.
6. Have the child copy the first problem from the subtraction table on the recording paper, placing a numeral or sign in each square except the red one.

7. Ask the child to read the problem and to place the bead bar representing the table near the bottom of the mat. For example, if it is the six table, take the six bar from the container. Starting at the right end of the bead bar and using the right hand, have the child count the beads to be subtracted, covering them with the fingers. The remaining exposed beads on the bar are counted with the index finger of the other hand and the number is written in the red square. Tell the child to restate the problem with the answer.
8. Have the child copy and read the next problem in the table, then count and cover as before the beads to be subtracted. Ask the child to count the exposed beads and record the answer as before.
9. Tell the child to continue with the exercise once it is apparent that the procedure is understood. Say that you will show the child how to check the answers after the table is completed, then leave and observe unobtrusively.
10. After the table is completed, demonstrate how to check the answers by placing the completed table to the left of the corresponding column on the subtraction summary table.
11. Tell the child to use the date stamp. The child's name is written on all papers.
12. The child returns all materials to the shelf and places the completed table in the file or container for individual work, depending upon the method used to store each child's work.

Control of Error:

The fixed number of beads on the bar representing the minuend
Printed subtraction tables for copying
A separate box on the paper for each numeral and each symbol
The red answer square
Subtraction summary tables

Observations:

Drawing of red squares
Copying of problems
Counting of beads

Use of summar table

Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Teach the terms minuend, subtrahend and difference by the three period lesson.
Provide subtraction problems not in table form, containing various combinations.
Use bead stairs to teach the child how to do subtraction with minuends eleven to eighteen by combining a ten bar with another bead bar.

Vocabulary:

subtraction table subtraction summary table minuend
subtrahend difference

Subtraction Strip Board

Purposes:

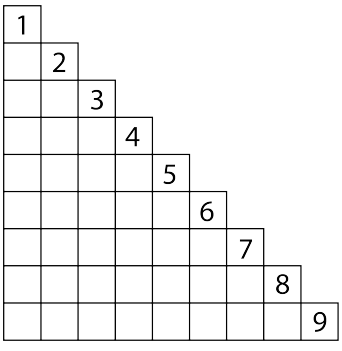
To further reinforce the learning of subtraction tables
To provide a systematic way in which to analyze subtraction tables To develop appropriate vocabulary

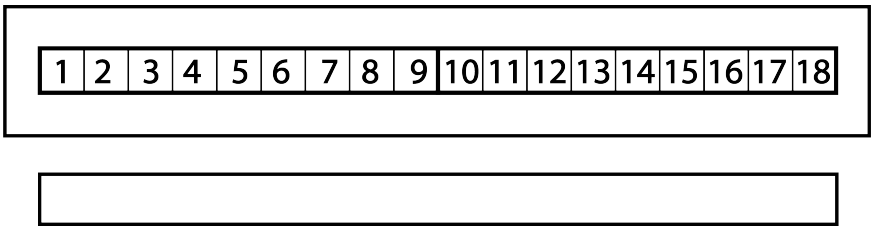
Preliminary Exercises:

Group exercises on subtraction without changing using golden beads
Subtraction with small red and blue rods
Subtraction with short bead stair
Experience with recording subtraction problems
Work with Seguin boards for learning eleven through nineteen

Materials:

Subtraction strip board with numerals one through nine in blue, ten through eighteen in red, each numeral in a square and a vertical blue line after the nine.
Note: The colors of the numerals and strips may be reversed by some suppliers.
Nine blue strips divided into squares the same dimensions as the squares on the strip board, graduated in size with red numerals one through nine written at the far right
Neutral strip the length of seventeen squares with no numerals or divisions
Note: The one strip is exactly the size of one square on the board, the two strip is exactly the size of two squares, etc.
Subtraction tables with minuends up to eighteen and subtrahends one through nine, squared paper for recording problems and answers, pencil
Subtraction problem tickets with minuends up to eighteen and subtrahends one through nine
Subtraction summary tables





Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done and take the materials to a table. Ask the child to choose any subtraction table problem paper. Place the subtraction strip board in front of the child with subtraction paper and pencil on the child's dominant side.
2. Ask the child to arrange the blue strips on the non-dominant side of the subtraction board, starting at the top with one and having the left sides of the strips even. The neutral strip is placed below the board.
3. Have the child copy the first equation on the paper and place the neutral strip on the board to cover all numerals on the right except the minuend. For example, if it is 15 - 6, the child covers 18, 17 and 16 with the neutral strip, then places the six strip to its left touching the neutral strip.
4. Tell the child to look at the numeral at the left of the six strip and to record that answer on the paper in the answer square, in the example given above, 9. Tell the child to restate the problem and the answer.
5. The child replaces the blue strip, copies the next equation and places the appropriate blue strip next to the neutral strip which remains constant for its table.
6. As soon as it is observed that the procedure is understood, remind the child to check the answers on the paper against those on the subtraction summary table once the exercise is completed.
7. Thank the child and say that any subtraction table may be chosen. Leave and observe unobtrusively. The child returns the materials to the shelf when finished.

Control of Error:

Numbered strips and numerals on the board Subtraction tables
Subtraction summary tables

Observations:

Arrangement of strips beside the board
Placement of strips on the board
Recording of answers
Use of subtraction summary tables
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use subtraction problem tickets for additional exercises with the subtraction strip board.

Note: Remind the child to check all work against the subtraction summary tables.

Vocabulary:

subtraction strip board neutral

Multiples on the Pegboard

Purposes:

To reinforce the learning of multiplication tables
To provide a means for learning skip counting
To prepare for work with fractions
To develop appropriate vocabulary

Preliminary Exercises:

Multiplication with short bead stair
Counting bead chains

Materials:

Large pegboard approximately thirty holes by thirty holes for a total of nine hundred
Two boxes of pegs, each box containing a different color of pegs
Small yellow-covered booklet labeled "Multiples" in which to record multiples, or recording paper
Pencil

Procedure:

1. Invite a child to the lesson once preliminary exercises have been done. Take the materials to a table, place them in front of child who is seated on your dominant side.
2. Remove the lids from the boxes and place open side up at the left of the pegboard, one above the other. The boxes of pegs are at the left of their open lids.
3. Tell the child to choose any number between two and ten, for example, four, and count that number from each box into its lid.
4. Starting at the top left corner, have the child place pegs from the top box lid vertically in the pegboard. Tell the child to record the number of pegs, in this case, 4.
5. Have the child place the pegs from the second box lid in the same column, count the total number and record. In this case, there would be 8.
6. Four more pegs from each box are counted into their lids as before. Pegs in the color first used are placed in the same column and the total number of pegs in the column recorded. Pegs in the second color are added to the same column and the total number of pegs recorded. Have the child continue to the board's lower edge.
8. Leave and observe unobtrusively.

Control of Error:

Change in color between each multiple

Observations:

Counting of pegs
Placement of pegs in lids and on board
Recording of multiples
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation and degree of interest and concentration

Variations:

Invite the child to use two other colors of pegs.

Vocabulary:

multiple peg pegboard

Least Common Multiple on the Pegboard

Purposes:

To determine least common multiples of two numbers
To prepare for work with fractions
To develop appropriate vocabulary

Preliminary Exercises:

Practice in finding multiples of numbers

Materials:

Large pegboard with approximately nine hundred holes
Four boxes of pegs, each box containing a different color of pegs
Small booklet with yellow cover, labeled "Least Common Multiples" in which to record work, or recording paper labeled Least Common Multiples
Pencil

Procedure:

1. Invite a child to the lesson once the prelfminary exercise has been done. Have the child take the materials to a table. Sit with the child on your dominant side.
2. Remove the lids from the boxes and place open side up at the upper left of the pegboard. The boxes of pegs are at the left of the open lids with boxes and lids one below the other.
3. Tell the child to choose any two numbers between two and ten, for example, three and four, and count three pegs from two different colors into their box lids, then count four from two other colors into their box lids.
4. Have the child place the pegs from the first quantity at the top left of the board in a column, then place pegs from the other quantity in a column to the right of the first. In this example, there would be three pegs in the first column and four in the second.
5. Tell the child to start placing pegs in the shorter column, using the second color. In this example, the three column would receive three more pegs of a different color. Have the child add pegs to the other column with a fourth color.
6. Ask the child to count pegs of the first two colors into the lids. In this example three of the first color and four of four's first color color.
7. Again the three's column has pegs added, then the four's column. Going back to the three's column, three more pegs are added. Since the two columns are now equal in length, no more are added to the four's column.
8. Tell the child to count the pegs in each column. That number is the least common multiple for the two numbers chosen. In this example, twelve is the least common multiple of three and four. The child records on the paper or in the booklet, "The least common multiple of 3 and 4 is 12." Have the child remove the pegs from the pegboard.
9. Thank the child and say that any two numbers may be chosen for finding least common multiples.

Control of Error:

Color of pegs
Visual awareness of equal length of columns once the common multiple is reached

Observations:

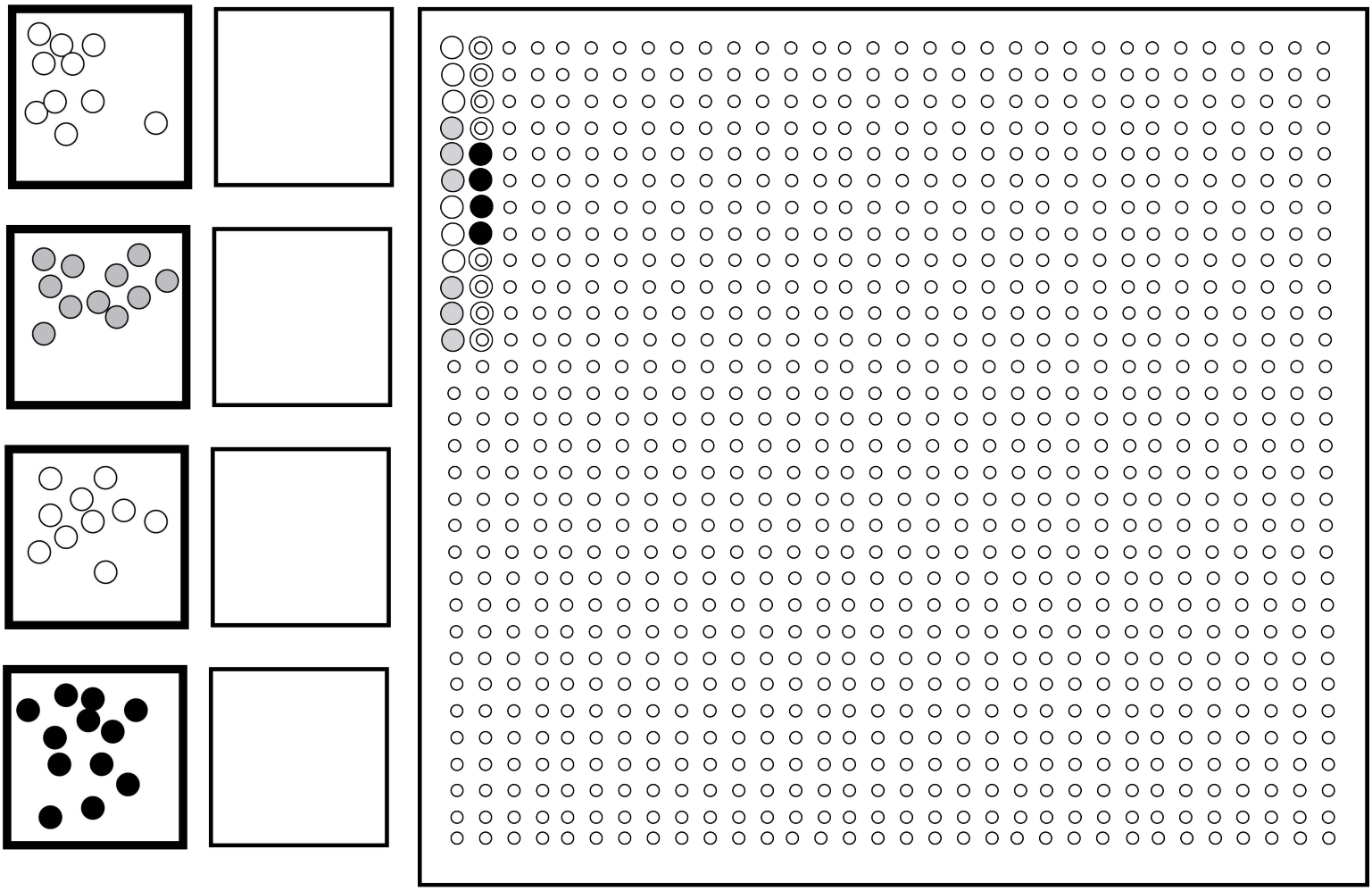
Counting of pegs
Placement of pegs in lids and on board
Changing of colors for the quantities chosen
Recording of common multiples
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Use four other colors of pegs.

Vocabulary:

least common multiple



Finding Multiples on "Multiples of Numbers" Paper

Purposes:

To determine common multiples of numbers through one hundred
To prepare for work with fractions
To develop appropriate vocabulary

Preliminary Exercises:

Work with materials for finding multiples and least common multiples on the pegboard
Practice in writing so that circles can be drawn

Materials:

Paper with numerals 1 to 100, arranged in rows of tens
Container of colored pencils in as many colors as possible

Procedure for Multiples:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the paper on a table in front of the child with the container of pencils on the child's dominant side.
2. Ask the child to choose any number between one and ten, for example, three, then to choose any color pencil. The chosen numeral is written at the top left of the paper in the chosen color.
3. Tell the child to count the numerals through three, then to make a circle around the 3 with the chosen color. Have the child count three more numerals and draw a circle in the same color. In this example, 6 would be circled. Tell the child to continue counting three more numerals and drawing circles as far as possible on the paper. In this example, 99 would be the last numeral circled.
4. Tell the child that other Multiples of Numbers paper may be used to find multiples of any number 2 through 10.
5. As soon as it is observed that the procedure is understood, leave and observe unobtrusively.

Procedure for Common Multiples

1. Invite a child to the lesson once the preceding exercise has been understood. Place the paper on a table in front of the child with the container of pencils on the child's dominant side.
2. Ask the child to choose any number between one and ten, for example, three, then to choose any color pencil. The chosen numeral is written at the top left of the paper in the chosen color.
3. Tell the child to count the numerals through three, then to make a circle around the 3 with the chosen color. Have the child count three more numerals and draw a circle in the same color. In this example, 6 would be circled. Tell the child to continue counting three more numerals and drawing circles as far as possible on the paper. In this example, 99 would be the last numeral circled.
4. Ask the child to choose another number and a different color, then to count and draw circles around numerals as before. The child will find that some of the numerals will have two circles. Explain that these are common multiples.

5. Tell the child that other Multiples of Numbers paper may be used to find the common multiples of any two numbers.
6. As soon as it is observed that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Different color to circle each numeral
Child's ability to count
Pattern made by circles drawn in different colors around numerals

Observations:

Choice of numerals and colors
Counting of numerals
Drawing of circles
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to find multiples of other pairs of numerals.
Invite the child to find multiples of more than two numbers on a single sheet of **Multiples of Numbers** paper.
Invite the child to find multiples of as many numbers as possible on the same **Multiples of Numbers** paper by carefully drawing the circles concentrically, each circle a different color.
Using a ruler, draw lines only through the numerals which have circles.
Invite the child to record, on a separate sheet of **Multiples of Numbers** paper, those numbers which have only one or no circles. This sheet is labeled **Prime Numbers.**

Vocabulary:

concentrically

Factors on the Pegboard

Purposes:

To develop an understanding of the process of factoring
To find factors of numbers up to thirty
To prepare for work with fractions
To develop appropriate vocabulary

Preliminary Exercises:

Counting the chains
Work with the division board

Materials:

Large pegboard with nine hundred holes
Eight boxes of pegs, each box containing a different color of pegs
Container with one set of one inch numeral cards 1 to 28.
Booklets with blue covers or cards with numbers to be factored, 18 and under for introductory exercises
Small blue-covered booklet labeled "The Factors of ------" in which to record work, or recording paper, pencil

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child take the materials to a table. Sit with the child on your dominant side.
2. Remove the lids from the boxes and place open sides up at the left of the pegboard, in a column. The boxes of pegs are at the left of their open lids.
3. Tell the child to choose a problem from the booklet or card, for example, twelve, then count twelve pegs of one color into a box lid.
4. Have the child place the pegs from the box lid in a row of ones , starting at the top left of the board. One is a factor so place the numeral 1 beside the last peg. Using a second color of pegs, have the child count twelve into the lid and place pegs in columns of twos under the first row. In the example given here, there will be two rows of six pegs in the same color, so two is a factor of twelve. Therefore, place the numeral 2 beside the last peg.
5. Using the third color of pegs, the child counts twelve into the lid, then places pegs in threes under the row of twos. In this example, there will be three rows of four pegs, so three is a factor of twelve. The numeral 3 is placed beside the last peg.
6. Using a fourth color of pegs, the child counts twelve into the lid, then places pegs in fours under the other rows. In this example, there will be four rows of three pegs. The numeral 4 is placed beside the last peg.
7. Have the child continue placing pegs in successive quantities up to the number chosen, in this example, up to twelve. Leave and observe unobtrusively.
8. Once all of the pegs have been arranged, have the child write the numeral for the number factored, then write the factors. In the example given here, the label on booklet would read, "The Factors of 12." Inside the booklet, the numerals 1, 2, 3, 4, 6, 12 would be written.

9. Tell the child that any booklet or card may be chosen to factor numbers up to eighteen.

Control of Error:

Different color of pegs for each factor
Counting the number of pegs into the lids b_efore factoring

Observations:

Use of different colors of pegs for each factor
Counting of pegs into lids
Placement of pegs on board
Recording of factors
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to factor any number up to the limit of the board, that is, the number of holes horizontally or vertically.

Vocabulary:

factor

Greatest Common Factors on the Pegboard

Purposes:

To develop an understanding of highest common factors
To prepare for work with fractions
To develop appropriate vocabulary

Preliminary Exercises:

Practice with factoring numbers

Materials:

Large pegboard with nine hundred holes
Ten boxes of pegs, each box containing a different color of pegs
Blue booklets or cards with problems which can be accommodated on the pegboard
Note: Do not use prime numbers (1, 3, 5, 7, 11, 13, 17, 19, 23)
Container with two sets of 1" numeral cards, each set having the numerals 1 through 28
Small blue booklets labeled "Greatest Common Factors" in which to record work Pencil

Procedure:

1. Invite a child to the lesson once the preliminary exercise has been done. Have the child take the materials to a table. Sit with the child on your dominant side.
2. Place five boxes of pegs at the left of the pegboard in a column and five boxes in a column on right side of the pegboard.
3. Tell the child to choose a problem, for example, eight and twelve.
4. Have the child factor one number on the left side of the board and the other number on the right side, beginning at the top of the pegboard.
5. Tell the child to place the appropriate small numeral card on the left beside the last peg of the each factor. The same procedure is repeated for the number on the right.
6. Have the child look for the highest numeral common to both of the factored numbers. In this example, 1, 2, and 4 are common factors. The numeral squares for the common factors are removed except for the greatest common factor, in this example, 4. In the booklet, the child writes on one page, "The greatest common factor of 8 and 12 is 4."
7. Tell the child that any of the problems in the booklets or on cards may be chosen to find the greatest common factor.

Control of Error:

Numeral squares
Different colors of pegs
Problem booklets or cards to suit the size of the pegboard

Observations:

Counting of pegs into lids
Placement of pegs on board
Different color for each factor Recording
of factors on squares of paper

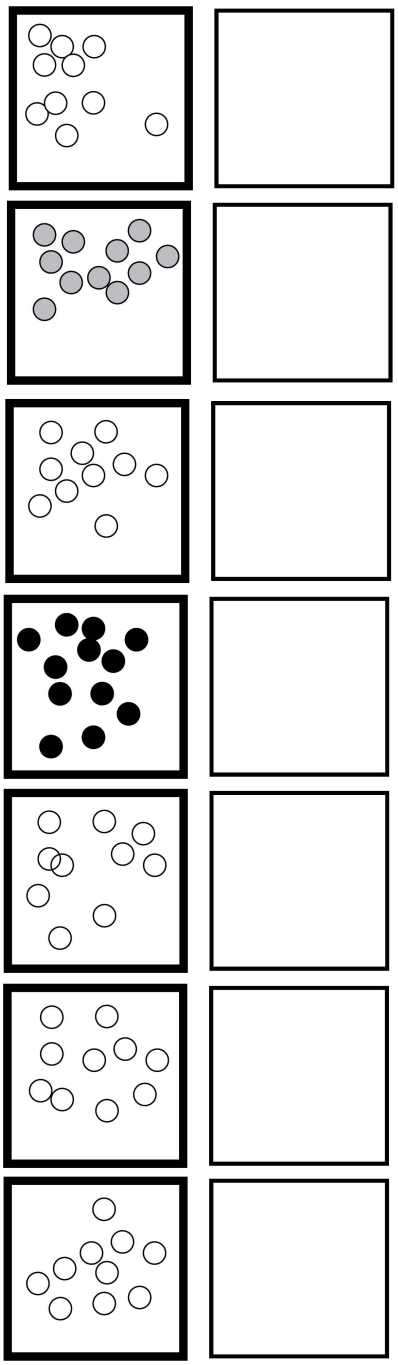
Recording of greatest common factor Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

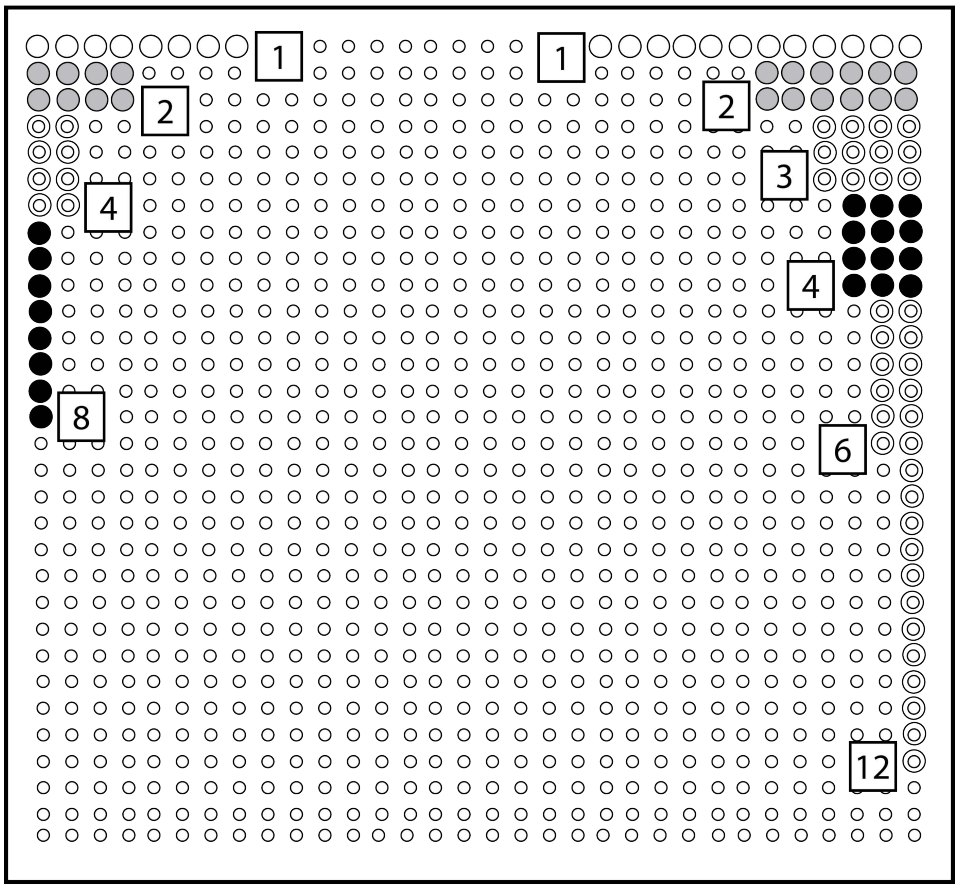
Variations:

None

Vocabulary:

greatest common factor





Charts

Purposes:

To provide further practice in memorizing number facts
To call attention to number patterns
To develop appropriate vocabulary

Preliminary Exercises:

Practice with addition, subtraction, multiplication and division boards

Materials:

Paper addition chart and control chart
Paper multiplication chart (Table of Pythagorus) and control chart
Paper subtraction chart and control chart
Paper division chart and control chart
Containers of problem tickets for each operation with addition on red, multiplication on yellow, division on blue and subtraction on green
Pencil

Addition Chart Procedure:

1. Invite the child to the lesson. Place the addition control chart in front of the child with the paper chart on the child's dominant side. Place the container of addition problem tickets above the control chart with the lid beside it, if there is one.
2. Ask the child to choose a problem ticket at random from the container, placing the ticket beside the container, for example, 3 + 5=.
3. On the control chart, show the child how to locate 3 in the left column with the index finger of the left hand and 5 in the top row with the index finger of the right hand.
4. Have the child move the left index finger across the row to the right, then move the right index finger down the column until they meet at the answer, in this example, 8.
5. Show the child how to find the answer space on the paper addition chart by repeating the previous directions.
6. Have the child record the answer in the appropriate answer space.
7. Have the child put the problem ticket just completed in the box lid or upside down behind the container before choosing another problem ticket.
8. The child continues selecting addition problem tickets, finding the answer on the control and recording it on the paper chart.
9. As soon as it is observed that the child understands the procedure, leave and observe unobtrusively.

Multiplication Chart Procedure:

I. Invite the child to the lesson. Place the multiplication control chart in front of the child with the paper chart on the child's dominant side. Place container of multiplication problem tickets above the control chart with the lid beside it, if there is one.
2. Ask the child to choose a problem ticket at random from the container, placing the ticket beside the container, for example, 2 x 4 =.
3. On the control chart, show the child how to locate 2 in the left column with the index finger of the left hand and 4 in the top row with the index finger of the right hand.
4. Have the child *move* the left index finger across to the right, then *move* the right index finger down the column until they meet at the answer, in this example, 8.
5. Show the child how to find the answer space on the paper multiplication chart by repeating the previous directions.
6. Have the child record the answer in the answer space.
7. Have the child put the problem ticket just completed in the lid or upside down behind the container before choosing another problem ticket.
8. The child continues selecting multiplication problem tickets, finding the answer on the control and recording it on the paper chart.
9. As soon as it is observed that the child understands the procedure, leave and observe unobtrusively.

Subtraction Chart Procedure:

1. Invite a child to the lesson. Place the subtraction control chart in front of the child with the paper chart on the child's dominant side. Place the container of subtraction problem tickets *above* the control charts with the lid beside it, if there is one.
2. Ask the child to choose a problem ticket at random from the container, placing the ticket beside the container, for example 5 - 3 =.
3. On the control chart, show the child how to locate 5 on the left side of the chart with the left index finger and -3 on the right side of the chart with the right index finger.
4. Have the child *move* the left finger down the column under the 5, then the right finger to the left across the row beside -3. The square where the fingers meet contains the answer, in this example, 2.
5. The child finds the corresponding space on the paper subtraction chart and writes the answer, in this example, 2.
6. The problem ticket just used is placed in the lid or behind the container upside down.
7. The child continues to select problem tickets and proceeds as before.
8. As soon as it is observed that the procedure is understood, leave and observe unobtrusively.

Division Chart Procedure:

1. Invite a child to the lesson. Place the division control chart in front of the child with the paper chart on the child's dominant side. Place the container of division problem tickets above the control charts with the lid beside it, if there is one.
2. Ask the child to choose a problem ticket at random from the container, placing the ticket beside the container, for example, 9 ..,.. 3 =.
3. On the control chart, show the child how to find 9 on the top row with the right index finger and ..,..3 on the left with the left index finger.
4. Have the child move the right index finger down the column under 9 and the left index finger to the right across the row by ..;-3. The square where the fingers meet contains the answer, in this example, 3.
5. The child finds the corresponding space on the paper division chart and writes the answer.
6. The problem ticket just used is placed in the lid or behind the container upside down.
7. The child continues to select problem tickets and proceeds as before.
8. As soon as it is observed that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Control Chart
Prepared problem tickets

Obserations:

Placement of fingers
Recording answers
Placement of problem tickets
Child's reaction to error
Length of work time and number of repetitions
Degree of interest and concentration

Variations:

Invite the child to use the problem tickets and fill in the paper charts without referring to the control until all work is completed. This indicates to the child those parts of the tables which have not yet been committed to memory and provides an incentive to be able to complete the entire chart, that is, to learn all the tables.

Vocabulary:

Chart

Stamp Exercise

Purposes:

To provide additional practice in addition, multiplication, division and subtraction
To assist in establishing a basis for future abstract operations
To prepare for pegboard operations in addition, multiplication, division and subtraction
To develop appropriate vocabulary

Preliminary Exercises:

Practice with individual golden bead operations

Materials:

Compartmented container with numeral squares (stamps) of two centimeters, fifty green printed with the numeral 1, fifty blue printed with 10, fifty red printed with 100 and a minimum of nine green printed with 1000
Container with nine small red skittles, nine small blue skittles, eighteen small green skittles
Container with nine red discs, nine blue discs, eighteen green discs
Note: Some containers have compartments for the discs and skittles as well as the stamps.
Four strips of gray felt, 8 1/2" long and 2 1/2" wide
Booklets or cards of problems without changing, answers on back
Booklets or cards of problems with changing, answers on back
Note: The problem booklets or cards provided for individual golden bead operations may be used.
Paper with symbolically colored vertical lines for recording; or red, green and blue pencils and ruler for drawing lines on regular lined paper
Pencil

Introductory Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Place the container(s) of numbered stamps, discs and skittles near the top of a table but within easy reach of the child who is seated on your dominant side. Explain to the child that the green squares printed with 1 represent units, the blue squares printed with 10 represent tens, the red squares printed with 100 represent hundreds and the green squares printed with 1000 represent thousands.
3. Place one strip before the child and together count nine green unit stamps as they are placed vertically, starting at the top. Say, "If we had one more, there would be ten." Slide the strip containing the unit stamps to your non-dominant side. Place another strip before the child and together count the nine ten stamps as before. Say, "If we had one more, there would be one hundred." Slide the strip to your non-dominant side. Place another strip before the child and together count the nine hundred stamps as before. Say, "If we had one more, there would be one thousand." Slide the strip to your non-dominant side. Repeat the procedure to count the nine thousand stamps.

4. Ask the child to lay out quantities with the stamps representing what the teacher has written on symbolically lined paper.
5. Invite the child to write on symbolically lined paper the quantities which the teacher has laid out with the stamps.

Procedure for Addition:

1. Once the introductory procedure is understood, have the child get or make the symbolically lined paper, a pencil and the booklet or card containing addition problems without changing. Tell the child to write the first problem on the symbolically lined paper, for example: 2345 + 5432.
2. The appropriate number of squares is arranged vertically under each place value. In this example, there are five green squares printed with 1, four blue squares printed with 10, three red squares printed with 100 and two green squares printed with 1000. The appropriate number of squares for the second addend is arranged vertically under each place value. In this example, there are two green squares printed with 1, three blue squares printed with 10, four red squares printed with 100 and five green squares printed with 1000.
3. Ask the child to move the stamps down as they are counted, starting at the top. Begin with the green unit stamps. Record the number of units on the problem paper in unit's place. The blue ten stamps are moved down, counted and recorded appropriately, then the red hundred stamps and finally, the green thousand stamps.
4. Tell the child to continue with the problems in the booklet or on cards, and leave as soon as it is apparent that the procedure is understood.

Procedure for Multiplication:

1. Invite a child to the lesson once addition exercises have been done.
2. Place the container(s) of stamps, discs and skittles near the top of a table but within easy reach of the child who is seated on your dominant side.
3. Have the child get or make the symbolically lined paper, a pencil and the booklet or *carq* containing multiplication problems without changing. Tell the child to write the first problem on the symbolically lined paper, for example: 1321 x 3.
4. Ask the child to place three green disks at the right vertically, representing the multiplier. The appropriate number of stamps is arranged vertically to the left of each disc according to each place value. In this example, there is one green stamp printed with 1, two blue stamps printed with 10, three red stamps printed with 100 and one green stamp printed with 1000. The appropriate number of squares is arranged vertically in each place value as before. Repeat the process a second and third time with the stamps.
5. Ask the child to count the green unit stamps by moving them down, starting at the top, then to record the number of units on the problem paper in unit's place. The same procedure is followed for tens, hundreds and thousands
6. Tell the child to continue with the problems in the booklet or on cards, and leave as soon as it is apparent that the procedure is understood.

Procedure for Division:

1. Invite a child to the lesson once the addition and multiplication exercises have been done.
2. Place the container(s) of stamps, discs and skittles near the top of a table but within easy reach of the child who is seated on your dominant side.
3. Have the child get or make the symbolically lined paper, a pencil and the booklet or card containing division problems without changing. Tell the child to write the first problem on the symbolically lined paper, for example: 3696 ÷ 3.
4. Invite the child to place small green skittles vertically at the left to form the dividend of the problem, for example, 3696 ÷ 3. These must be spaced to allow room for the stamps. The appropriate number of squares representing the dividend is arranged according to place value above the skittles. In this example, there are six green stamps printed with 1, nine blue stamps printed with 10, six red stamps printed with 100 and three green stamps printed with 1000. 5. Have the child give each skittle a green thousand stamp. The child records the answer in the thousands place. Give each skittle a red hundred stamp until all have been divided. The child records the answer in hundreds place. Repeat the process with the blue ten stamps and the green unit stamps.
6. Tell the child to continue with the problems in the booklet or on cards, and leave as soon as it is apparent that the procedure is understood.

Procedure for Subtraction:

1. Invite a child to the lesson once the previous exercises have been done.
2. Place the container(s) of numbered stamps, discs and skittles near the top of a table but within easy reach of the child who is seated on your dominant side.
3. Have the child get or make the symbolically lined paper, a pencil and the booklet or card containing subtraction problems without changing. Tell the child to write the first problem on the symbolically lined paper.
4. Invite the child to lay out the stamps to form the minuend of the problem, for example, 7895 - 5432. In this example, there are five green stamps printed with 1, nine blue stamps printed with 10, eight red stamps printed with 100 and seven green stamps printed with 1000.
5. Have the child subtract the amount of the subtrahend by counting and moving down the stamps below the minuend. In this example, two green unit stamps, three blue ten stamps, four red hundred stamps and five green thousand stamps are counted as they are moved down.
6. Ask the child to move the stamps at the top of the table below the stamps representing the subtrahend, count and record the answer on the problem paper.
7. Tell the child to continue with the problems in the booklet or on cards, and leave as soon as it is apparent that the procedure is understood.

Control of Error:

Answers on the back of cards or booklets
Color of numeral squares, vertical lines on paper

Observations:

Copying of problems onto paper
Counting of numeral squares and recording of answers
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to do problems involving changing.

Vocabulary:

stamp exercise numeral squares

Fractions Materials Used Sensorially

Purposes:

To assist visual perception of size and shape
To provide a sensorial basis for the future study of fractions To heighten attentiveness in carrying out systematic operations To associate abstract representation with concrete objects
To develop appropriate vocabulary

Preliminary Exercises:

Practical life exercises
Practice with geometric cabinet and cards
Work with number rods and spindle box for number-numeral concepts one through ten

Materials:

Ten fraction circles, one undivided, the others divided into equal parts from two to ten, set into frames of a contrasting color (Usually fractions are red, the frames green.) Two trays, each long enough to accommodate five frames of fractions in a row

Procedure:

1. Invite a child to the lesson. Take the tray of fractions up to fifths and place on the table between you and the child who is seated on your dominant side. Check to be sure the fractions are in sequence.
2. Using the first three fingers of the dominant hand, grasp the knob and remove the fractional pieces from each frame, one frame at a time, placing them vertically in front of each corresponding frame.
3. Ask the child to replace the parts, starting on the left with the one.
4. Stand, replace chair and return fractions to the shelf. Thank the child and say, "This fraction tray may be chosen," or say , "We will have another lesson later," if the child has not understood the procedure.

Control of Error:

Placement of fractional parts directly in front of frames
Improper fit of fractions causing overlapping or space between the parts Visual recognition of shape of fractional parts

Observations:

Handling of materials
Placement of fractions upon removal and replacement
Child's reaction to error
Length of work time and number of repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to repeat the exercise using sixths through tenths fractional parts.

Using the tray of fractions through fifths, invite the child to remove the fractional parts, placing them in random order at the front edge of the table. Have the child sort according to size with each set of fractional parts being placed vertically in front of a frame. Tell the child to replace the fractional parts into the frame in front of which they have been placed.

Using the three period lesson, teach the names of the fractional parts. Place the frames containing the halves and the tenths on your non-dominant side. An empty frame is acquired by removing the whole or one inset and placing it out of sight on your non-dominant side. The empty frame is placed at the bottom edge of the table.

The part being named in the three period lesson is placed into the empty frame. Teach the names of all the fractions by the three period lesson according to the previous procedure.

Note: For the second period of the lesson when both fractional parts are presented, leave space between the fractional parts.

Invite the child to place fractional parts in the empty frame according to verbal directions, for example, "Place four fifths in the empty frame," or according to written commands. These fractional parts are removed before continuing with other placements.

Vocabulary:

fraction names of fractional parts vertical

Introduction of Fraction Materials for Mathematics

Purposes:

To associate abstract representation of fractional parts with concrete objects
To develop an understanding of operations with fractions
To provide the skills needed to do fraction work
To provide the basis for application of fractions in telling time, food preparation, science
To develop appropriate vocabulary

Preliminary Exercises:

Use of fractions as sensorial material
Exercises with cards and counters

Materials:

Two trays of fraction materials in frames
Set 1: white fraction cards 1" by 1 1/2", one card with the numeral 1 written on it, two cards with 1/2, three cards with 1/3, continuing to ten cards with l/10, contained in a clear plastic box with ten compartments
Set 2: white fraction cards 1" by 1 1/2", on which are written 1/2, 2/2; 1/3, 2/3, 3/3;1/4, 2/4, 3/4, 4/4, up to 1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10, contained in a clear plastic box with ten compartments
Note: Insert into each compartment a laminated piece of paper the width of the compartment to form an inclined plane. This permits easy removal of cards.

Procedure for Learning Symbols:

1. Invite a child to the lesson once the names of the fractional parts previously presented as sensorial material are known. Have the child sit on your dominant side.
2. Take the first set of fraction cards and place on a table on your non-dominant side.
3. By the three period lesson, teach the child the terms numerator and denominator.
4. By the three period lesson, teach the names of the written symbols, two at a time, as long as interest is evident.
5. Stand, replace chair, thank child and say that other lessons will be given until all symbols are learned. Return material to the shelf.

Control of Error:

The teacher

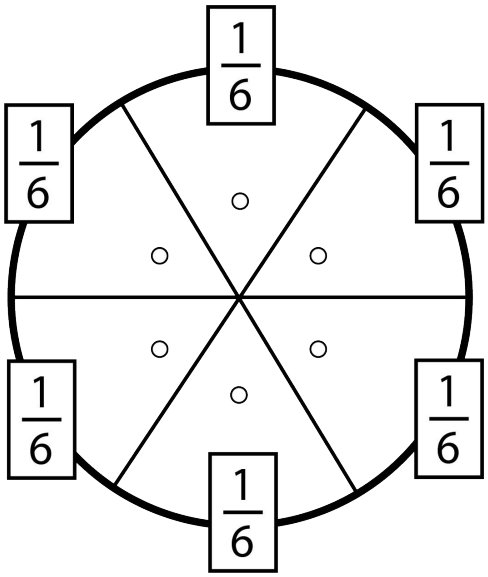
Procedure for Matching:

1. Invite a child to the lesson once the names and symbols of fractions are known. Have the child place both trays of fraction materials at the center of a table and sit on your dominant side.
2. Ask the child to remove the frames containing fractions and place in sequence near the front of the table.

3. Get the first set of fraction cards and ask the child to place the cards on each appropriate fractional part, leaving the parts in their frames.
4. Stand, replace chair, thank child and say that the fractions and symbols may be chosen. Return material to the shelf.

Control of Error:

Number of cards equals number of fractional parts for each fraction



Procedure for Combining Fractions and Symbols:

1. Invite a child to the lesson once the fractions and symbols have been matched. Have the child place the fraction material at the center of a table and sit on your dominant side.
2. Get the second set of fraction cards and place on the child's dominant side near the front of the table.
3. Place the empty frame near the front of the table. Remove several fractional parts from the frame and place in the empty frame. For example, remove four of the fifths.
4. Ask the child to determine what the fractional parts are, in this example fifths, and to count the number of fractional parts which have been placed in the frame, in this example, four fifths.
5. Ask the child to find the card that corresponds to the number of fractional parts set into the empty frame, in this example, the card that has 4/5 written on it. Have the child place the card in front of the frame. Replace fractions and cards.
6. If the first example presents no problems to the child, continue to have the child remove fractional parts, place in the empty frame and locate the fraction card that indicates the number of parts displayed.
7. It is not necessary to remain with the child once it is determined that the exercise is understood. The child knows to return the materials to their proper places.

Control of Error:

Child's ability to count and to read fraction cards

Observations:

Placement of fraction cards on fractions
Placement of fractions in frame
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Reverse the procedure for combining fractions and symbols by showing the child a fraction card and requesting that the fraction parts be placed in the empty frame. For example, show the card with 3/4 written on it and ask the child to find the corresponding fractional material which will be placed in the empty frame with the card placed below it.

Vocabulary:

numerator denominator

Note: Do not prepare any problem booklets or cards without first using the fractional materials to ensure that they are appropriate.

Equivalent Fractions

Purposes:

To prepare for addition, multiplication, division and subtraction operations with fraction material

To facilitate understanding of equivalent fractions

To develop appropriate vocabulary

Preliminary Exercises:

Practice with sandpaper numerals

Work with matching fractions and fractional cards Exercises for finding factors and multiples on the pegboard

Materials:

Two trays of fraction materials in frames

Booklets or cards with exercises for forming equivalent fractions, color coded tan or beige

Containers for cards, color coded appropriately

Small booklet, color coded as above in which equivalent fractions may be recorded by the child

Rule booklet or card color coded to match exercise cards or booklet

Note: Booklet cover or card mounting should be 4 1/2" by 3".

Paper with rules is cut to 4" by 2 1/2".

Procedure for Finding Equivalent Fractions:

1. Invite a child to the lesson once fractions and symbols have been combined. Have the child place the fraction material at the center of a table and sit on your dominant side.
2. Place the whole near the front of the table, remove the whole or one and place to the side of the frame. Ask the child to place other fractional parts in the empt frame to exactly fit. For example, two fourths and one half will fit exactly into the empty frame.
3. Invite the child to try many combinations of fractions to form the whole and to record them in the small booklet.

..4. Remind the child that each fraction frame contains fractional parts equal to one.

5. As soon as it is observed that the child understands the procedure, leave and observe unobtrusively.

Control of Error:

Size of empty frame Exactness of fractional parts Child's visual perception of size

Observations:

Placement of fractional parts in the empty frame
Recording of results
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use small booklets or cards with names of fractions for which equivalents are to be found, for example, "Find the equivalent fractions for 1/2."

Provide additional small booklets or paper for recording results: 1/2 = 2/4, 1/2 = 3/6, 1/2 = 4/8, 1/2 = 5/10.

Invite the child to read a small booklet or card with a rule for the concepts which the child has deduced from work with equivalent fractions.

Invite the child to find equivalent fractions without using the fraction materials by referring to the definitions.

Note: Rules are given in the white pages.

Vocabulary:

whole number

Reduction of Fractions

Purposes:

To prepare for addition, multiplication, division and subtraction operations with fraction material

To facilitate understanding of reduction of fractions

To develop appropriate vocabulary

Preliminary Exercises:

Finding and recording equivalent fractions

Materials:

Two trays of fraction materials

Booklets or cards with exercises for reduction of fractions, color coded gray

Note: In booklets, there is one fraction per page to be reduced.

Gray container with individual booklets for recording reduction of fractions and rule cards for reduction of fractions, color coded gray

Procedure Reduction of Fractions:

1. Invite a child to the lesson once finding equivalent fractions has been practiced. Have the child place the fraction material at the center of a table and sit on your dominant side.
2. Present a "Reduction of Fractions" booklet or card. Tell the child to read the fraction on the first page and remove the appropriate frame with fractional parts to the front of the table.
3. Have the child remove the appropriate fractional parts from the frame and place to the side of the table. Ask the child to find the largest fractional part which exactly fits the space in the frame. For example, if the fraction written on the first page of the booklet is 4/8, the child removes four 1/8 fractional parts and replaces them with the 1/2.
4. Ask the child to record the results in the small individual booklet. In this example, the child would write 4/8 = 1/2.
5. Tell the child to continue with the other fractions in the booklet once it is observed that the procedure is understood. Leave the table and observe unobtrusively.

Control of Error:

Size of empty space in the frame

Exactness of fractional parts

Booklet with only fractions which can be reduced Exact match of smaller fractional parts with a larger one

Observations:

Placement of fractional parts in the frame
Recording of results
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Introduce the small booklet or card with rules for concepts which the child understands from work with reduction of fractions. Invite the child to reduce fractions without using the fraction materials by referring to the definitions.
Note: Rules are given in the white pages.

Vocabulary:

reduction

Addition of Fractions with Like Denominators

Purposes:

To facilitate understanding of addition of fractions
To prepare for abstract addition of fractions
To develop appropriate vocabulary

Preliminary Exercises:

Practice with short bead stair addition
Exercises in reduction of fractions

Materials:

Two trays of fraction materials
Container of booklets with red covers or of red cards as a color code for addition, containing problems for addition of fractions with like denominators, having answers on the back
Note: For the introductory exercises, provide problems whose answers do not require reduction.
Red card with rule for addition of fractions with like denominators kept in container with other rule cards
Booklets with red covers for recording problems and answers, pencil

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child place the fraction material at the center of a table and sit on your dominant side.
2. Place an "Addition of Fractions" booklet or card and the booklet for recording answers on the child's dominant side. Tell the child to read the fraction addition problem on the first page and place the fractional parts into the empty frame. For example, if the problem is 1/7 + 3/7 = , the child places the one 1/7 fractional part in the empty frame, then places three 1/7 fractional parts together in the frame, not touching the first 1/7.
3. The fractional parts are moved together and counted. The child writes the problem and answer in a small booklet with a red cover and reduces the fraction if necessary.
4. Tell the child to continue with the problems for addition of fractions and leave as soon as it is observed that the procedure is understood. Observe unobtrusively.

Control of Error:

Answers on back of card or back page of booklet

Observations:

Placement of fractional parts in the empty frame
Recording of results
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use addition of fraction booklets or cards whose answers require reducing.
Introduce a card with the rule which the child understands from work with addition of fractions.
Invite the child to add fractions without using the fraction materials by referring to the rule.
Note: Rules are given in the white pages.

Vocabulary:

like denominators

Subtraction of Fractions with Like Denominators

Purposes:

To facilitate understanding of subtraction of fractions
To prepare for abstract subtraction of fractions
To develop appropriate vocabulary

Preliminary Exercises:

Practice with short bead stair subtraction
Exercises in reduction of fractions

Materials:

Two trays of fraction materials
Container of booklets with green covers or of green cards as a color code for subtraction, containing problems for subtraction of fractions with like denominators, having answers on the back
Note: For introductory exercises, provide problems whose answers do not require reduction.
Green card with rule for subtraction of fractions with like denominators kept in container with other rule cards
Booklets with green cover for recording problems and answers

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child place the fraction material at the center of a table and sit on your dominant side.
2. Place a "Subtraction of Fractions" booklet or card and the booklet for recording answers on the child's dominant side. Tell the child to read the fraction subtraction problem on the first page and place the fractional parts for the minuend into the empty frame. The fractional part or parts representing the subtrahend are removed from the frame and placed below it. For example, if the problem is 6/8 - 1/8 = , the child places six 1/8 fractional parts into the empty frame, then removes one 1/8, placing it below the frame.
3. The fractional parts remaining in the frame are counted. The child writes the problem and answer in a small booklet with a green cover.
4. Tell the child to continue with the problems for subtraction of fractions and leave as soon as it is observed that the procedure is understood. Observe unobtrusively.

Control of Error:

Answers on back of card or back page of booklet

Observations:

Placement of fractional parts in the empty frame Removal of the fractional part representing the subtrahend
Recording of results
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use subtraction of fraction booklets or cards whose answers require reduction.

Introduce the card with the rule which the child understands from work with subtraction of fractions.

Invite the child to subtract fractions without using the fraction materials by referring to the rule.

Note: Rules are given in the white pages.

Vocabulary:

No new vocabulary

Multiplication of Fractions by Whole Numbers

Purposes:

To facilitate understanding of multiplication of fractions
To prepare for abstract multiplication of fractions
To develop appropriate vocabulary

Preliminary Exercises:

Practice with short bead stair multiplication
Exercises in reduction of fractions

Materials:

Two trays of fraction materials
Container of booklets with yellow covers or of yellow cards as a color code for multiplication, containing problems for multiplication of fractions by whole numbers, having answers on the back
Note: For introductory exercises, provide problems whose answers do not require reduction.
Yellow card with rule for multiplication of fractions by whole numbers kept in container with other rule cards
Booklets with yellow covers for recording problems and answers

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child place the fraction material at the center of a table and sit on your dominant side.
2. Place a "Multiplication of Fractions" booklet or card and the booklet for recording answers on the child's dominant side. Tell the child to read the fraction multiplication problem on the first page and place the number of identical fractional parts designated by the multiplier into the empty frame with the fractional parts not touching. For example, to solve the problem 1/8 X 3 =, tell the child to place three 1/8 fractional parts into the empty frame.
3. Have the child count the number of identical fractional parts in the frame. The problem and answer are written in the small booklet.
4. Tell the child to continue with the problems for multiplication of fractions and leave as soon as it is observed that the procedure is understood. Obsere unobtrusively.

Control of Error:

Answers on back of card or back page of booklet

Observations:

Placement of fractional parts in the empty frame
Recording of results
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use multiplication of fractions booklets or cards whose answers require reduction.

Introduce the card with the rule which the child understands from work with multiplication of fractions.

Invite the child to multiply fractions by whole numbers without using the fraction materials by referring to the rule.

Note: Rules are given in the "white pages."

Vocabulary:

whole number divisible

Division of Fractions by Whole Numbers

Purposes:

To facilitate understanding of division of fractions
To prepare for abstract division of fractions
To develop appropriate vocabulary

Preliminary Exercises:

Practice with division board
Exercises in reduction of fractions

Materials:

Two trays of fraction materials
Container of booklets with blue covers or of blue cards as a color code for division, containing problems for division of fractions by whole numbers, having answers on the back, set one having dividends which do not require changing before being divided, set two having dividends which require changing
Note: For introductory exercises, provide problems whose answers do not require reduction.
Blue card with rule for division of fractions by whole numbers kept in rule container
Nine large green skittles in a container
Booklets with blue covers for recording problems and answers

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child place the fraction material at the back of a table and sit on your dominant side.
2. Place a "Division of Fractions" booklet or card and the booklet for recording answers on the child's dominant side. Tell the child to read the fraction division problem on the first page and place the number of identical fractional parts designated by the dividend in a row near the front of the table.
3. Have the child place the number of skittles indicated by the divisor in a column at the left above the row of fractional parts. Each skittle is given a fractional part, starting with the first skittle at the top.
4. Have the child count the number of fractional parts which any skittle received. The problem and answer are written in the small booklet. The answer is the quantity placed beside one skittle.
5. Tell the child to continue with the problems for division of fractions and leave as soon as it is observed that the procedure is understood. Observe unobtrusively.

Control of Error:

Answers on back of card or back page of booklet

Observations:

Placement of fractional parts and skittles
Recording of results
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use division of fractions booklets or cards whose answers require reduction.

Introduce the card with the rule which the child understands from work with division of fractions.

Invite the child to divide fractions by whole numbers without using the fraction materials by referring to the rule.

Invite the child to do problems with fractions not divisible by whole numbers, therefore requiring that the fraction be exchanged for equivalent, identical fractional parts.

Vocabulary:

No new vocabulary

Ordinal Number

Purposes:

To develop an understanding of ordinal number
To relate ordinal number to the visual perception of position
To develop appropriate vocabulary

Preliminary Exercises:

Rolling mats and strips
Work with cards and counters
Exercises with beginning fractions to learn names of fractional parts
Practice with phonogram materials

Materials:

Set 1- White container labeled **Ordinal Number** with white control strip 2" by 15", divided into ten equal sections labeled 1st, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th; white mute strip 2" by 15" divided into ten equal sections; separate white cards 1 1/2" wide by 2" high, labeled with the words first through tenth on one side and with ordinal numerals 1st through 10th on reverse

Set 2- White container labeled **Ordinal Number Exercise** with white control strip 2" by 15", divided into ten equal sections, each section having a circle of a different color to match counters; ten counters of identical size but each a different color; command cards or booklet with directions for placement of counters; control strip for multiple placement of counters

Note: Using small letters, label the first section left and the tenth section right to ensure correct placement.

Container of large wooden or plastic beads with string, command cards giving sequence of placement with control drawings on the reverse

Preliminary Procedure:

1. Take every opportunity to use ordinal terms, for example, "The smallest cylinder is first, the largest is last,"or, "John is first in line, Amy is second, etc."
2. Give verbal directions with indication for placement of objects, for example, "Place the thousand cubes first."

Procedure for Set 1 :

1. Invite a child to the lesson once the preliminary exercises have been done. Place the materials of Set 1 on a table in front of the child who is seated on your dominant side.
2. Ask the child to unroll the strips from left to right with the control above the mute strip, then to read the control and place the separate word cards appropriately on the mute strip. Tell the child to turn the card over to check placement once the exercise has been completed. Leave as soon as it is observed that the procedure is understood and observe unobtrusively.

Procedure for Set 2:

1. Invite a child to the lesson once the exercise with Set 1 has been done. Place the materials of Set 2 and the control and mute from Set 1 on a table in front of the child who is seated on your dominant side.
2. Ask the child to unroll the strips from left to right with the control from Set 1 above the mute. Have the child place the container of counters in front of the strips and conveniently arrange the command cards or booklet for reading.
3. Tell the child to read the commands, for example, "Place the red counter on the second section,"and to place the counters appropriately on the mute strip, referring to the control strip from Set 1 if necessary.
4. When the commands have been completed, have the child unroll the control from Set 2 (with the colored circles) in front of the strip on which the counters have been placed and check placement of counters by looking at the colors in each section.
5. Leave as soon as it is observed that the procedure is understood and observe unobtrusively.

Procedure for Beads:

1. Invite a child to the lesson once the preliminary exercise has been done. Place the container of beads, commands and controls in front of the child, near the center of the table.
2. Invite the child to read and execute all the commands on one card, for example, "Place the blue bead that is a cube first on the string. Place the yellow bead that is a sphere second," etc.
3. After all the commands on one card have been done, have the child turn the command card over to check the placement of beads.

Control of Error:

Control strip with ordinal numerals
Cards with ordinal numerals on reverse
Control strip with colored circles
Diagrams on reverse of commands for bead stringing
Counters of different colors
Control drawings of bead placement

Observations:

Handling of materials
Placement of strips, beads, cards and counters
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use the commands and place the counters on the mute strip without referring to the ordinal control strip. Once the work is completed, it is checked with the control strip containing the colored circles.

Provide additional commands and control strip which require placement of several counters, for example, "Place a red counter and a pink counter in the sixth position."

Vocabulary:

ordinal number ordinal numeral first second third fourth fifth sixth seventh eighth ninth tenth 1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th

Symbols

Purposes:

To provide a means for understanding mathematical symbols To prepare for doing mathematical operations using the symbols To develop appropriate vocabulary

Preliminary Exercises:

Individual work in mathematics involving addition, multiplication, division and subtraction with golden beads

Exercises using addition, subtraction, multiplication and division boards

Materials:

White container with white control cards, mute cards and labels with symbols for greater than, less than, not equal

Note: Control cards are 4 1/4" by 2 5/8". Mute cards and labels are made by cutting a second set of cards into two equal pieces.

Paper and pencil

Cards or booklets with mathematics problems having missing symbols, answers on back

Procedure for Symbols:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the container of cards with symbols for greater than, less than, not equal in front of the child who is seated on your dominant side.
2. Tell the child to lay out the control cards in a row from left to right, then to place the mute cards below them with labels matched to appropriate cards.
3. Once it is observed that the procedure is understood, leave and observe unobtrusively.

Procedure for Missing Symbols:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the container of materials in front of the child who is seated on your dominant side. Place a problem card or booklet on the child's dominant side.
2. Ask the child to copy the first problem and to draw the appropriate symbol in the blank space in order to complete the equation. Tell the child that any of the mathematics materials may be used in solving the problem.
3. Once it is observed that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Control cards

Answers for missing symbol problems

Observations:

Handling of materials
Placement of cards and labels
Placement of symbols in problems
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration
Variations:
Invite the child to place the labels on mute cards without referring to the control until the work is complete.
Invite the child to use the booklet or cards with mathematics problems involving the symbols for greater than, less than and not equal.

Vocabulary:

greater than less than not equal

Money

Names of Currency Denominations

Purposes:

To learn the names of coins
To provide a basis for further work, leading to an understanding of the monetary system
To develop appropriate vocabulary

Preliminary Exercises:

Practice with materials which enhance size discrimination, such as sorting by size, work with geometric cabinet
Use of golden bead material to develop an understanding of place value for units, tens, hundreds and thousands
Exercises with Seguin boards for learning eleven through ninety-nine
Work with phonograms for use of control and mute cards

Materials:

For children who cannot read, white container with one each of the following: penny, nickel, dime, quarter, half-dollar, dollar coin and dollar bill
Small felt mat
For children who read, white container of white control cards each card showing both sides of each coin with its name written beneath it, mute cards with both sides of coins, separate labels
. Note: Labels may be copied from the white page containing the money chart and cut 3/4" by 3 1/2". Coin cards are 21/2" by 3 1/2".
White control and mute cards with labels to teach the symbols for dollar and cent
Note: Cards are 2" wide by 2 1/2" high, labels are 3/4" by 2".

Procedure for Nonreaders:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the container of money on the table on your non-dominant side. The child is seated on your dominant side. Spread the felt mat in front of the child.
2. Select the penny and the dollar bill. By the three period lesson, teach their names.
3. Stand, replace chair, thank the child and say that another lesson may be requested.
4. Continue to teach the names of all the coins over a period of time.

Procedure for Readers:

1. Invite a child to the lesson once the preliminary exercises have been done. Place the container on the table in front of the child who is seated on your dominant side.
2. Have the child lay out the coin control cards in the center of and parallel to the front edge of the table, then match and place the mute cards beneath the control. Ask the child to read the labels and lay them out under the appropriate mute coin cards.
3. As soon as it is apparent that the child understands the procedure, leave and observe unobtrusively.

Control of Error:

The teacher for the non-reading exercise
Control cards for the reading exercise

Observations:

Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to lay out the mute coin cards and place the appropriate labels without referring to the control cards until the work is complete.

Teach the symbols$ and¢ by the three period lesson, using the mute cards for the child who cannot read, or with control cards, mute cards and labels for the child who can read.

Invite several children who can read labels and have learned the names of the coins to participate in a money "banking game." On one table, have them lay out the coin labels. On another table, place coins on a felt mat. Ask each child in turn to bring either the coin or label requested by the teacher. For example, "Please bring me the nickel," "Please bring me the label for quarter." The "banking game" may be played by giving a label to each child and having them bring the appropriate coin, or giving a coin for which the appropriate label is brought.

In geography, introduce the monetary systems of other countries for which money, pictures or rubbings of coins are available.

In history, provide information booklets with pictures concerning forms of "money" used in early and/or primitive civilizations.

Vocabulary:

penny nickel dime quarter half-dollar dollar cent denomination currency monetary system

Values of Denominations of Currency

Purposes:

To acquaint the child with the value of coins
To provide a basis for further work, leading to an understanding of the monetary system
To develop appropriate vocabulary

Preliminary Exercises:

Work with phonograms

Materials:

White control chart with outline and name denominations in the left section and their values and symbols in the second, with the largest denomination at the top, the smallest at the bottom
White container of mute cards with denominations, white cards with value and white cards with symbol to be matched with mute cards
Paper, coin stamps, ink pad
Container of currency of all denominations

Preliminary Procedure: Learning names of coins

1. Invite the child to the lesson and teach the names of currency by the three period lesson.

Procedure:

1. Invite a child to the lesson once the names of denominations are known. Take the materials to a table and seat the child on your dominant side. Have the child place the control at left, then match and arrange the mute cards at the right of the control with the value and symbol card placed at the right of the mute card.
2. As soon as it is apparent that the exercise is understood, leave and observe unobtrusively.

Control of Error:

Control chart
Name of denomination of currency

Observations:

Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to lay out the mute cards and place the appropriate value and symbol card, without referring to the control chart until the work is complete.

Provide paper for making small bookiets, coin stamps and an ink pad so that a different coin can be stamped on each page with the name and value of the coin written beneath. This becomes the child's personal property.

Commands for equivalents in larger amounts

Vocabulary:

vaiue

Composition of a Dollar

Purposes:

To learn the equivalent amounts of denominations that compose a dollar
To provide a basis for further work, leading to an understanding of the monetary system
To develop appropriate vocabulary

Preliminary Exercises:

Learning values of denominations of currency

Materials:

White control chart with all the denominations of currency (See directions for preparation of control chart which follow.)
Felt mat the same size as the control
White container with six compartments to hold all coins shown on the control chart, each denomination in a separate compartment (one dollar, two half-dollars, four quarters, ten dimes, twenty nickels, one hundred pennies)
White paper eleven by seventeen, coin stamps, ink pad

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Place the chart on the table in front of the child who is seated on your dominant side. Have the child place the felt mat and the container of coins at the right of the chart.
3. Point out on the control that two half-dollars are equivalent to one dollar, as are four quarters, ten dimes, twenty nickels, one hundred pennies.
4. Invite the child to lay out the coins from the container first on the chart, then on the felt mat beside the control by consulting the control chart.
5. As soon as it is apparent that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Control chart

Observations:

Handling of materials
Placement of coins
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to lay out the coins without referring to the control chart until the work is complete.

Invite the child to use command cards which direct activities to determine equivalent amounts of lesser denominations in each denomination possible, for example, two quarters, five dimes, ten nickels or fifty pennies in a half-dollar.

Provide paper eleven by seventeen inches, coin stamps and ink pad so that the child can prepare a personal "composition of a dollar" chart

Vocabulary:

equivalent

Changing Coins for Equivalent Amounts

Purposes:

To understand the changing of money for equivalent amounts To establish a basis for using money in real situations
To provide the foundation for doing calculations involving money
To develop appropriate vocabulary

Preliminary Exercises:

Work with values of coins and composition of a dollar

Materials:

White container of coins (two nickels, ten pennies) which can be used to compose the equivalent amount of a dime with a control card on which is drawn coin outlines indicating equivalent amounts
White container of coins (two dimes, six nickels, twenty-five pennies) which can be used to compose the equivalent amount of a quarter with a control card on which is drawn coin outlines indicating equivalent amounts
White container of coins (two quarters, five dimes, ten nickels, fifty pennies) which can be used to compose the equivalent amount of a half-dollar with a control card on which is drawn coin outlines indicating equivalent amounts
White container of coins (two half-dollars, four quarters, ten dimes, twenty nickels, one hundred pennies) which can be used to compose the equivalent amount of a dollar with a control card on which is drawn coin outlines indicating equivalent amounts
Note: Control cards are 3" by 4".
Container of money counting cards with combinations of coins stamped on one side and the combined amount written on the reverse, small booklet for recording
Container with money "bingo" card composed of twenty-five squares, each square having amounts in different arrangements; a pair of dice, each surface of which has a picture of a coin; a pair of dice, each surface of which has the name of a coin; small container of discs or squares as covers for the bingo card; container of coins
Money command cards with control on reverse
"Purchase game" with basket of items on which are attached price tags from one cent to one dollar, a change purse containing money and control cards naming each item's price with the correct change for one dollar
Booklets of problems involving money

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Place the container of coins for the dime on the table in front of the child who is seated on your dominant side.
3. Ask the child to find all the combinations with the coins in the container that will make the amount equivalent to a dime, then check the work with the control card.
4. Once it is apparent that the procedure is understood, tell the child that any of the containers may be chosen. Leave and observe unobtrusively.

Control of Error:

Control card in each container

Observations:

Handling of materials
Combinations of coins
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite a child who has learned equivalent amounts to use the money counting cards by counting the amount represented by the combinations of coins pictured on the front, recording the answer, then checking it against the control on the reverse of the card.

Invite a child who has learned equivalent amounts to play money "bingo". A card is placed on the table with the container of cover squares or discs and the container of money on the dominant side. The child rolls either pair of dice, counts with the coins the amount indicated by the dice, then covers the appropriate space on the card.

Invite the child to use money command cards.

Invite two children to play the purchase game. One child chooses an item and counts out the exact amount which is checked by the other child. At a later time, one child chooses an item and pays for it with one dollar while the other child gives back change. Control cards are used to check the amount of change.

Invite the child to use the booklets to solve problems involving money.

Vocabulary:

combinations

Measurement

Quantitative Measurement

Purposes:

To associate quantity or relative quantity with collective terms
To introduce vocabulary which denotes collections

Preliminary Exercises:

Practice in opening and closing containers
Exercises for transfer of objects
Work with golden beads and cards for recognition of numbers up to at least 500

Materials:

Two identical egg holders for one dozen eggs each, one dozen stone or wooden eggs in one holder
Set 1: Gold container of pictures mounted on gold paper to illustrate pair, couple, dozen, baker's dozen, few, many, gross, ream
Set 2: Gold container of pictures mounted on gold paper to illustrate the term, pair (scissors, pliers, glasses, trousers, earrings, gloves, shoes, socks)
Note: Mount pictures on cards which are uniform in size, but large enough to accommodate the largest picture.

Procedure for Dozen:

1. Invite a child to the lesson once the preliminary exercises have been done.
2. Have the child help take the eggs and egg holders to a table. Place them in front of the child who is seated on your dominant side. The holder with eggs is at the left of the empty holder with the long sides of the holders parallel to the front edge of the table.
3. Say, "This is one dozen."
4. Ask the child to transfer the eggs to the empty holder. Indicate that the eggs on the top row are transferred in sequence, starting with the top left egg, then the eggs on the bottom row are transferred in sequence, starting with the egg on the far left.
5. Once the procedure is understood, leave and observe unobtrusively.

Control of Error:

Exact number of eggs to make a dozen
Twelve depressions in each egg holder

Observations:

Placement of egg holders
Sequence of transfer process
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to count the eggs as they are transferred if able to count to twelve.

Vocabulary:

egg holder depression dozen

Procedure for Set 1 :

1. Invite the child to the lesson once the preliminary exercises have been done. Place the container of pictures labeled Set 1 on the table between you and the child who is seated on your dominant side.
2. Remove the pictures from the container which is placed on your non-dominant side. Show the pictures, one at a time. Give the term appropriate for each picture.
3. Return the pictures to the container, stand, replace chair and return the container to the shelf. Thank the child and say that these pictures may be chosen.

Control of Error:

The teacher

Observations:

Placement of pictures
Handling of materials
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

By the three period lesson, teach the terms depicted in the pictures.
Invite the child to locate quantities of objects to match the pictures, for example, a ream of paper, a gross of pencils, etc.

Vocabulary:

names of objects in pictures pair couple dozen baker's dozen gross ream few many

Procedure for Set 2:

1. Invite the child to the lesson once the preliminary exercises have been done. Place the container of pictures labeled Set 2 on the table between you and the child who is seated on your dominant side.
2. Remove the pictures from the container which is placed on your non-dominant side. Show the pictures, one at a time. Give the term appropriate for the picture.
3. Return the pictures to the container, stand, replace chair and return the container to the shelf. Thank the child and say that these pictures may be chosen.

Control of Error:

The teacher

Observations:

Placement of pictures
Handling of materials
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to locate pairs of objects in the classroom or at home. If able to write, invite the child to make a list of all the pairs of items located.

Vocabulary:

names of pairs of items in pictures or on lists

Volumetric Measurement

Purposes:

To give practice in the measurement of volume
To assist indirectly in the understanding of fractions
To provide experiences needed for food preparation and for experimental science
To develop appropriate vocabulary

Preliminary Exercises:

Use of an apron
Use of sponge, table crumber, dustpan and brush
Carrying a tray
Folding
Removing and replacing lids
Practice with pouring exercises
For recording, practice in writing numerals and words
Practice with fractions and fraction symbols

Materials:

Tray with small lidded bowl of grits or parakeet gravel on left, identical empty lidded bowl on right, separate measuring spoons for 1/8, 1/4, 1/2 and 1 teaspoon at center front of tray, small broad blade spatula beside spoons, on child's non-dominant side
Twoidentical measuring cups with handles, two identical one pint measures with handles, one quart measure with handle, peck measure, bushel basket

Note: Cup, pint and quart measures for dry materials must have the calibration for the greatest amount even with the rim.
Tray with two identical clear measuring cups with handles, two identical clear one pint measures with handles
Clear quart measure with handle
One gallon calibrated container
Graduated cylinders for ten milliliters (cubic centimeters) ten centiliters, one liter
Note: Containers for measuring liquids must have pouring lips, handles (except for cylinders) and the calibration for the greatest amount below the rim
Towel for drying measurement equipment
Apron
Container of white control cards, mute cards and separate labels for abbreviations of teaspoon, tablespoon, cup, pint, quart, gallon
Container of tan control cards, mute cards and separate labels for abbreviations of milliliter, cubic centimeter, centiliter, liter
Box with 250 cubes 2 by 2 by 2 centimeters which exactly fill the box
Box with 1000 cubes 1 by 1 by 1 centimeters which exactly fill the box
Container of colored 1" cubes
Container of three dimensional drawings of configurations of cubes in varying degrees of difficulty, numbered in the lower right corner to show difficulty, with one indicating the easiest; number of cubes in each construction (volume) on back of drawing
Command cards for exercises in volumetric measurement
Mathematics problems involving volumetric measurement

Pictures, time lines and booklets with information concerning the history of volumetric measurement

Note: These are located with the history materials.

Procedure for Dry Materials:

1. Invite a child to the lesson once preliminary exercises have been done. Remove the tray of materials with measuring spoons from the shelf and place on a table in front of the child who is seated on your dominant side.
2. Remove lids from containers and place behind them on the tray.
3. With the dominant hand, pick up any measuring spoon, dip it into the dry material in the container so that it is heaped on the spoon. Hold the spoon over the container.
4. Pick up the spatula in the non-dominant hand and scrape excess material from the spoon by drawing the edge of the spatula across the top of the spoon.
5. Transfer the measured amount from the spoon to the other container.
6. Repeat the procedure one more time. Replace the lids on the containers. Return spoon and spatula to the tray. Stand, replace chair.
7. Check tray, table and floor for spilled material. If cleanup is needed, use the table crumber on the tray and/or table, the dustpan and brush on the floor. Make sure the equipment has been cleaned free of dry material before putting away.
8. Return materials to the shelf. Thank the child and say that measuring with spoons may be chosen.

Control of Error:

Level appearance of dry material on spoon No material spilled on tray, table, chair or floor

Observations:

Manner in which tray is carried
Handling of spoon and spatula
Condition of tray, table and floor upon completion of exercise
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use separate measuring cups for 1/4, 1/3, 1/2, 1 cup, leveling the dry material as with spoons.

Invite the child to use the one teaspoon to fill the one cup measure and record the number of teaspoons in one cup.

Invite the child to use the box with 250 cubes and the box with 1000 cubes for further work with volume.

In history, introduce the materials relative to the history of volumetric measurement.

Vocabulary:

one-fourth one-third one-half teaspoon
name of material being measured spatula

Procedure for Liquids:

1. Invite a child to the lesson once preliminary exercises have been done. Put on an apron, remove the tray with cups and pints from the shelf and place toward the center of the table in front of the child who is seated on your dominant side. Have the child put water in the quart measure to the one quart mark and place it on the dominant side in front of the tray.
2. Remove the two one cup measures from the tray and place on the non-dominant side in front of the tray.
3. With the dominant hand, pick up the quart measure and pour water into one cup, then the other up to the one cup calibration in each.
4. Demonstrate the procedure for determining accuracy in measurement by looking at the cup on the table so that the water is at eye level. This involves kneeling beside the table. Explain that the bottom of the meniscus must be exactly on the calibration line denoting one cup. Add or pour water back into the quart measure until accuracy is achieved in both cups.
5. Move the one pint measures to the non-dominant side in front of the tray and pour the water from each cup into a one pint measure. Pour the remaining water from the quart measure into the other pint.
6. Pour the water back into the quart measure. At the conclusion of the exercise, empty the water, dry the measures with the towel.
7. Check tray, table and floor for spills. If cleanup is needed, use the table sponge on the tray and/or table, the wet mop on the floor. Return the items to the tray. Remove apron.
8. Return materials to the shelf. Thank the child and say that measuring water may be chosen.

Control of Error:

Calibrations of measuring equipment
Reading of the meniscus
No water on tray, table or floor

Observations:

Manner in which tray is carried
Handling of measuring equipment
Technique of pouring
Reading of the meniscus
Condition of tray, table and floor upon completion of exercise
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to record the number of cups in a pint and pints in a quart in a small booklet labeled **Measurement.**

Invite the child to use quart and gallon measures and record the number of quarts in a gallon in the same booklet.

Invite the child to use all of the measures and to determine equivalencies.

Provide materials for metric measurement of liquids and have the child proceed as before. Call attention to the equivalency of milliliter and cubic centimeter. Introduce materials relative to the history of measurement.

Vocabulary:

calibration calibrated cubic meniscus cup pint quart gallon
milliliter centimeter centiliter deciliter liter

Procedure with Cubes:

1. Invite a child to the lesson once preliminary exercises have been done. Place the container of cubes at the center of the table with the container of drawings beside it. The child is seated on your dominant side.
2. Select a drawing marked one and place it on the child's left. Ask the child to examine the drawing and, at its right, to construct it with the cubes.
3. Have the child count the cubes in the construction to determine the volume.
4. Tell the child that any of the drawings marked one may be chosen for construction and determination of volume. Leave and observe unobtrusively.

Control of Error:

Degree of difficulty indicated by numeral on each drawing
Drawing to show construction
Number of cubes (volume) on back of drawing

Observations:

Construction of figures represented in drawings
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use the drawings in sequence of difficulty as indicated by the numeral.
Invite the child to make constructions without the drawings and to determine the volume of each by counting the cubes.
Invite the child to use cubes whose dimensions are metric.

Vocabulary:

volume

Linear Measurement

Purposes:

To learn the techniques of linear measurement
To develop understanding of appropriate measuring devices to be used in various situations
To apply linear measurement in the environment
To develop appropriate vocabulary

Preliminary Exercises:

Work with mathematics materials to develop concept of number
Practice with sandpaper numerals for writing and reading numerals
Exercises with Seguin boards for 11 through 99

Materials:

Note: Materials which can be used in preparation of the following items are: 1" wide white and tan Velcro; 1" wide flat steel ruler, with magnetized rubber strips 1" wide; 1" wide plastic ruler and peel-off plastic strips 1" wide.

US Customary System

White container with a 12" strip or ruler, labeled one foot, marked in twelve 1" segments, twelve separate 1" pieces, labeled one to twelve inches
Twelve-inch ruler
White container with a 36" strip or yardstick, labeled one yard, marked in thirty-six 1" segments with longer, heavier marks at twelve, twenty-four and thirty-six inches, three separate white 1" pieces labeled one foot, two feet, three feet
White container of control cards, mute cards and separate labels, all white, for abbreviations and symbols for inch, foot, yard and mile
Container of white command cards 3" by 5" with directions for measuring items in the environment, using each type of measuring device, booklet for recording measurements
Yardstick
Tape measure (60 inch)
Folding rule
Measuring wheel
Cloth or vinyl reel tape with handle for rewinding (16 foot preferred)
Inside calipers
Outside calipers
Container of paper cut into a variety of sizes
Note: Abbreviation cards are 2 1/2 " by 3" high. Word cards are 2 1/2" by 1 1/2" high with word label 2" by 1". These dimensions apply to Metric system as well.

Metric system

Tan container with tan 1 decimeter strip, labeled decimeter, marked in ten 1 centimeter segments,. ten separate tan 1 centimeter pieces, labeled from one to ten centimeters with the first centimeter marked in ten millimeter segments

Decimeter ruler

Tan container with 1 meter strip, labeled one meter, marked in ten 1 decimeter segments, ten separate 1 decimeter pieces, labeled from one to ten decimeters

Tan container of control cards, mute cards and separate labels, all tan, for abbreviations of millimeter, centimeter, decimeter, meter, kilometer

Meter stick

Container of command cards 3" by 5" with directions for measuring items in the environment, using each type of measuring device, booklet for recording

Booklets of mathematics problems in measurement, involving all operations with whole numbers as well as fractions and mixed numbers, a separate booklet for each type of problem

Note: Booklets have tan covers for metric problems, white for U.S. Customary System

Pictures, time lines and booklets with information on the history of linear measurement.

Note: These are placed with history materials.

Procedure:

1. Invite a child to the lesson. Place the container with the 12" strip and the separate 1" pieces before the child at the front of a table. Have the child sit on your dominant side.
2. Ask the child to lay out the the 12" strip, then place the twelve 1" segments in sequence on the long strip, beginning on the left.
3. Have the child remove the 1" segments and return them to the container.
4. By the three period lesson, teach the terms one foot and one inch, using the twelve-inch strip and a one-inch segment.
5. At the conclusion of the lesson, thank the child, say that lessons may be requested. Stand, replace chair and return materials to the shelf.

Control of Error:

Length of strip and component pieces

Number of component segments

Observations:

Handling of materials

Child's reaction to error

Length of work time and number of repetitions

Length of period of contemplation

Degree of interest and concentration

Variations:

Invite the child to use the material for foot and yard.
Give the three period lesson with material for feet and yard as with material for inches and foot.
Invite the child to use the materials for names, abbreviations and symbols for the previous materials.
Introduce materials for metric measurement in the same manner as for the U.S. Customary System.
Give three period lessons on the names of each measuring device.
Demonstrate the use of each measuring device.
Invite the child to use commands involving practical measurement exercises within the environment, using each of the measuring devices. (See white page of commands.)
Invite the child to solve the mathematics problems in measurement, for example,
1' 6" + 3' 9" =
Provide materials on historic measurement such as palm, span, cubit, hand, foot, pace.
Invite the child to use time lines on the history of measurement.

Vocabulary:

metric U.S. Customary System millimeter centimeter decimeter meter kilometer inch foot yard mile ruler yardstick meter stick tape measure folding rule measuring wheel

Measurement of Area

Purposes:

To learn the techniques of measurement of area
To apply measurement of area in the environment
To develop appropriate vocabulary

Preliminary Exercises:

Practice with linear measurement
Use of geometric cabinet

Materials:

Container of 1" squares made of white plastic or matte board
Container of command cards or booklet with drawing on back of card or page showing arrangement of squares and answer
Booklet for recording answers

Procedure:

1. Invite a child to the lesson. Place the container with the 1" squares at the center of a table and the container of command cards or booklet in front of the child. Have the child sit on your dominant side.
2. Ask the child to read a command, place the squares accordingly, count the squares and record the area in the booklet.
3. As soon as it is apparent that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Control drawing and answer on back of card or page

Observations:

Placement of squares
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to construct figures without referring to commands and find areas of each. Invite the child to use the metric system materials for finding area.

Vocabulary:

area

Measurement of Perimeter

Purposes:

To learn the techniques of measurement of perimeter
To apply measurement of perimeter in the environment
To develop appropriate vocabulary

Preliminary Exercises:

Practice with measurement of area
Use of geometric cabinet

Materials:

Container of calibrated figure cards with measurement of perimeter on reverse
Command cards for finding perimeter
Booklet for recording answers

Procedure:

1. Invite a child to the lesson. Place the container of figures at the center of a table in front of the child. Have the child sit on your dominant side.
2. Ask the child to choose any figure.
3. Starting at the arrow, show the child how to count the spaces between the calibrations along the edge of the figure. Have the child record the perimeter in the booklet.
4. As soon as it is apparent that the procedure is understood, leave and observe unobtrusively.

Control of Error:

Answer on back of card or page

Observations:

Counting of spaces, recording of perimeter
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to measure the perimeter of the straight-edged figures in the geometric cabinet and record the name of each figure and its perimeter.

Vocabulary:

perimeter

Gravimetric Measurement

Purposes:

To learn the techniques of gravimetric measurement
To develop understanding of appropriate gravimetric scales to be used in various situations
To apply gravimetric measurement in the environment
To develop appropriate vocabulary

Preliminary Exercises:

Work with mathematics materials to develop concept of number
Practice with sandpaper numerals for writing and reading numerals
Exercises with Seguin boards for 11 through 99
Use of baric tablets

Materials:

Two pan balance
Two 1-pound weights in a container; sixteen separate 1 ounce weights in a container; tour 1/4 ounce weights, two 1/2 ounce weights, four 4-ounce weights, two 8 ounce weights in a container (Avoirdupois weight)
One kilogram weight; one hundred 1-decagram weights in a container, ten 1-gram weights in a container (metric system)
White container of white control cards, mute cards and separate labels for abbreviations: ounce, oz.; pound, lb.; ton, t.
Tan container of tan control cards, mute cards and separate labels for abbreviations: milligram, mg; gram, g; kilogram, kg,
Other types of scales or drawings of them (balance scale, spring balance, pull-spring scale, triple beam balance)
Scales to take apart and reassemble
Time line, drawings and information booklets concerning the history of gravimetric systems in past civilizations (with history material)
Sealed transparent containers completely filled with materials such as rice, dried beans, plastic packing material, sand, feathers, etc.
Note: All containers must be identical in size and in weight before filling.
Command cards for activities involving use of various types of scales in the environment

Procedure:

1. Invite a child to the lesson, take a 1 pound weight and a 1 ounce weight to a table and teach the terms, pound and ounce, by the three period lesson.
2. Once the terms are learned, take the balance to a table and have the child bring the container of sixteen 1 ounce weights, then get the container of two 1 pound weights. The balance is placed in front of the child who is seated on your dominant side. The container of pound weights is placed before the pan on the left, the 1-ounce weights on the right.
3. Ask the child to place one of the 1 pound weights on the left pan and observe what happens. Tell the child to place the other 1 pound weight on the right pan and observe what happens.
4. Have the child return to the container the 1 pound weight from the right pan, then add 1 ounce weights, one at a time to the right pan so that the two pans balance each other. Leave as soon as it is observed that the procedure is understood.

Control of Error:

Level appearance of the pans of the balance

Observations:

Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to find other combinations of weights equivalent to one pound and to record these in a booklet.
Invite the child to weigh items in the environment which are of a size to fit on the balance pans.
Invite the child to weigh sealed transparent containers of materials.
Invite the child to use metric weights according to the previous procedures.
Invite the child to use the materials for learning abbreviations.
Invite the child to take apart scales available for such work.
Invite the child to use times lines and the accompanying bookiets on the history of gravimetric measurement.(With history materials)
Invite the child to use command cards for gravimetric exercises.

Vocabulary:

gravimetric ounce pound milligram gram kilogram
Avoirdupois weight metric system triplebalance scale spring balance
pull-spring scale beam balance names of parts of scales

Thermic Measurement

Purposes:

To learn the techniques of temperature measurement
To develop understanding of appropriate thermometers to be used in various situations
To apply temperature measurement in the environment
To develop appropriate vocabulary

Preliminary Exercises:

Work with mathematics materials to develop concept of number
Practice with sandpaper numerals for writing and reading numerals
Exercises with Seguin boards for 11 through 99
Use of thermic tubes and thermic tablets
Science experiments showing expansion and contraction of liquids and metals upon heating and cooling

Materials:

Liquid thermometer with Fahrenheit scale to measure air temperature
Liquid thermometer with Centigrade (Celsius) scale to measure air temperature
Metal thermometer with Fahrenheit scale to measure air temperature
Metal thermometer with Centigrade (Celsius) scale to measure air temperature
Liquid crystal thermometer for measuring body temperature
Digital electronic thermometer
Metal thermometer for food preparation
Booklets showing how each type of thermometer functions
Booklet on the development of the Fahrenheit scale
Booklet on the development of the Centigrade (Celsius) scale
Weather page from the daily newspaper
Time line and booklet on the history of thermometry.(with the history materials)
Booklets of science experiments which produce changes of temperature
Command cards for activities involving use of various types of thermometer

Procedure:

1. Invite a child to the lesson and go to the thermometer which should be mounted in the room in an accessible place.
2. Say the name of the scale {Fahrenheit, Centigrade or Celsius) and point out the lowest numeral as well as the highest on the scale. Show how to read the temperature. There should be more than one type of thermometer in the room with which to repeat the procedure.
3. Invite the child to read the appropriate booklets concerning the functioning of the thermometers introduced, if able to read phonograms, or you may read the information as the child examines the diagrams.
4. Thank the child and say that the thermometers may be read at any time.

Control of Error:

Equivalent temperature reading on each thermometer within the room

Observations:

Reading of thermometers
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Demonstrate the use of the other types of thermometers and invite the reading of appropriate booklets which explain the functioning of each type.

Invite the child to read the booklets on the development of the Fahrenheit scale and the Centigrade (Celsius) scale.

In history invite the child to use the time line and booklet on the history of thermometry.

Invite the child to do science experiments involving changes of temperature using directions in the booklet.

Invite the child to use command cards for activities involving use of various types of thermometer.

Invite the child to use booklets or cards with mathematics problems involving temperature.

Initiate as a class project the practice of recording room and outside temperatures first thing in the morning, midway of the session and at the end of class. Use the data to make a graph.

Invite each child who is interested to choose a foreign city, record its temperatures daily from the weather page of the newspaper, then compare with the local temperature. The latitude of each city should be noted at the beginning of the project.

Vocabulary:

thermometer thermometry Fahrenheit Centigrade
Celsius liquid crystal digital

Chronology

Purposes:

To provide a means for learning to tell time with clocks and calendars
To introduce the history of chronology
To give information on the functioning of various kinds of clocks
To arouse awareness of chronology in astronomy, geology, archaeology, history
To introduce concepts of many types of time: ephemeris, equinoctial, atomic, light year
To develop appropriate vocabulary

Preliminary Exercises:

Work with mathematics materials to develop concept of number
Practice with sandpaper numerals for writing and reading numerals
Exercises with Seguin boards for 11 through 99

Materials:

Teaching clock with synchronized hands which the child can move
Container with control and mute cards with separate labels, one for each clock hour and one for each half hour
Container with control and mute cards with separate labels for minutes
Container with control and mute cards with separate labels for half past, quarter past, quarter to
Note: The preceding is introduced only after fractions are understood.
Unlabeled calendar with containers of separate labels for year, month, day, numerals one through thirty-one.
Regular calendar with all the months of the year shown.
Blank paper calendar.
Note: See Language for materials for days and months labels.
Examples of time measuring devices or pictures such as hour glass, water clock, sun dial, etc., and directions for preparation of those devices when possible
Container of pictures of clocks as art forms
Time line for decade, century and millennium
Time line of calendars of ancient civilizations with information on the reckoning of the year for each (with history materials)
Time line depicting measurement of daily time from ancient cultures to present with information booklets (with history materials)
Time line and information booklets on geological eras and periods as a measure of time
Note: Times lines are covered in history.
Definition booklet for terms related to time such as daily, weekly, yearly, monthly, decade, century, millennium, centennial, bicentennial, sesquicentennial, semester, trimester, bimensal or bimenstrial
Clocks to take apart
Diagrams and labels for parts of the following clocks: tower, electric, quartz crystal, atomic, digital, pendulum, spring operated

Procedure for Telling Time:

1. Invite a child to the lesson and place the synchronized teaching clock in front of the child. Show how to turn the knob at the back to move the hands in the correct direction, and invite the child to practice turning.
2. Set the hands to show 12 o'clock and say, "12 o'clock," then set it to show 1 o'clock and say, "1 o'clock." Repeat this first period of the lesson several times.
3. For the second period of the lesson, use the mute cards for the two times being taught, in this example, 12 and 1. Say, "Show me 12 o'ciock." "Show me 1 o'clock.". Repeat the second period of the lesson several times, randomly using 12 and 1 o'clock.
4. For the third period of the lesson, set the clock for either 12 or 1 o'clock and ask the child to identify it. Randomly repeat the third period of the lesson.
5. Teach all of the hours by the three period lesson over a span of time according to the child's rate of learning. At the end of each session with the clock, thank the child and say that the lesson with the clock may be requested.

Procedure for Calendar:

1. Invite a child to the lesson and place the calendar materials at the front of a table with the control at the left, mute to its right and the containers of labels in front of the unlabeled calendar.
2. Ask the child to locate the current month on the control calendar and to place the labels on the unlabeled calendar appropriately.
3. Invite the child to copy the month, year, days, and dates on the blank paper calendar by referring to the calendar on which labels have been placed.

Control of Error:

The teacher
Calendar control

Observations:

Handling, placement, and copying of calendar materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to use control and mute cards with separate labels, one for each clock hour, followed by control and mute cards with separate labels for minutes, for half past, for quarter after, tor quarter to. Match half past cards with the 30 minutes past cards.

Invite the child to prepare a calendar with each month of any chosen year.

Invite the child to take apart clocks and reassemble them.

Invite the child to examine examples or pictures of time measuring devices such as hour glass, water clock, etc.

Invite the child to examine diagrams which show parts and function of each for the tower clock, electric clock, quartz crystal clock, atomic clock, digital clock, pendulum clock, spring operated clock.

Invite the child to look at pictures of clocks as art forms.

Invite the child to use the time line of calendars of ancient civilizations and to readnformation on the reckoning of the year for each. (in history)

Invite the child to use the time line depicting measurement of daily time from ancient cultures to present and to read information booklets. (in history)

Invite the child to use the time line and to read information booklet on geological eras and periods as a measurement of time. (in history)

Make candle clocks, water clocks and a sundial (Related to experimental science)

Vocabulary:

hand clock hour minute half past quarter past quarter to o'clock hour glass water clock tower clock electric clock quartz crystal clock atomic clock digital pendulum clock spring operated clock calendar daily astronomy clock rotation revolution solar time weekly yearly chronology decade century millennium centennial monthly twenty-four hour semester trimester bimensal bimenstrial clock bicentennial sesquicentennial

Geometry

Advanced Work with the Geometric Cabinet

Purposes:

To provide written labels for the geometric shapes previously learned by the three period lesson
To give written definitions tor the concepts learned through the sensorial use to the geometric cabinet
To reinforce reading skills
To develop appropriate reading vocabulary

Preliminary Exercises:

Practice feeling all the shapes in the geometric cabinet
Use of phonograms in preparation for reading labels and definitions

Materials:

Geometric cabinet
White container of labels for each shape in the geometric cabinet
White container of definitions tor each shape in the geometric cabinet
Container of white control cards with a label and its definition for each shape
White container of commands involving the use of the geometric cabinet

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child place any drawer from the geometric cabinet on a table with the corresponding container(s) of labels, definitions and controls on the dominant side.
2. Ask the child to remove the figures from the drawer and place on the table to the right.
3. Ask the child to place appropriate labels in front of each shape.
4. Have the child read the definitions and lay them under the label, referring to the control if necessary.
5. If the child is observed to understand the procedure, say that any drawer may be chosen, leave and observe unobtrusively.

Control of Error:

Control cards

Observations:

Handling of materials
Placement of labels and definitions
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to lay out the labels and definitions without the geometric cabinet drawers and controls.

Invite the child to make a booklet with the drawing of the shape on the left page and the label and definition on the right page.

Invite the child to use the command cards with the geometric cabinet.

Vocabulary

closed plane closed curve component

Advanced Work with Constructive Triangles

Purposes:

To provide further activities leading to the understanding of geometry
To reinforce reading skills
To develop appropriate reading vocabulary

Preliminary Exercises:

Beginning work with constructive triangles
Use of phonograms in preparation for reading labels and commands

Materials:

White container of commands tor constructive triangles with diagram or answers on the reverse to act as controls
Constructive triangles

Procedure:

1. Invite a child who can read to the lesson.
2. Ask the child to place any box of constructive triangles toward the back of a table as for previous exercises with the triangles.
3. Have the child place, on the dominant side, the container of commands for the constructive triangle box just chosen.
4. Invite the child to read a command and to carry out the action. Once a command is completed, the card is turned face down behind the container. Leave as soon as it is observed that the procedure is understood and observe unobtrusively.

Control of Error:

Diagram or answer on reverse of command card

Observations:

Appearance of figures formed with the constructive triangles
Checking of work against the control on reverse of command
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

None

Vocabulary:

congruent segment protractor compass

Geometric Solids

Purposes:

To extend the study of geometry from sensorial to more abstract levels
To develop understanding for future work in geometry
To provide the basis for geometric calculations
To develop appropriate vocabulary

Preliminary Exercises:

Work with pairing and naming geometric solids
Practice with phonogram reading materials

Materials:

Geometric solids used in sensorial exercises
Container of control drawings of geometric solids; mute drawings and labels
Container of labels for each classification: regular solids; straight edged semi-regular solids; flat based, curved surface semi-regular solids; curved surface solids
Separate booklet with drawings and definitions for each classification of solids
Container of definition cards for each solid

First Procedure:

1. Invite a child to the lesson. Take the container with one of each pair of solids, the matching drawings and a felt mat to the table. Have the child sit on your dominant side.
2. Indicate that the child is to place the container of solids near the back of the table and to spread the mat near the front of the table with the container of drawings on the child's dominant side. Tell the child to remove the drawings and place in a stack in front of the container.
3. Ask the child to place the geometric solids in a row at the back of the mat, then to put one drawing at a time in front of its matching appropriate geometric solid.
4. Leave as soon as it is apparent that the procedure is understood and observe unobtrusively.

Second Procedure:

1. Invite a child to the lesson. Take the container with one of each pair of geometric solids, the labels and a felt mat to a table. Have the child sit on your dominant side.
2. Indicate that the child is to place the container of solids near the back of the table and to spread the mat near the front of the table with the container of labels on the child's dominant side.
3. Tell the child to read a label, lay it on the mat at the left center, then find the appropriate solid to place above the label.
4. Have the child read the next label, lay it to the right of the one previously placed, then find the appropriate solid and place it above its label.
5. Leave as soon as it is apparent that the procedure is understood and observe unobtrusively.

Control of Error:

Child's previous knowledge of the names of the solids
Control cards for drawings of geometric solids
Definition booklet

Observations:

Placement of drawings with solids Placement
of labels and solids
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to classify the solids, using the booklets as a control, by placing the classification label at the top of the mat, then arranging the solids and their labels appropriately in front of the classification.

Invite the child to place definition cards in front of the solids, then to use the booklet for checking the work.

Vocabulary:

regular solids straight-edged, semi-regular solids curved-surface solids
flat based, curved surface, semi-regular solids

The Equation Of The Square Of The Binomial

Purposes:

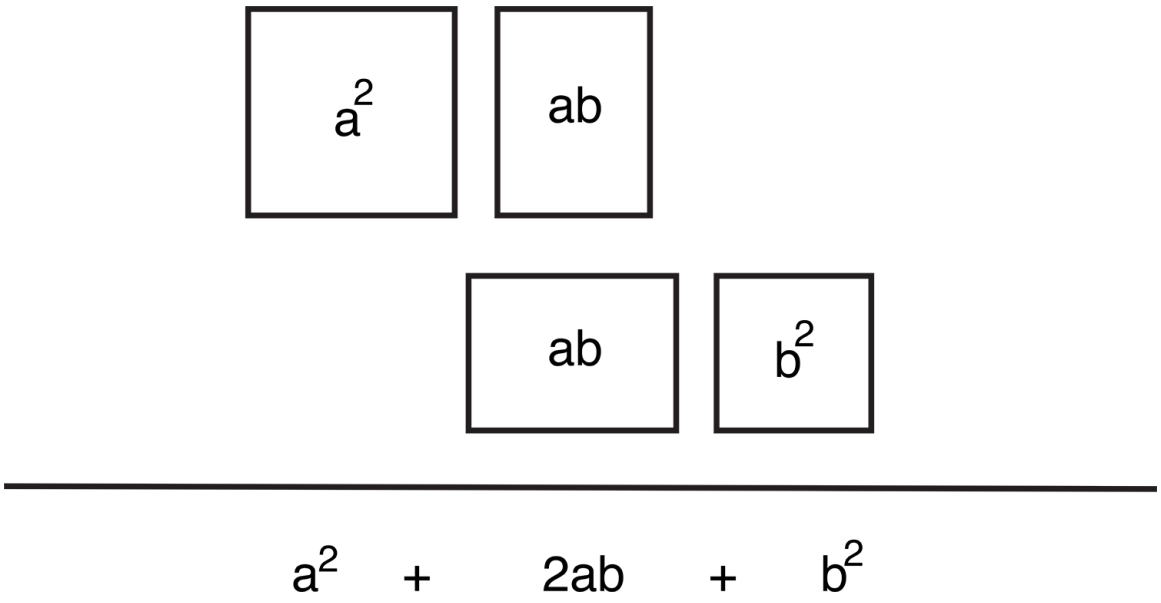
To assist in the perception of mathematical patterns
To prepare for operations with algebraic equations
To develop understanding of the expansion of the binomial square equation
To develop appropriate vocabulary

Preliminary Exercises:

Binomial square used sensorially

Materials:

Container with red square, blue square and two black rectangles which exactly match the faces of the binomial cube material
Unlabeled control chart for layout of the equation of the square of the binomial
Labeled control chart for the layout of the equation of the square of the binomial with each component labeled and the equation written across the bottom under the appropriate forms: $a^2 + 2ab + b^2$
Container of separate labels for the figures which compose the square of the binomial to show that the red square is a^2, the blue square is b^2, and the two black rectangles are ab, and the equation of the binomial square cut into pieces to match the labeled chart
Note: Do not attempt to make these materials until after the lesson on preparation of material has been completed.



Procedure:

1. Invite a child to the lesson. Place the binomial square material on a table between you and the child who is seated on your dominant side.
2. Place the unlabeled control chart to the right of the container of binomial square materials. The labeled control chart and the container of labels remain out of the way on your left side.
3. Ask the child to remove the squares and rectangles from the container and place in a row in front of the unlabeled chart.
4. Have the child place the red square on the unlabeled control, then a black rectangle to the right of the red square, followed by the second black rectangle below the first rectangle and the blue square to the right of the second black rectangle.
5. Place the labeled control chart *above* the unlabeled chart on which the squares and rectangles have been placed. Place the container of labels in front of the child. Using the labeled chart as a control, tell the child to place the labels on the squares and rectangles on the unlabeled chart.
6. Ask the child to place the components of the equation in order across the bottom of the unlabeled chart to match the labeled chart.
7. Return all materials to appropriate containers.
8. Stand, replace chair and return materials to the shelf.
9. Thank the child and say that the binomial square equation chart may be chosen.

Control of Error:

Control charts
Color and shape of components

Observations:

Handling of materials
Sequence of placement
Placement of components on the control chart
Placement of labels on components
Placement of components of the equation on the chart
Child's reaction to error
Length of work time and number of repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to build the equation of the square of the binomial without the control chart.
Invite the child to substitute number values for a and b and work out the equation.

Vocabulary

binomial square rectangle component control chart
a^2 b^2 ab

The Equation of the Square of the Trinomial

Purposes:

To assist in the perception of mathematical patterns
To prepare for operations with algebraic equations
To develop understanding of the expansion of the trinomial square
To develop appropriate vocabulary

Preliminary Exercises:

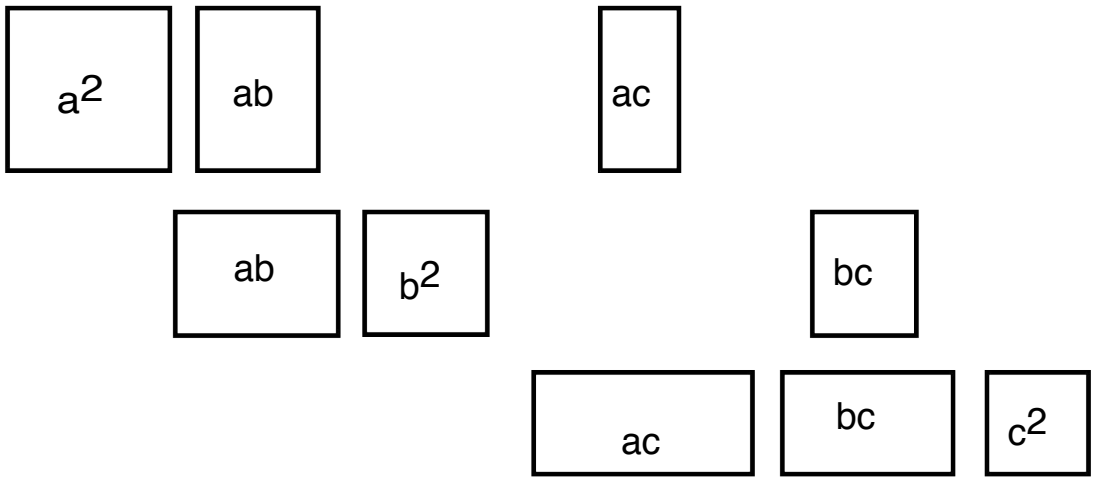
Geometric cabinet and cards
Constructive triangles
Square of the trinomial
Equation of the square of the binomial

Materials:

Container with red square, blue square, yellow square and six black rectangles which exactly match the faces of the trinomial cube material
Unlabeled control chart for layout of the equation of the square of the trinomial
Labeled control chart for the layout of the equation of the square of the trinomial with each component labeled and the equation written across the bottom under the appropriate forms: $a^2 + 2ab + 2ac + b^2 + 2bc + c^2$
Container of separate labels for the figures which compose the square of the trinomial to show that the red square is a^2, the blue square is b^2, the yellow square is c^2, the two large black rectangles are ab, the two medium black rectangles are ac and the two small black rectangles are be
The equation of the trinomial square cut into pieces to match the labeled chart Note:
Do not attempt to make these materials until after the lesson on preparation of material has been completed.

Procedure:

1. Invite a child to the lesson. Place the trinomial square material on a table between you and the child who is seated on your dominant side.
2. Place the unlabeled control chart to the right of the container of trinomial square materials. The labeled control chart and the container of labels remain out of the way on your left side.
3. Ask the child to remove the squares and rectangles from the container and place in a row in front of the unlabeled chart.
4. Tell the child that the procedure for the equation of the trinomial square is the same as for the equation of the binomial square except that there are more pieces. Indicate that each row is built on the control chart from left to right.



$$a^2 + 2ab + b^2 + 2ac + 2bc + c^2$$

Control of Error:

Control charts
Color and shape of components

Observations:

Handling of materials
Sequence of placement
Placement of components on the control chart
Placement of labels on components
Placement of components of the equation on the chart 2
Child's reaction to error
Length of work time and number of repetitions Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to build the equation of the square of the trinomial without the control chart.
Invite the child to substitute number values for a, b and c and work out the equation.

Vocabulary

trinomial

The Equation of the Cube of the Binomial

Purposes:

To assist in the perception of mathematical patterns
To prepare for operations with algebraic equations
To develop understanding of the expansion of the binomial cube equation
To develop appropriate vocabulary

Preliminary Exercises:

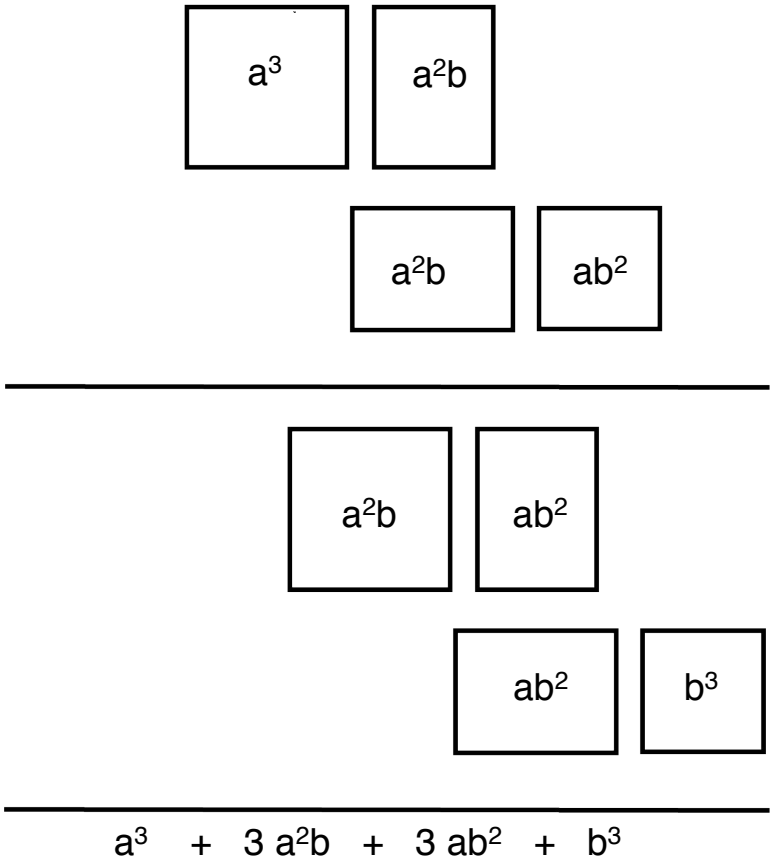
Use of the binomial cube
Practice with the materials for the equation of the square of the binomial

Materials:

Lidded box hinged at the bottom of two sides, containing components of the binomial
Note: Dimensions of the box and its contents vary among different manufacturers. Dimensions given by Montessori were 6 cm for the red cube and 4 cm for the blue so that a cube of 10 cm is formed.
Unlabeled chart for the layout of the equation of the cube of the binomial
Control chart for the layout of the equation of the cube of the binomial with each component labeled and the equation written across the bottom under the appropriate forms: $a^3 + 3a^2 b + 3ab2 + b^3$
Container of separate labels for the components which compose the cube of the binomial, and the equation of the binomial cube cut into component parts to match the labeled chart
Note: Do not attempt to make these materials until after the lesson on preparation of material has been completed.
Two light colored felt mats large enough to accommodate the cube and prisms of each layer.

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child take the binomial cube, felt mats, and the charts to a table, then sit on your dominant side.
2. Tell the child to place the box, unlabeled control chart and felt mats according to the procedure followed in the sensorial exercise.
3. Ask the child to open the box and place the components of the binomial cube on the unlabeled chart as previously learned sensorially.
4. With the labeled control chart above the unlabeled chart, have the child place the separate labels on the components which have been placed on the unlabeled chart.
5. Ask the child to place the components of the equation in order across the bottom of the unlabeled chart to match the labeled chart.
6. Tell the child to return the materials to the shelf when the work is finished, and say that the binomial cube equation chart may be chosen.



$$a^3 + 3\,a^2b + 3\,ab^2 + b^3$$

Control of Error:

Control charts
Lid of box
Height of assembled cube and prisms on each level
Colors of cubes and prisms

Observations:

Placement of charts., box and lid
Placement of cubes and prisms on felt mats and on the unlabeled chart
Placement of labels on cubes and prisms
Placement of components of the equation on the chart
Return of materials
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

Variations:

Invite the child to place the labels on the cubes and prisms without referring to the labeled control chart.
Invite the child to substitute number values for a and b and work out the equation.

Vocabulary:

binomial	algebraic terms for binomial expansion

The Equation of the Cube of the Trinomial

Purposes:

To assist in the perception of mathematical patterns
To prepare for operations with algebraic equations
To develop understanding of the expansion of the binomial cube equation
To develop appropriate vocabulary

Preliminary Exercises:

Use of the equation of the binomial cube with control chart
Practice with materials for the equation of the trinomial square

Materials:

Lidded box hinged at the bottom of two sides, containing components of the binomial
Unlabeled chart for the layout of the components of the cube of the trinomial labeled and the equation written across the bottom under the appropriate components:

$$a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$$

Separate labels for the components of the cube of the trinomial, and the equation of the trinomial cube cut into pieces to match the labeled chart
Note: Do not attempt to make materials until after preparation of material lesson. Three light colored felt mats large enough for the pieces of the trinomial cube

Procedure:

1. Invite a child to the lesson once the preliminary exercises have been done. Have the child take the trinomial cube, felt mats, and the charts to a table, then sit on your dominant side.
2. Tell the child to place the box, unlabeled control chart and felt mats according to the procedure followed in the sensorial exercise.
3. Ask the child to open the box and place the components of the trinomial cube on the unlabeled chart as previously learned sensorially.
4. With the labeled control chart above the unlabeled chart, have the child place the separate labels on the components which have been placed on the unlabeled chart.
5. Ask the child to place the components of the equation in order across the bottom of the unlabeled chart to match the labeled chart.
6. Tell the child to return the materials to the shelf when the work is finished, and say that the trinomial cube equation chart may be chosen.

Control of Error:

Control charts
Lid of box
Height of assembled cubes and prisms on each level
Color of cubes and prisms

Observations:

Placement of control charts, box and lid
Placement of cubes and prisms on felt mat and on control chart
Return of material to lid, then into box
Placement of labels on cubes and prisms
Placement of parts of the equation on the chart
Handling of materials
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration
Variations:
Invite the child to place the labels on the cubes and prisms without referring to the labeled control chart.
Invite the child to substitute number values for a, b and c and work out the equation.

Vocabulary:

trinomial	algebraic terms for trinomial expansion

$a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^2$

Labels for the Powers of Two

Purposes:

To provide a sensorial basis for understanding exponential concepts
To assist in the perception of mathematical patterns
To give indirect preparation for algebra
To show relationships among geometric solids
To develop appropriate vocabulary

Preliminary Exercises:

Geometric cabinet and cards
Stereognostic exercises with geometric solids
Learning names of geometric solids

Materials:

Box with two, 2 centimeter yellow cubes; a white oblong prism, 2 by 2 by 4 centimeters; a green square prism, 4 by 4 by 2; a yellow 4 centimeter cube; a white oblong prism, 4 by 4 by 8; a green square prism, 8 by 8 by 4, all representing 23 through 2^9.

Note: To facilitate removal of the large green prism, a ribbon 3/4" wide by 8" long is attached to the center of the box near the hinge with the ribbon running under the green prism to the front of the box. By lifting the end of the ribbon, which extends at the front, the green prism can be removed easily.

Control chart labeled with the values of the cubes and prisms in ascending order
Container with separate labels: 23, 23, 24, 2s, 26, 27, 2s, 2^9
Felt mat large enough to accommodate the cubes and prisms

Procedure:

1. Invite the child to the lesson. Have the child take the power of two box, charts, labels and felt mat to a table and sit on your dominant side.
2. Have the child place power of two box toward the left center of the table with the unlabeled control chart to the right of the box and the labeled chart behind the unlabeled chart. Place felt mat directly in front of the box and the labels in front of the felt mat.
3. Ask the child to open the box, remove the prisms and place them on the felt mat as in in the sensorial presentation.
4. Beginning with the small yellow cubes, have the child transfer the cubes and prisms to the unlabeled control chart.
5. Using the labeled control chart as a reference, tell the child to place the labels on the cubes and prisms of the power of two box, except for 2^9.
6. Ask the child to return the labels to their container, except for 2^9.
7. Have the child place the cubes and prisms in their box as previously learned and place the label 2^9 on top of the box. Return label to container.
8. Stand, replace chair and return the materials to the shelf.
9. Thank the child and say the the labels for the powers of two may be chosen.

Control of Error:

Control charts
Color of cubes and prisms

Observations:

Placement of control charts, box and felt mat
Placement of cubes and prisms on felt mat and on control chart
Placement of labels on cubes and prisms
Handling of material
Child's reaction to error
Length of work time and number of repetitions
Length of period of contemplation
Degree of interest and concentration

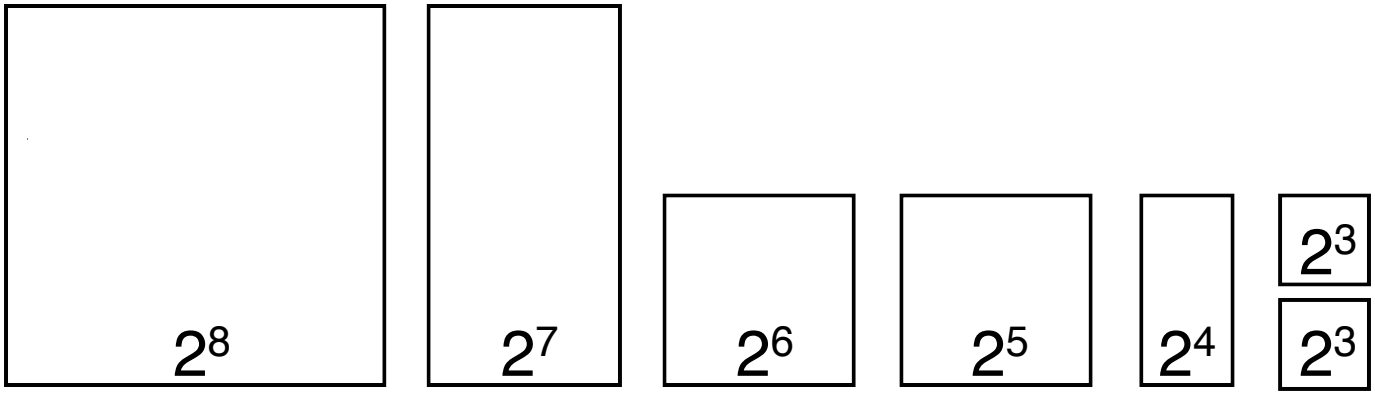
Variations:

Invite the child to place the labels on the cubes and prisms without referring to the labeled control chart.

Invite the child to solve for the values of the powers of two.

Vocabulary:

no new vocabulary



Materials Preparation

Preparation of "Sandpaper" Numerals

1. Cut nine pieces of green matte board to a uniform size 6" by 71/2" OR nine pieces of tempered Masonite 6" by 71/2". (If Masonite is used, sand the corners to be rounded instead of sharp, wipe the surface with paint thinner or any organic solvent to remove the oily finish, spray paint green.)
2. Cut piece of blue matte board 7" by 71/2" OR tempered Masonite 7" by 71/2". (If Masonite is used, prepare as above and spray paint blue.)
   Note: Some suppliers of Masonite will cut it to size upon request.
3. Turn the board so that the shorter side is parallel to the front edge of the table. Measure 1 inch from the bottom of each board and draw a straight line parallel to the bottom edge, first in pencil, then with black marker or paint pen. The line must be straight and accurately drawn, so practice on other material first and use the ruler or a straight edge to produce a clean line no thicker than 1/8 inch. Allow to dry completely.
4. Using the patterns supplied in the "White Pages" trace the numerals on the reverse of fine grade sandpaper or gray anti-slip strip, being sure to reverse the letter pattern as well. Arrange the numerals to the best advantage for space use.
   Note: The anti-slip strip or stair tread is available in hardware stores. It is made by 3-M and has an adhesive back.
5. Cut out the numerals.
6. Measure the line drawn on the board and mark the center with a pencil.
7. Peel the backing from a cut-out numeral or apply glue to the back of a sandpaper numeral, then carefully place in correct orientation to the line, centering the numeral on the mark.
8. With marker or pen, make a small dot to show where to begin feeling the numeral. Use white if the numerals are made of dark material, black if made of light material.

Preparation of Control Chart for Values of Denominations

1. Make two copies on white paper from the "White Pages" with pictures of denominations and mount on white cover stock. Cut one page into separate pictures. Laminate the control and the separate pictures.
2. Make two copies on cover stock from the white page with words and numerals for the value of each denomination. Cut one into eighteen separate labels 3/4" high and 2 3/4" wide and mount on white cover stock. Laminate the control and the separate labels. Place in a white container.

Preparation of Control Chart for Currency

1. Cut white matte board or canvas board approximately 12" by 17". Obtain two sets of each of the following: one dollar bill, two half dollars, four quarters, ten dimes, twenty nickels, one hundred pennies
   Note: Imitation coins may be used provided that they are the same size, color and design as real coins.
2. Place the matte or canvas board so that the greatest dimension parallels the front edge of the table.
3. Lay out the coins to determine the spacing with the following arrangement: one dollar bill at the top center; two half-dollars on second row arranged one fourth of the distance from each side; four quarters on the third row arranged so that two quarters are spaced under each half-dollar; ten dimes on the fourth row arranged to show that there are five dimes in a half-dollar; twenty nickels on the fifth row arranged in a row to show that there are two nickels in a dime; one hundred pennies arranged in columns of five to show that there are five in each nickel. Once the spacing has been accurately determined, make light pencil marks to maintain the spacing, then glue the coins in place, starting with the pennies.
4. Laminate the front of the board to keep coins from being removed, allowing enough laminating material to wrap around all four edges of the board.
5. Place the other set of coins in their compartments in the white coin container with six compartments sized to hold the number of coins in each denomination.

Preparation of Materials for Measurement Abbreviations

1. Make two copies of words and abbreviations for measurement and cut each to 5/8" by 3 5/8" with equal margins on each end.
2. Cut 28 strips of white cover stock 2" by 5 1/4".
3. Mount each work with its abbreviation on tan if related to the metric system and mount the remaining words and abbreviations on white.
4. Cut one of each pair 3 1/2" from the left edge to separate the word from its abbreviation.
5. Separate cards and place in labeled containers as follows:

Volumetric Measurement: teaspoon, tablespoon cup, pint, quart, gallon container	white
Volumetric Measurement: milliliter, cubic centimeter, centiliter, liter container	tan
Linear Measurement: inch, foot, yard, mile container	white
Linear Measurement: millimeter, centimeter, decimeter, meter, kilometer container	tan
Gravimetric Measurement: ounce, pound, ton container	white
Gravimetric Measurement: milligram, gram, kilogram container	tan

Thermic Measurement: Fahrenheit container	white
Thermic Measurement: Celsius container	tan

Preparation of Materials for Quantitative Measurement

Set 1

1. Cut eight pieces of gold cover stock 5 1/4" by 5 1/4".
2. Mount pictures illustrating pair, couple, dozen, baker's dozen, few, many, grass, ream on white paper 4 1/4" by 4 1/4".
3. Mount pictures on white paper on gold cover stock with a 1/2" border on all sides.
4. Place in a gold container labeled "Quantitative Measurement Set 1."

Set 2

1. Follow the same procedure as above using the following pictures: scissors, pliers, glasses, trousers, earrings, gloves, shoes, socks.
2. Place in a gold container labeled "Quantitative Measurement Set 2."

Preparation of Cards and Labels for Geometric Figures

1. Make one copy of definitions and labels. Copy the geometric cards with thin outlines in a uniform reduced size to obtain the plain figures needed.
2. Cut cards with plain figures 4 1/4" by 4 1/4".
3. Cut definitions 2 1/2" by 4 1/4".
4. Cut labels 1" high. Because of the format of labels on the white pages, each is to be cut with a 1/2" space on either end of the word or words, but is mounted on standard size cover stock, as given in step 5.
5. Mount on white cover stock each definition on 3 1/2" by 5 1/4", each plain figure on 6 3/4" by 5 1/4" and each label on 2" by 5 1/4".
6. Laminate each mounted label, definition and plain figure.

Preparation of Booklets for Geometric Figures

1. Make one copy of definitions and labels. Copy the geometric cards with thin outlines in a uniform reduced size to obtain the plain figures needed.
2. Make booklet cover from white cover stock 11" by 7" , folded to 5 1/2" by 7".
3. Label the cover of the booklet Plane Geometric Figures and laminate the cover.
4. Cut ten pieces of white paper 10 1/2" by 6 1/2" and fold to 5 1/4" by 6 1/2". Arrange the folded pages for each booklet temporarily to ensure correct placement of figures with their appropriate definitions.
5. Mount the definition on the right-hand page with the corresponding figure facing it on the left-hand page. Mount the label under the the drawing.
6. Place the prepared pages in their labeled cover and sew together as previously instructed.

Preparation of Cards and Labels for Geometric Solids

1. Make one copy of definitions and labels and one copy of the geometric solids drawings.
2. Cut drawings 4 1/4" by 4 1/4".
3. Cut definitions 2 1/2" by 4 1/4".
4. Cut labels 1" high. Because of the format of labels on the white pages, each is to be cut with a 1/2" space on either end of the word or words, but is mounted on standard size cover stock, as given in step 5.
5. Mount on white cover stock each definition on 3 1/2" by 5 1/4", each drawing on 6 3/4" by 5 1/4" and each label on 2" by 5 1/4".
6. Laminate each mounted label, definition and drawing.

Preparation of Booklets for Geometric Solids

1. Make one copy of definitions and labels and one copy of the geometric solids drawings.
2. There will be four booklets, one for each classification of solids, so make four booklet covers from white cover stock, each 11" by 7" , folded to 5 1/2" by 7".
3. Mount a copy of a label on the cover of each booklet according to the classification to be contained in it. Laminate the cover.
4. Cut white paper 10 1/2" by 6 1/2" and fold to 5 1/4" by 6 1/2" as follows: seven pieces, Regular Solids; six pieces, Semi-regular Solids with Straight Edges; four pieces, Semi-regular Solids with Straight Edges and Curved Surfaces; six pieces,Curved Surface Solids. Arrange the folded pages for each booklet temporarily to ensure correct placement of drawings with their appropriate definitions.
5. Mount the definition on the right-hand page with the corresponding drawing facing it on the left-hand page. Mount the label under the the drawing.
6. Place the prepared pages in their labeled cover and sew together as previously instructed.

Book Preparation for Books with Spines

1. If there are more than ten pages, count the number of pictures to determine how many pages will be need. Then divide into groups of no more than six pages. Fold the paper to make groups of pages with not more than six pages per group. Determine the sequence of drawings and information. Mount on the pages appropriately and laminate both sides of each page. Reestablish sequence of pages in each group and the sequence of groups.
2. Place the groups of pages together and measure the thickness at the folded edge to determine the width of the spine.
3. Cut a front and a back cover 7" by 51/2", adding on to the 51/2"" dimension the width of the spine which is folded so that both covers are now 7 1/2" by 5 1/2".
4. Label the front cover. Overlap the two folded edges and glue together to form a spine.
5. After the glue has dried laminate both sides of the cover.
6. Refold to form the spine again.
7. Sew the first group of folded pages into the spine near the left side of the spine. Repeat with other groups of pages, keeping the groups in sequence. There will be parallel line of stitches showing alone the spine.

Preparation of Materials for Terms Related to Time

1. Make two copies on white cover stock of the terms related to time.
2. Laminate one page for the control.
3. Semester, trimester and bimensal or bimestrial are cut into cards 1" x 2'. The number of month are cut into cards 1 112" x 2' and fractions of years are each cut into cards 1" x 2'. Mount on white cover stock which has been cut to 1 112" and laminate. Definitions for the preceding are cut into cards 1" x 2 3/4". Mount on white cover stock which has been cut to 1 112" x 3 1/4".