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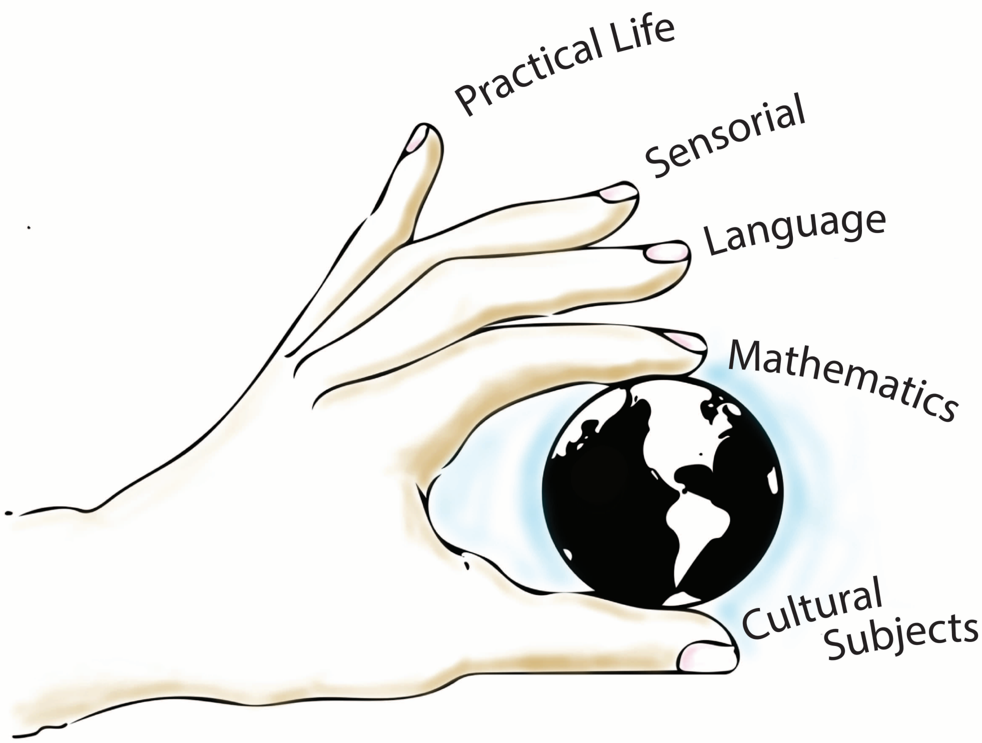

Mathematics

Elementary

Lesson Preparation Materials

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MEI, Inc Mathematics Elementary

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1\. 7958
2\. 6796
3\. 5555
4\. 5555
5\. 6958

3\. 1234
\+ 4321

4\. 5230
\+ 325

5\. 4806
\+ 2152

2\. 2351
\+ 4445

1. 3725
   2133
+ 2412

1. 8270
2. 4206
3. 6215
4. 7057
5. 7661

3. 1874
+ 4341

4. 5730
+ 1327

5. 4806
+ 2855

2. 2361
+ 1845

1. 6359
- 4126

1. 2233
2. 5233
3. 1110
4. 3133
5. 1013

3. 3751
- 2641

4. 3589
- 456

5. 5326
- 4313

2. 8235
- 3002

1. 1391
2. 4587
3. 2943
4. 1109
5. 253

1.
```
   4632
-  3241
```

3.
```
   4867
-  1924
```

4.
```
   5555
-  4446
```

5.
```
   925
-  672
```

2.
```
   7821
-  3234
```

1. 3699
2. 6060
3. 8264
4. 9969
5. 8642

$$\begin{array}{r} 1233 \\ \times \quad 3 \\ \hline \end{array}$$
(1.)

3.
$$\begin{array}{r} 4132 \\ \times \quad 2 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 3323 \\ \times \quad 3 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 4321 \\ \times \quad 2 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 3030 \\ \times \quad 2 \\ \hline \end{array}$$

1. 2432
x 3

1. 7296
2. 2528
3. 252
4. 1539
5. 8506

3. 63
x 3

4. 513
x 3

5. 4253
x 2

2. 1264
x 2

1. $4\overline{)8084}$

1. 2021
2. 4321
3. 1322
4. 3312
5. 4021

3. $3\overline{)\,3966}$

4. $3\overline{)\,9936}$

5. $2\overline{)\,8042}$

2. $2\overline{)\,8642}$

1. $4\overline{)5684}$

1. 1421
2. 3216
3. 2227
4. 1130
5. 2352

3. $3\overline{)6681}$

4. $5\overline{)5650}$

5. $4\overline{)9408}$

2. $3\overline{)9648}$

1. 341
x 10

1. 3410
2. 2260
3. 4110
4. 3620
5. 1530

3. 411
x 10

4. 362
x 10

5. 153
x 10

2. 226
x 10

1. 6690
2. 8260
3. 8560
4. 1610
5. 7350

1.
```
  223
x  30
-----
```

3.
```
  214
x  40
-----
```

4.
```
   23
x  70
-----
```

5.
```
  245
x  30
-----
```

2.
```
  413
x  20
-----
```

1. 232
x 42

1. 9744
2. 6413
3. 5400
4. 4862
5. 7356

3. 225
x 24

4. 143
x 34

5. 613
x 12

2. 121
x 53

1. 13
x 100

1. 1300
2. 2700
3. 3200
4. 1400
5. 2100

3. 32
x 100

4. 14
x 100

5. 21
x 100

2. 27
x 100

1. 6900
2. 2800
3. 6400
4. 3900
5. 4800

1. 23
x 300

3. 32
x 200

4. 13
x 300

5. 24
x 200

2. 14
x 200

1. 13
x 221

1. 2873
2. 8424
3. 7776
4. 7168
5. 9051

3. 32
x 243

4. 14
x 512

5. 21
x 431

2. 27
x 312

1. 247
x 324

1. 80,028
2. 170,768
3. 131,580
4. 365,442
5. 312,094

3. 340
x 387

4. 1,243
x 294

5. 817
x 382

2. 821
x 208

1. 36
x 23

1. 828
2. 448
3. 504
4. 546
5. 713

3. 21
x 24

4. 13
x 42

5. 31
x 23

2. 14
x 32

1. 111
2. 211
3. 314
4. 303
5. 312

1. $26\overline{)2886}$

3. $12\overline{)3768}$

4. $31\overline{)9393}$

5. $31\overline{)9672}$

2. $41\overline{)8651}$

1. 22
2. 12
3. 32
4. 21
5. 42

1. $122\overline{)\,2684}$

3. $231\overline{)\,7392}$

4. $121\overline{)\,2541}$

5. $211\overline{)\,8862}$

2. $123\overline{)\,1476}$

2	2	2	2	3	3	3	3
5	5	5	7	7	7	11	11
37	31	29	23	19	17	13	13
41	43	47	53	59	61	67	71
			97	89	83	79	73

1. 2,2,2,2,2,3,5

2. 2,3,3,5,7

3. 2,2,3,5,5,11

4. 3,5,7,7,11

5. 2,2,2,2,5,5,5

1. 480 =

3. 3300 =

4. 8085 =

5. 2000 =

2. 630 =

1. 45

2. 77

3. 33

4. 54

5. 50

1. The HCF of 315 and 270 =

3. The HCF of 66 and 231 =

4. The HCF of 54 and 270 =

5. The HCF of 150 and 350 =

2. The HCF of 1155 and 154 =

1. 864

2. 330

3. 924

4. 54

5. 450

1. The LCM of 96 and 27 =

3. The LCM of 84 and 132=

4. The LCM of 54 and 18=

5. The LCM of 50 and 45=

2. The LCM of 66 and 165=

Mathematical Properties

7 + -7 = 0

-a + a = 0

Property of Opposites

Property of Opposites

$(m + n) + y = m + (n + y)$

$(4 + 3) + 7 = 4 + (3 + 7)$

$(f \times g) \times h = f \times (g \times h)$

$(8 \times 3) \times 2 = 8 \times (3 \times 2)$

Associative Property
of Addition

Associative Property
of Addition

Associative Property
of Multiplication

Associative Property
of Multiplication

$8 + 5 = 5 + 8$

$m + n = n + m$

$m \times n = n \times m$

$44 \text{ X } 3 = 3 \text{ X } 44$

Commutative Property of Addition

Commutative Property of Addition

Commutative Property of Multiplication

Commutative Property of Multiplication

$n \times 1 = n$

$32 \times 1 = 32$

$a + 0 = a$

$7,230 + 0 = 7,230$

Multiplicative Property of One or Identity Property of One

Multiplicative Property of One or Identity Property of One

Additive Property of Zero or Identity Property of Zero

Additive Property of Zero or Identity Property of Zero

$$m \times (n + p) = (m \times n) + (m \times p)$$

$$2 \times (3 + 4) = (2 \times 3) + (2 \times 4)$$

$$m \times (n - p) = (m \times n) - (m \times p)$$

$$6 \times (8 - 2) = (6 \times 8) - (6 \times 2)$$

Distributive Property of
Multiplication over Addition

Distributive Property of
Multiplication over Addition

Distributive Property of
Multiplication over Subtraction

Distributive Property of
Multiplication over Subtraction

Mathematical Properties

A number and its opposite equal zero. 1+ -1 = 0

-6 = 6 = 0

When the grouping of addends is changed, the sum is the same.

(2 + 3) + 4 = 2 + (3 + 4)

When the grouping of factors is changed, the product is the same.

(1 x 2) x 3 = 1 x (2 x 3)

(a x b) x c=a x (b x c)

When the order of addends is changed, the sum is the same.

1 + 2 = 2 + 1

a + b = b + a

When the order of factors is changed, the product is the same

1 x 2 = 2 x 1

a + b = b + a

Any quantity multiplied by one is the same quantity.

2 x 1 = 2
a x 1 = a

The sum of any quantity and zero is the same quantity

2 + 0 = 2
a + 0 = a

The sum of two or more quantities may be multiplied by another quantity, or each addend may be multiplied by a common quantity, then the products added.

1 x (2 + 3) = (1 x 2) + (1 x 3) ax
(b + c) = (a x b) + (a x c)

The difference between two quantities may be multiplied by another quantity, or each quantity may be multiplied by a common quantity, then the products subtracted.

1 x (3 - 2) = (1 x 3) - (1 x 2)
a x (b - c) = (a x b) - (a x c)

Property of Opposites

Associative Property of Addition

Associative Property of Multiplication

Commutative Property of Addition

Commutative Property of Multiplication

Multiplicative Property of One or
Identity Property of One

Additive Propertyof Zero or
Identity Property of Zero

Distributive Property of
Multiplication over Addition

Distributive Property of
Multiplication over Subtraction

Property of Opposites

A number and its opposite equal zero.
1 + -1 = 0
-6 = 6 = 0

Associative Property

When the grouping of addends is changed, the sum of Addition is the same. (2+3) +4=2+(3+4)

Associative Property of Multiplication

When the grouping of factors is changed, the product is the same.
(1 X 2) X 3 = 1 X (2 X 3)
(a X b) X C = a X (b X c)

Commutative Property of Addition

When the order of addends is changed, the sum is the same.
1 + 2 = 2 + 1
a + b = b + a

Commutative Property of Multiplication

When the order of factors is changed, the product is the same.
1 x 2 = 2 x 1
a + b = b + a

Multiplicative Property of One or Identity Property of One

Any quantity multiplied by one is the same quantity.

2 x 1 = 2
a x 1 = a

Additive Property of Zero or Identity Property of Zero

The sum of any quantity and zero is the same quantity.

2 + 0 = 2
a + 0 = a

Distributive Property of Multiplication over Addition

The sum of two or more quantities may be multiplied by another quantity, or each addend may be multiplied by a common quantity, then the products added.
1 x (2 + 3) = (1 x 2) + (1 x 3)\
a x (b + c) = (a x b) + (a x c)

Distributive Property of Multiplication over Subtraction

The difference between two quantities may be multiplied by another quantity, or each quantity may be multiplied by a common quantity, then the products subtracted.
1 x (3-2) = (1 x 3) - (1 x 2)
a x (b - c) = (a x b) - (ax c)

1. $\frac{7}{9}$

2. $\frac{5}{8}$

3. $\frac{5}{6}$

4. $\frac{5}{10} = \frac{1}{2}$

5. $\frac{3}{4}$

1. $\frac{4}{9} + \frac{1}{3} =$

3. $\frac{1}{6} + \frac{2}{3} =$

4. $\frac{3}{10} + \frac{1}{5} =$

5. $\frac{1}{2} + \frac{1}{4} =$

2. $\frac{3}{8} + \frac{1}{4} =$

1. $\frac{3}{8}$
2. $\frac{1}{4}$
3. $\frac{4}{9}$
4. $\frac{3}{6} = \frac{1}{2}$
5. $\frac{3}{10}$

1. $\frac{7}{8} - \frac{1}{2} =$

3. $\frac{7}{9} - \frac{1}{3} =$

4. $\frac{5}{6} - \frac{1}{3} =$

5. $\frac{7}{10} - \frac{2}{5} =$

2. $\frac{1}{2} - \frac{1}{4} =$

1. $\frac{1}{8}$
2. $\frac{2}{9}$
3. $\frac{2}{5}$
4. $\frac{1}{8}$
5. $\frac{1}{9}$

1. $\frac{1}{2} \times \frac{1}{4} =$

3. $\frac{1}{2} \times \frac{4}{5} =$

4. $\frac{1}{4} \times \frac{1}{2} =$

5. $\frac{1}{3} \times \frac{1}{3} =$

2. $\frac{2}{3} \times \frac{1}{3} =$

1. $\frac{4}{10} = \frac{2}{5}$

2. $\frac{4}{6} = \frac{2}{3}$

3. $\frac{2}{4} = \frac{1}{2}$

4. $\frac{4}{4} = 1$

5. $\frac{4}{9}$

1. $\frac{3}{10} \div \frac{3}{4} =$

3. $\frac{1}{4} \div \frac{1}{3} =$

4. $\frac{3}{4} \div \frac{3}{4} =$

5. $\frac{2}{9} \div \frac{1}{2} =$

2. $\frac{1}{2} \div \frac{3}{4} =$

1. $\frac{5}{2}$
2. $\frac{17}{5}$
3. $\frac{7}{4}$
4. $\frac{19}{8}$
5. $\frac{11}{7}$

1. $2\frac{1}{2} =$

3. $1\frac{3}{4} =$

4. $2\frac{3}{8} =$

5. $1\frac{4}{7} =$

2. $3\frac{2}{5} =$

1. $2\frac{3}{4}$

2. $3\frac{1}{2}$

3. $2\frac{4}{5}$

4. $1\frac{8}{9}$

5. $2\frac{2}{3}$

1. $\frac{11}{4} =$

3. $\frac{14}{5} =$

4. $\frac{17}{9} =$

5. $\frac{8}{3} =$

2. $\frac{7}{2} =$

1. $3\frac{5}{9}$

2. $7\frac{1}{6}$

3. $4\frac{7}{10}$

4. $2\frac{9}{10}$

5. $3\frac{7}{8}$

1. $2\frac{1}{3} + 1\frac{2}{9} =$

3. $3\frac{2}{5} + 1\frac{3}{10} =$

4. $1\frac{4}{5} + 1\frac{1}{10} =$

5. $2\frac{3}{8} + 1\frac{1}{2} =$

2. $5\frac{1}{2} + 1\frac{2}{3} =$

1. $2\frac{9}{10}$

2. $3\frac{1}{2}$

3. $2\frac{5}{6}$

4. $1\frac{1}{4}$

5. $2\frac{1}{9}$

1. $5\frac{4}{5} + 2\frac{9}{10} =$

3. $4\frac{1}{2} + 1\frac{2}{3} =$

4. $2\frac{1}{2} + -1\frac{2}{3} =$

5. $3\frac{2}{3} + 1\frac{5}{9} =$

2. $6\frac{1}{3} + 2\frac{5}{6} =$

1. $7\frac{1}{2}$

2. $4\frac{1}{2}$

3. $14\frac{6}{7}$

4. $7\frac{7}{8}$

5. $3\frac{6}{7}$

1. $1\frac{2}{3} + 4\frac{1}{2} =$

3. $3\frac{1}{4} + 4\frac{4}{7} =$

4. $1\frac{4}{5} + 4\frac{3}{8} =$

5. $1\frac{1}{8} + 3\frac{3}{7} =$

2. $2\frac{1}{2} + 1\frac{4}{5} =$

1. $1\frac{9}{10}$

2. $2\frac{1}{3}$

3. $7\frac{3}{5}$

4. $\frac{1}{2}$

5. $\frac{2}{4}$

1. $4\frac{1}{2} \div 2\frac{1}{2} =$

3. $9\frac{1}{2} \div 1\frac{1}{4} =$

4. $3\frac{1}{3} \div 6\frac{2}{3} =$

5. $1\frac{3}{4} \div 2\frac{1}{3} =$

2. $2\frac{2}{3} \div 1\frac{1}{7} =$

Fractions

Reduction of fractions is the process of producing equivalent fractions by dividing the same whole number into both the numerator and denominator.

Example:

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Like fractions are fractions having a common denominator.

Examples:

$$\frac{1}{4} \qquad \frac{3}{4}$$

Common denominators are denominators which are the same in two or more fractions.

Examples:

$$\frac{1}{8} \qquad \frac{7}{8}$$

Fraction Rules

Fraction Rules: Addition

Convert unlike fractions to equivalent like fractions.

Add the denominators and place over the common denominator.

Reduce to the lowest terms if necessary.

Fraction Rules: Subtraction

Convert unlike fractions to equivalent like fractions.
Subtract the denominators and place over the common denominator.

Reduce to the lowest terms if necessary.

Fraction Rules: Division

Invert the second fraction (the divisor).

Multiply the numerators.

Multiply the denomiators.

Reduce to the lowest form if necessary.

Fraction Rules: Multiplication

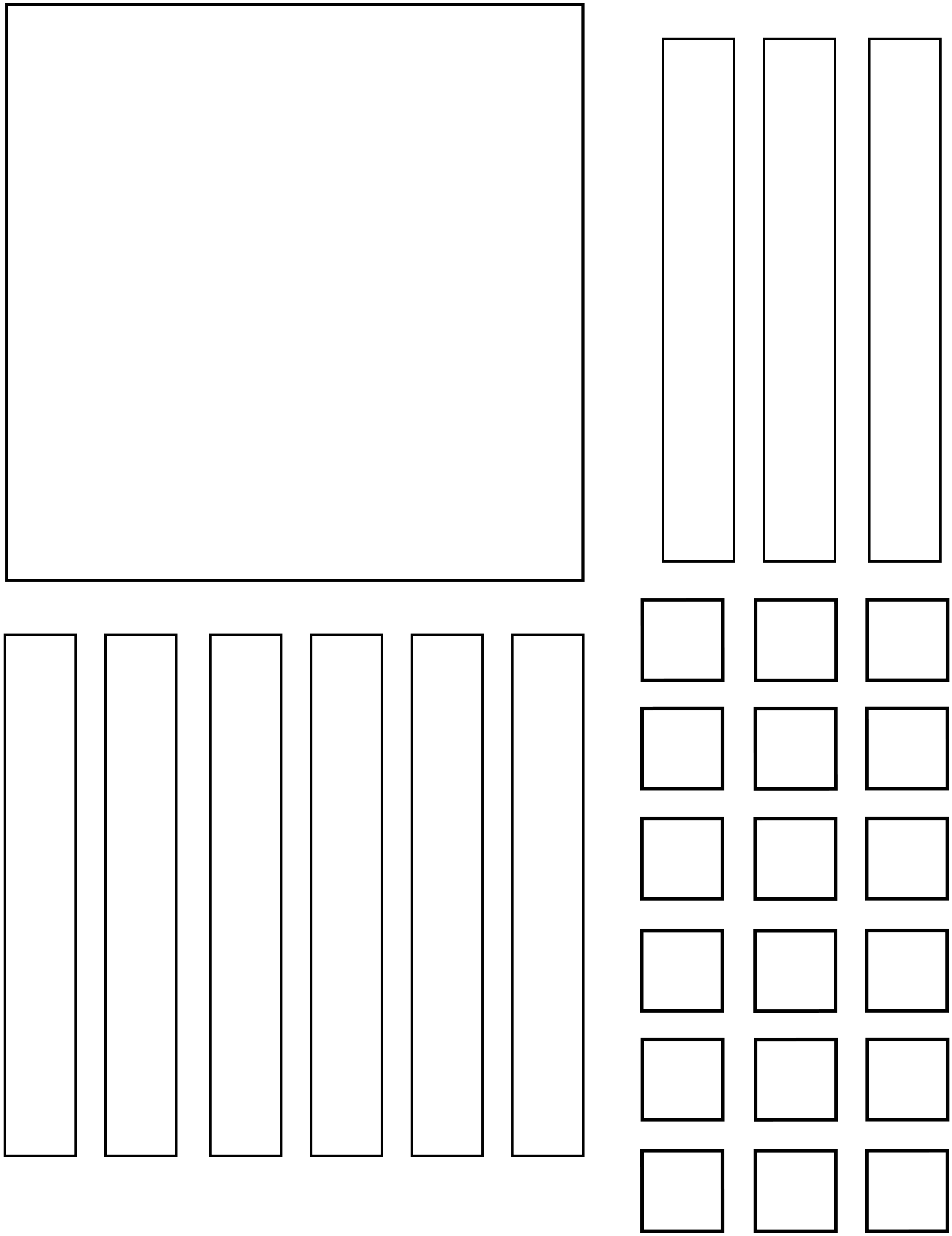
Multiply the numerators to get the numerator of the answer, then multiply the denominators to get the denominator of the answer.

Reduce to the lowest terms if necessary.

Fraction Rules: Cross Reduction

Reduce the numerator of the first fraction and the denominator of the second fraction by dividing both by the highest common factor.

Reduce the denominator of the first fraction and the numerator of the second fraction by dividing both the the highest common factor.



1. 5.37
2. 6.59
3. 4.96
4. 4.82
5. 5.55

1. 3.26
+ 2.11

3. 4.07
+ 0.89

4. 3.17
+ 1.65

5. 3.92
+ 1.63

2. 3.67
+ 2.92

1. 1.24
2. 1.17
3. 5.80
4. 1.11
5. 2.01

1. 2.75
 \- 1.51

3. 6.97
 1.17

4. 7.89
 \- 6.78

5. 3.62
 \- 1.61

2. 3.78
 \- 2.61

1. 3.21
x 3

1. 9.63
2. 8.44
3. 4.86
4. 4.84
5. 3.39

3. 2.43
x 2

4. 1.21
x 4

5. 1.13
x 3

2. 2.11
x 4

1. 2$\overline{)\,4.86}$

1. 2.43
2. 4.31
3. 2.31
4. 1.22
5. 3.02

3. 3$\overline{)\,6.93}$

4. 4$\overline{)\,4.88}$

5. 3$\overline{)\,9.06}$

2. 2$\overline{)\,8.62}$

1. 2.623
+ 4.241

1. 6.864
2. 2.9262
3. 5.3963
4. 8.99449
5. 5.975

3. 1.0231
+ 4.3732

4. 5.80014
+ 3.19435

5. 1.963
+ 4.012

2. 1.8130
+ 1.1132

1. 3.0186
- 0.0152

1. 3.003
2. 2.1342
3. 1.1111
4. 5.8030
5. 1.2433

3. 7.8912
+ 6.7801

4. 6.9731
- 1.1701

5. 2.7536
- 1.5103

2. 8.6374
- 6.5032

1. 2.13301
x 3

1. 6.39903
2. 2.4086
3. 8.8624
4. 4.8408
5. 6.90369

3. 4.4312
x 2

4. 1.2102
x 4

5. 2.30123
x 3

2. 1.2043
x 2

1. 2.211
x 0.4

1. 0.8844
2. 0.39639
3. 0.8408
4. 0.9936
5. 0.40246

3. 2.102
x 0.4

4. 3.312
x 0.3

5. 2.0123
x 0.2

2. 1.3213
x 0.3

1. 1.426
2. 4.3123
3. 2.031
4. 2.102
5. 3.123

1. $2\overline{)2.846}$

3. $3\overline{)6.093}$

4. $4\overline{)8.408}$

5. $3\overline{)9.369}$

2. $2\overline{)8.6246}$

1. $.2\overline{).4826}$

1. 2.413
2. 4.3124
3. 3.2212
4. 12.021
5. 3.123

3. $3\overline{)\ .96636}$

4. $.4\overline{)4.8084}$

5. $.3\overline{).9369}$

2. $.2\overline{).86248}$

1. $3.1\overline{)9.672}$

1. 3.12
2. 32.12
3. 2.2
4. 12.102
5. 1.11

3. $1.22\overline{)2.684}$

4. $4.1\overline{)49.6182}$

5. $3.4\overline{)3.774}$

2. $2.1\overline{)67.452}$

1. $\frac{3}{1}$

2. $\frac{1}{5}$

3. $\frac{1}{4}$

4. $\frac{9}{10}$

5. $\frac{7}{20}$

1. 75% =

3. 25%=

4. 90%=

5. 35%=

2. 20%=

1. 25%

2. 60%

3. 180%

4. 50%

5. 30%

1. $\frac{1}{4}$ =

3. $\frac{14}{5}$ =

4. $\frac{1}{2}$ =

5. $\frac{3}{10}$ =

2. $\frac{3}{5}$ =

1. 37.5%

2. 63%

3. 1%

4. 250%

5. 25%

1. 0.375 =

3. 0.01=

4. 2.5 =

5. 0.25 =

2. 0.63 =

1. 0.03

2. 0.22

3. 0.25

4. 0.902

5. 1.65

1. 3% =

3. 25% =

4. 90.2% =

5. 165% =

2. 22% =

1. Books are on sale for $5 each. If you buy four books with a 10% discount, what will the total cost be?

1. $18.00
2. $361.25
3. $2475
4. 33.3%
5. 12.5%

3. The tuition for one child in a school is $900 a month. A second child in the family receives a 10% discount. A third child receives a 15% discount. What is the total monthly tuition for three children?

4. If a customer paid $30 for a shirt which was originally priced at $45, what discount was received?

5. The airline offers a fare for $800 roundtrip to Mexico. By ordering the ticket two weeks in advance and staying
over a Saturday night, $100 can be saved. What is the discount?

2. A television set cost $500 last month. It was reduced by 15% last week. Yesterday the price was reduced by another 15%. What is the cost of the television set today?

1. What is the interest paid on a $21,000 loan at 7.5% for two years?

1. $3150
2. 1%
3. $280.33
4. $2994.88
5. $8100

3. The credit card company charges 18% per year on unpaid balances. What is the amount of interest due on $3000 left unpaid for six months? Note: First determine the monthly interest.

4. How much interest is received on an investment of $19,000 at 5% when the interest is allowed to remain in the investment?

5. What is the interest on $45,000 at 6% over three years?

2. If $50,000 is invested for one year and makes a profit of $500, what is the rate of interest?

Percent Rules

Changing Percents to Common Fractions

Drop the percent symbol(%) and write the numeral as a numerator for the common fraction.
Write a denominator of one hundred and reduce the fraction if possible.

Example: The common fraction for 75% is 75/100 or 3/4.

Changing Percents to Declmal Fractions

Drop the per cent symbol(%).
Place a decimal point two places to the left of the existing decimal point.
Example: The decimal fraction for 75.5% is 0.755.

Changing Common Fractions to Percents

Divide the numerator by the denominator to get a decimal fraction.
Move the decimal two places to the right and add the per cent symbol at the right.

Example: The per cent for 3/4 is 4+3 = 0.75 or 75%.

Changing Declmal Fractions to Percents

Move the decimal point two place to the right.
Add the per cent symbol at the right.

Example: 0.75 = 75 %

Addition of Declmals

Copy the problem with decimal points in alignment. Insert zeros to complete place value columns. Add each place value column, beginning at the far right.

Example: 0.2 + 0.03 + 1.036 =

```
  0.200
  0.030
+ 1.039
-------
  1.269
```


Subtraction of Declmals

Copy the problem with decimal points in alignment. Insert zeros in the minuend or subtrahend if needed. Subtract each place value column, beginning at the far right.

Example: 2.975 - 0.86 =

```
  2.975
- 0.860
-------
  2.115
```


Multlpllcatlon of Decimals by Whole Numbers

Copy the problem without regard to decimal point placement. Multiply the multiplicand by the multiplier. Count the number of decimal places in the multiplicand. Starting at the right of the product, count the same number of decimal places as in the multiplicand, moving to the left, and insert a decimal point in the product.

Example: 4.321 x 2 =

```
  4.321  (Three places)
x     2
-------
  8.642  (Three places)
```

Multiplication of Decimals by Decimals
Copy the problem without regard to decimal point placement. Multiply the multiplicand by the multiplier. Count the number of decimal places in the multiplicand. and in the multiplier. Starting at the right of the product, count the same number of decimal places as in the multiplicand and multiplier, moving to the left, and insert a decimal point in the product..

Example: 2.312 x 0.3 =

```
  2.312
x   0.3
-------
 0.6936
```


Division of Decimals by Whole Numbers
Copy the problem and divide the dividend by the divisor. Insert the decimal point in the quotient directly above the decimal point in the dividend.

Example: 6.39 ÷ 3 =

```
    2.13
3) 6.39
```


Division of Decimals by Decimals
Copy the problem, count and move the decimal point in the divisor to the right so that it becomes a whole number. Move the decimal point in the dividend the same number of places to the right. After dividing the dividend by the divisor, insert a decimal point in the quotient directly above the decimal point in the dividend.

Example: 0.46 + 2.3

```
     0.2
23 ) 4.6
```

Hundreds	Color this section red.
Tens	Color this section blue.
Units	Color this section green.
Tenths	Color this section light blue.
Hundredths	Color this section pink.
Thousandths	Color this section light green.
Ten Thousandths	Color this section light blue.
Hundred Thousandths	Color this section pink.
Millionths	Color this section light green.

1. 0.452
x 315

1. 142.38
2. 2951.7393
3. 32668.93
4. 972.5942
5. 463.736

3. 4.3362
x 7534

4. 2.3102
x 421

5. 1.352
x 343

2. 1.2043
x 2451

1. 45^2

1. 2025
2. 1296
3. 144
4. 1764
5. 3721

3. 12^2

4. 42^2

5. 61^2

2. 36^2

1. 53,824
2. 18,496
3. 20,164
4. 186,624
5. 132,496

1. 232^2

3. 142^2

4. 432^2

5. 364^2

2. 136^2

1. 16
2. 14
3. 21
4. 25
5. 18

1. $\sqrt{256}$

3. $\sqrt{441}$

4. $\sqrt{625}$

5. $\sqrt{324}$

2. $\sqrt{196}$

1. $\sqrt{2025}$

1. 45
2. 36
3. 42
4. 61
5. 33

3. $\sqrt{1764}$

4. $\sqrt{3721}$

5. $\sqrt{1089}$

2. $\sqrt{1296}$

1. $\sqrt{15129}$

1. 123
2. 231
3. 342
4. 415
5. 121

3. $\sqrt{116964}$

4. $\sqrt{172225}$

5. $\sqrt{14641}$

2. $\sqrt{53361}$

Find the square root.

$\sqrt{25}$

$\sqrt{9}$

$\sqrt{16}$

Find the square root.

$\sqrt{36}$

$\sqrt{100}$

$\sqrt{64}$

Find the square root.

$\sqrt{144}$

$\sqrt{441}$

$\sqrt{625}$

Find the square root.

$\sqrt{4}$

$\sqrt{49}$

$\sqrt{144}$

Find the square root.

$\sqrt{81}$

$\sqrt{121}$

$\sqrt{1}$

Find the square root.

$\sqrt{256}$

$\sqrt{196}$

$\sqrt{576}$

Find the square root.

$\sqrt{2025}$

$\sqrt{1296}$

$\sqrt{1764}$

Find the square root.

$\sqrt{6889}$

$\sqrt{3969}$

$\sqrt{676}$

Find the square root.

$\sqrt{2809}$

$\sqrt{4489}$

$\sqrt{2916}$

Find the square root.

$\sqrt{3721}$

$\sqrt{1089}$

$\sqrt{2601}$

Find the square root.

$\sqrt{1024}$

$\sqrt{1156}$

$\sqrt{1764}$

Find the square root.

$\sqrt{17424}$

$\sqrt{15129}$

$\sqrt{53361}$

T^2	T x h	T x t	T x U
T x h	h^2	h x t	h x U
T x t	h x t	t^2	t x U
T x u	h x U	t x U	u^2

GREEN	RED	BLUE	GREEN
RED	BLUE	GREEN	RED
BLUE	GREEN	RED	BLUE
GREEN	RED	BLUE	GREEN

Grid Color Scheme

Definitions for Exponents

The base is a number or symbol which is to be multiplied the number of times indicated by the exponent

Example: The base of 2^3 is 2

1

The exponent is the number or symbol located at the upper right of another number or symbol which indicates the number of times the base is to be used as a factor.

Example:
The exponent of 2 is 3, so two is multiplied or used as a factor three times. 2 x 2 x 2 = 8.

2

Calculator exponential notation uses symbols such as

^, x^y, y^x to indicate raising to a power on the calculator display.

3

Power can refer to the product obtained when the base is multiplied the number of times indicated by the exponent.

Example:
Eight is the third power of two in 2^3.

4

Power can be the reference to the exponent as in 2 to the third power equals 8.

5

Scientific notation refers to a number expressed as a product of two factors, the first factor having exactly one digit at the left of the decimal point and the second factor being a power of ten.

Example:
531.5 in decimal notation is **5.315** x $\mathbf{10}^3$ in scientific notation

6

Calculator scientific notation uses

E or · to indicate multiplication of a number times ten to a power because of limited number of digits which can be displayed.

Example:
2^{34} = **1.717986918E10**
(Actual value is **17,179,869,184** which cannot be displayed on some calculator screens.)

7

Scientific Notation Rules

To change a scientific notation to standard form, move the decimal the number of places indicated by the power of ten. Move the decimal to the right if the power of ten is positive, or to the left if it is negative.

1

To change a number from standard form to scientific notation, move the decimal until there is one non-zero digit to its left.

To the right of this number, write x 10 to the power which is indicated by the number of places the decimal was moved.

If the decimal was moved to the left, the power of ten is positive. If the decimal was moved to the right, the power of ten is negative.

2

Exponential Form and Logarithmic Forms of the Powers of Ten

Exponential Form	Logarithmic Form
$10^3 = 1000$	log 1000 = 3 log
$10^2 = 100$	100 = 2
$10^1 = 10$	log 10 = 1
$10^0 = 1$	log 1 = 0
$10^{-1} = 0.1$	log 0.1 = -1
$10^{-2} = 0.01$	log 0.01 = -2 log
$10^{-3} = 0.001$	0.001 = -3

Procedure for Finding the Antilogarithm of a Number

1. Find the mantissa in the tables.

   (The characteristic is not part of the mantissa.)

1

2. Locate the significant figure that corresponds to the mantissa first in the N column, then in the row of whole numbers across the top.

2

3. Write as a decimal number between 1 and 10.

3

4. Multiply the significant figure by 10 raised to the power indicated by the characteristic.

Example: Find the antilog of 3.3874.

1. Locate the mantissa, .3874, in the tables.
2. Find the significant figure corresponding to the mantissa in the N column which will be 24 and in the row of whole numbers which will be 4. The significant figure is 244.
3. Write these as a decimal number, 2.44.
4. Multiply the significant figure 2.44 by 10 to the third power since 3 is the characteristic.

   $2.44 \times 10^3 = 2{,}440.$

   Therefore, the antilog of 3.3874 = 2,440.

Procedure for Raising to a Power
(Involutlon)

1. Find the log of the number which is raised to a power.

   For example,

   in the problem 14^3,

   the log of 14 is 1.1461.

2. Multiply the log by the power according to the exponent.

   In this example,

   the power is 3,

   so 1.1461 x 3 =

   3.4383.

3. Find the antilog of the product resulting from the multiplication of the log of the number by its power and record the answer.

   In this example,

   the antilog of 3.4383 is 2744,

   so $\log_{10} 14^3 = 2744$.

Procedure for Finding the Logarithm of a Number

1. Write the number in scientific notation.

2. Write the characteristic.

3. Locate the mantissa corresponding to the significant figures of the number in the log table.

4. Add the mantissa to the characteristic to produce the logarithm of the number.

Example: Find the log of 232.

1. The significant figure is 232.
2. The scientific notation is 232 = 2.32×10^3
3. The characteristic is 2.
4. Locate the first two numbers of the significant figure, which is 23, in the Table of Logarithms in column N.
5. Find the third number of the significant figure in the column labeled 2, move down that column until the row marked 23 is reached, and record the mantissa which is 3655. Place a decimal point to the left of the mantissa.
6. Add the characteristic, 2, to the mantissa, .3655 to obtain the log of 232 which is 2.3655.

Procedure for Extracting a Root
(Evolution)

1. Find the log of the number.

   For example, in the problem $\sqrt[2]{25}$

   $\log_{10}25 = 1.3979$.

1

10

2. Divide the log by the index.

In this example,

the index is 2,

so $1.3979 \div 2 = 0.6989$.

2

3. Find the antilog of the quotient resulting from the division of the log of the number by its exponent.

   In this example,

   the antilog of 0.6989 is 5,

   so $\sqrt[2]{25} = 5$

3

Laws of Logarithms
(Also Laws of Exponents)

To multiply of two numbers with the same fixed base, add their logarithms.

Find the antilog of the sum.

To divide one number by another, determine the logarithm of each.

Subtract the log of the divisor from the log of the dividend.

Find the antilog of the difference.

To raise a number to a power, determine the log of the number.

Multiply the log by the power to which the number is to be raised.

Find the antilog of the product.

To find the root of a number, determine the log of the number.

Divide the log by the root of the number.

Find the antilog of the quotient.

Information about Logarithms

A logarithm is the exponent to which a fixed base must be raised to yield a given number.

For example $10^2 = 100$ is

$\log_{10} 100 = 2$ or $\log 100 = 2$.

The base is 10, the exponent is 2 and the logarithm is 2.

1

Common logarithms are logarithms to base ten, also known as Briggsian logs, accurate to as many as twenty decimal places.

Any number may be used as a base, but base ten has the most advantages.

The base is assumed to be ten unless otherwise stated.

2

The characteristic is the whole number part of a log.

In $\log 360 = 2.5563$, the characteristic is 2.

The characteristic is always positive for numbers greater than one.

The characteristic is always negative for numbers less than one.

3

The mantissa is the decimal part of a log.

In log 360 = 2.5563, the mantissa is .5563.

Logarithmic tables list only the mantissas of logarithms for 1 to 10.

The mantissa is always kept positive.

The antilogarithm is determined by using the table to convert the logarithm to the number associated with it.

History of Logarithms

In 1614 after working for twenty years, John Napier of Scotland published tables for his invented logarithms. These helped scientists in solving problems with very large numbers. Napier set up ratios in which one term came from a geometric series and the other from arithmetic series.

A Swiss mathematician, Jobst Burgi, invented logarithms at the same time yet independently of Napier. Napier published his tables first so credit is given to him, not Burgi.

Henry Briggs, an English mathematician, suggested that logarithms should be based on ten. In 1624, Briggs published his table of base 10 or common logarithms for 1 to 20,000 and 90,000 to 100,000. He also introduced the term mantissa for the decimal part of the logarithm.

After Briggs died, Adriaen Vlacq, a Dutch mathematician, completed Briggs' tables. Vlacq was the first to use the term characteristic for the whole number part of a log.

4

Since about 1620, logarithmic scales have been used to make computing devices. An English mathematician, Edmund Gunter, devised a logarithmic scale with a pair of dividers to multiply and divide. In 1622, William Oughtred developed two scales, one of which slid along the other. This was the basis for the modern slide rule which has both logarithmic and antilog scales.

5

It was not until the end of the seventeenth century that exponential notation and laws of exponents were invented.

Then mathematicians recognized logarithms as exponents. In Tables of Logarithms, the more places in the mantissa, the greater the accuracy.

6

Logarithmic scales are used to make delicate measurements of quantities such as sound intensity (decibels), magnitude of earthquakes (Richter readings) and musical sounds (frequency of vibrations).

7

Base 2	Group of 2^3	Group of 2^3	Group of 2^1	Units
	8	4	2	1

Base 3	Group of 3^3	Group of 3^2	Group of 3^1	Units
	27	9	3	1

Base 4	Group of 4^3	Group of 4^2	Group of 4^1	Units
	64	16	4	1

Base 5	Group of 5^3	Group of 5^3	Group of 5^3	Units
	125	25	5	1

Base 6	Group of 6^3	Group of 5^2	Group of 6^1	Units
	216	36	6	1

Base 7	Group of 7^3	Group of 7^2	Group of 7^1	Units
	343	49	7	1

Base 8	Group of 8^3	Group of 8^2	Group of 8^1	Units
	512	64	8	1

Base 9	Group of 9^3	Group of 9^2	Group of 9^1	Units
	729	81	9	1

Base 10	Group of 10^3	Group of 10^2	Group of 10^1	Units
	1000	100	10	1

Base 2					
group of 2^5	group of 2^4	group of 2^3	group of 2^2	group of 2	units
32	16	8	4	2	1

Base 3					
group of 3^5	group of 3^4	group of 3^3	group of 3^2	group of 3	units
243	81	27	9	3	1

Base 4					
group of 4^5	group of 4^4	group of 4^3	group of 4^2	group of 4	units
1024	256	64	16	4	1

Base 5					
group of 5^5	group of 5^4	group of 5^3	group of 5^2	group of 5	units
3125	625	125	25	5	1

Base 6					
group of 6^5	group of 6^4	group of 6^3	group of 6^2	group of 6	units
7776	1296	216	36	6	1

Base 7					
group of 7^5	group of 7^4	group of 7^3	group of 7^2	group of 7	units
16,807	2401	343	49	7	1

Base 8					
group of 8^5	group of 8^4	group of 8^3	group of 8^2	group of 8	units
32,768	4096	512	64	8	1

Base 9					
group of 9^5	group of 9^4	group of 9^3	group of 9^2	group of 9	units
59,049	6561	729	81	9	1

Base 10					
group of 10^5	group of 10^4	group of 10^3	group of 10^2	group of 10	units
100,000	10,000	1,000	100	10	1

1. 11010_2 = ______$_{10}$

1. 26
2. 23
3. 15
4. 31
5. 21

3. 1111_2 = ________ $_{10}$

4. 11111_2 = ________$_{10}$

5. 10101_2 = ________$_{10}$

2. 10111_2 = _________$_{10}$

1. 101
2. 242
3. 122
4. 212
5. 44

1. $10202_3 =$ ________$_{10}$

3. $11112_3 =$ ________$_{10}$

4. $21212_3 =$ ________$_{10}$

5. $1122_3 =$ ________$_{10}$

2. $22222_3 =$ ________$_{10}$

1. 377
2. 287
3. 978
4. 678
5. 478

1. $11321_4 = \_\_\_\_\_\_\_\__{10}$

3. $33102_4 = \_\_\_\_\_\_\_\__{10}$

4. $22213_4 = \_\_\_\_\_\_\_\__{10}$

5. $13132_4 = \_\_\_\_\_\_\_\__{10}$

2. $10133_4 = \_\_\_\_\_\_\_\__{10}$

1. 1791
2. 873
3. 3077
4. 1574
5. 156

1. $24131_5 =$ ________ $_{10}$

3. $44302_5 =$ ________ $_{10}$

4. $22244_5 =$ ________ $_{10}$

5. $1111_5 =$ ________ $_{10}$

2. $11443_5 =$ ________ $_{10}$

1. 3233
2. 1508
3. 5419
4. 3088
5. 2583

1. $22545_6 =$ ________$_{10}$

3. $41031_6 =$ ________$_{10}$

4. $22144_6 =$ ________$_{10}$

5. $15543_6 =$ ________$_{10}$

2. $10552_6 =$ ________$_{10}$

1. 1237
2. 6912
3. 481
4. 146
5. 9662

2. $3415_7 =$ ________ $_{10}$

3. $1255_7 =$ ________ $_{10}$

4. $266_7 =$ ________ $_{10}$

5. $40112_7 =$ ________ $_{10}$

2. $26103_7 =$ ________ $_{10}$

1. 10,127
2. 12,601
3. 20,791
4. 20,109
5. 5061

1. $23617_8 = \_\_\_\_\_\_\_\__{10}$

3. $50467_8 = \_\_\_\_\_\_\_\__{10}$

4. $47215_8 = \_\_\_\_\_\_\_\__{10}$

5. $11705_8 = \_\_\_\_\_\_\_\__{10}$

2. $30471_8 = \_\_\_\_\_\_\_\__{10}$

1. 21,041
2. 27,020
3. 44,626
4. 7931
5. 34,434

1. $31768_9 = \_\_\_\_\_\_\_\__{10}$

3. $67184_9 = \_\_\_\_\_\_\_\__{10}$

4. $11782_9 = \_\_\_\_\_\_\_\__{10}$

5. $52210_9 = \_\_\_\_\_\_\_\__{10}$

2. $41052_9 = \_\_\_\_\_\_\_\__{10}$

Convert these to base 10:

1. $1101_2 =$ ________$_{10}$
2. $11011_2 =$ ________$_{10}$
3. $1111_2 =$ ________$_{10}$
4. $11111_2 =$ ________$_{10}$
5. $10101_2 =$ ________$_{10}$
6. $100_2 =$ ________$_{10}$
7. $10001_2 =$ ________$_{10}$

Convert these to base 10:

1. $11321_4 =$ ________$_{10}$
2. $10133_4 =$ ________$_{10}$
3. $33102_4 =$ ________$_{10}$
4. $22213_4 =$ ________$_{10}$
5. $13132_4 =$ ________$_{10}$
6. $10333_4 =$ ________$_{10}$
7. $22310_4 =$ ________$_{10}$

Convert these to base 10:

1. $21221_3 =$ ________$_{10}$
2. $11122_3 =$ ________$_{10}$
3. $1122_3 =$ ________$_{10}$
4. $21212_3 =$ ________$_{10}$
5. $11112_3 =$ ________$_{10}$
6. $22222_3 =$ ________$_{10}$
7. $10202_3 =$ ________$_{10}$

Convert these to base 10:

1. $24131_5 =$ ________$_{10}$
2. $11443_5 =$ ________$_{10}$
3. $44302_5 =$ ________$_{10}$
4. $22244_5 =$ ________$_{10}$
5. $1111_5 =$ ________$_{10}$
6. $42000_5 =$ ________$_{10}$
7. $24200_2 =$ ________$_{10}$

Convert these to base 10:

1. $22545_6 = \_\_\_\_\_\_\_\__{10}$
2. $10552_6 = \_\_\_\_\_\_\_\__{10}$
3. $41031_6 = \_\_\_\_\_\_\_\__{10}$
4. $22144_6 = \_\_\_\_\_\_\_\__{10}$
5. $15543_6 = \_\_\_\_\_\_\_\__{10}$
6. $32152_6 = \_\_\_\_\_\_\_\__{10}$
7. $11515_6 = \_\_\_\_\_\_\_\__{10}$

Convert these to base 10:

1. $23617_8 = \_\_\_\_\_\_\_\__{10}$
2. $30471_8 = \_\_\_\_\_\_\_\__{10}$
3. $50467_8 = \_\_\_\_\_\_\_\__{10}$
4. $47215_8 = \_\_\_\_\_\_\_\__{10}$
5. $11705_8 = \_\_\_\_\_\_\_\__{10}$
6. $66317_8 = \_\_\_\_\_\_\_\__{10}$
7. $16657_8 = \_\_\_\_\_\_\_\__{10}$

Convert these to base 10:

1. $3415_7 = \_\_\_\_\_\_\_\__{10}$
2. $26103_7 = \_\_\_\_\_\_\_\__{10}$
3. $1255_7 = \_\_\_\_\_\_\_\__{10}$
4. $266_7 = \_\_\_\_\_\_\_\__{10}$
5. $40112_7 = \_\_\_\_\_\_\_\__{10}$
6. $30055_7 = \_\_\_\_\_\_\_\__{10}$
7. $22165_7 = \_\_\_\_\_\_\_\__{10}$

Convert these to base 10:

1. $31768_9 = \_\_\_\_\_\_\_\__{10}$
2. $41052_9 = \_\_\_\_\_\_\_\__{10}$
3. $67184_9 = \_\_\_\_\_\_\_\__{10}$
4. $11782_9 = \_\_\_\_\_\_\_\__{10}$
5. $52210_9 = \_\_\_\_\_\_\_\__{10}$
6. $26883_9 = \_\_\_\_\_\_\_\__{10}$
7. $11111_9 = \_\_\_\_\_\_\_\__{10}$

1. -2 + -4 =

1. -6
2. -9
3. -7
4. -13
5. -12

3. -2 + -5 =

4. -6 + -7 =

5. -5 + -7 =

2. -6 + -3 =

1. -2
2. +4
3. -5
4. -1
5. +2

1. +2 + -4 =

3. +8 + -13 =

4. +6 + -7 =

5. +9 + -7 =

2. +8 + -4 =

1. -2
2. -3
3. -4
4. -7
5. -2

1. 6 - -4 =

3. -9 - -5 =

4. -8 - -1 =

5. -9 - -7 =

2. -6 - -3 =

1. -2 - -9 =

1. +7
2. +2
3. +1
4. +5
5. +3

3. -4 - -5 =

4. -3 - -8 =

5. -4 - -7 =

2. -1 - -3 =

1. -6 - +4 =

1. -10
2. -9
3. -14
4. -9
5. -16

3. -9 - +5 =

4. -8 - +1 =

5. -9 - +7 =

2. -6 - +3 =

1. +12
2. +4
3. +9
4. +11
5. +11

1. +3 - -9 =

3. +4 - -5 =

4. +3 - -8 =

5. +4 - -7 =

2. +1 - -3 =

1. -36
2. -18
3. -27
4. -40
5. -28

1. -6 x 6 =

3. -9 x 3 =

4. -8 x 5 =

5. -4 x 7 =

2. -6 x 3 =

1. +8
2. +4
3. +2
4. +3
5. +2

1. -16 ÷ -2 =

3. -10 ÷ -5 =

4. -24 ÷ -8 =

5. -14 ÷ -7 =

2. -12 ÷ -3 =

1. -8
2. -2
3. -3
4. -2
5. -4

1. $-16 \div 2 =$

3. $-9 \div 3 =$

4. $-18 \div 9 =$

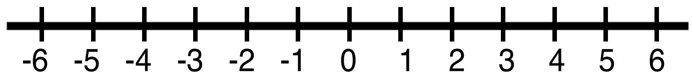
5. $-28 \div 7 =$

2. $-6 \div 3 =$

Negative Numbers

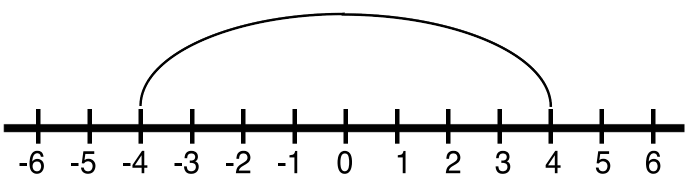
number line

continuous sequence of numbers with positive numbers to the right of zero and negative numbers to the left of zero



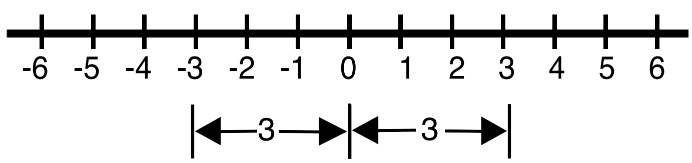
opposites

signed number paired with another signed number the same distance from zero in the opposite direction



absolute value

the distance from zero without regard to direction



Rules of Signs

Addition

To add two numbers with the same sign, add and keep the sign common to both.

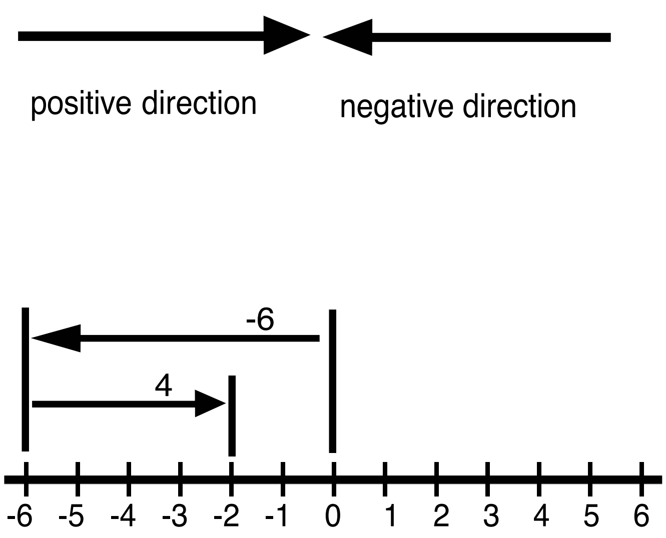
(+2) + (+4) = +6 (-5) + (-3) = -8

To add two numbers with different signs, find the difference and use the sign of the greater absolute value.

(+ 13) + (-16) = -3 (+18) + (-7) = +11

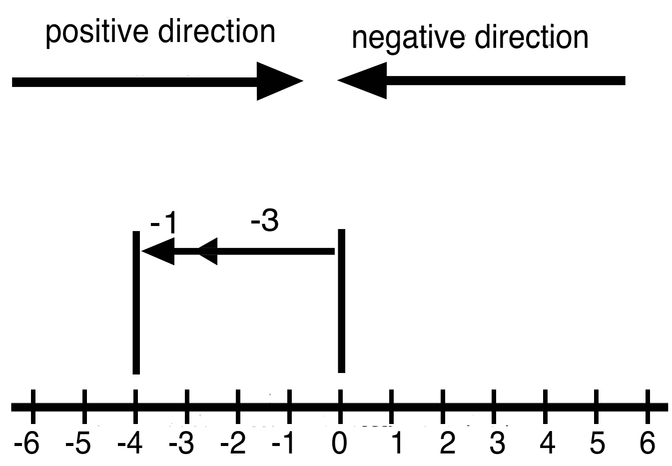
To add a positive number, move that many units in a positive direction forward to the right

-6 + 4 = -2



To add a negative number, move the number of units indicated in a negative direction backward to the left.

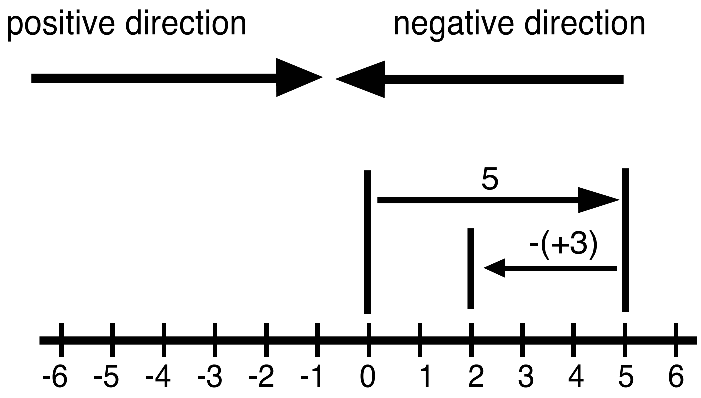
(-3) + (-1) = -4



Subtraction

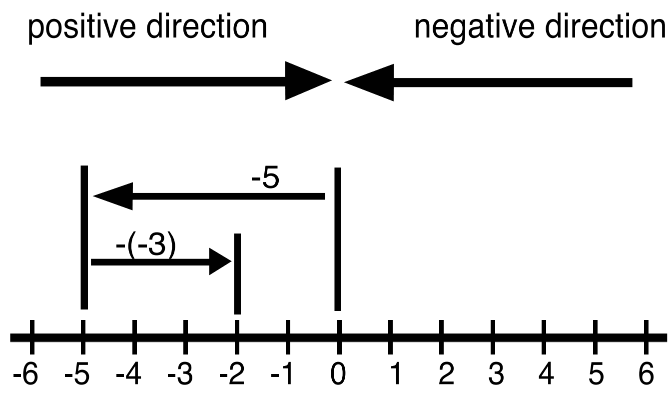
To subtract a positive number, move backward to the left in a negative direction.

$(+5) - (+3) = +2$



To subtract a negative number, move forward to the right in a positive direction.

$(-5) - (-3) = -2$



To subtract one signed number from another, replace the subtrahend with its opposite number and add.

$(+9) - (-6) = (+9) + (+6) = +15$

$(+23) - (+16) = (+23) + (-16) = +7$

$(-41) - (-16) = (-41) + (+16) = -25$

Multlpllication

When multiplying two numbers having the same sign, give the product a positive sign.

$(+4) \times (+2) = +8$

$(-6) \times (-3) = +18$

When multiplying two numbers with different signs, give the product a negative sign.

(+5) x (-2) = -10

(-3) x (+4) = -12

Division

When dividing numbers having the same sign, give the quotient a positive sign.

(+16) ÷ (+4) = +4

(-12) ÷ (-3) = +4

Division

When dividing numbers with different signs, give the quotient a negative sign.

(+8) ÷ (-2) = -4

(-25) ÷ (+5) = -5

number that is either positive or negative

number greater than zero

number less than zero

number which is neither positive nor negative

number assumed to be positive when no sign is indicated

quantitative size of a number

signed number +6 or - 6

positive number +9

negative number -9

zero 0

addition sign indicating positive number

minus sign indicating negative number

signed number +6 or - 6
number that is either positive or negative

positive number + 9
number greater than zero

negative number - 9
number less than zero

zero 0
number which is neither positive nor negative

unsigned number 9 number
assumed to be positive when no sign is indicated

magnitude 12
quantitative size of a number

plus +

addition sign indicating positive number

minus

minus sign indicating negative number

unsigned number 9

magnitude 12

plus +

minus −

Surface Area

cube	$SA = 6 \times e^2$
cylinder	$SA = 2\pi r^2 + 2\pi rh$
rectangular prism	$SA = 21w + 21h + 2wh$ $SA = 2(lw + lh + wh)$

Perimeter

equilateral triangle	$P=3s$
circle	$C = \pi d$ $C = 2\pi r$ $(d = 2r)$
rectangle	$P = 2l + 2w$ $P = 2(l + w)$

square

$P = 4s$

scalene triangle

$P = s_1 + s_2 + s_3$

rectangle

$A = l \times w$

square

$A = s^2$
$A = s \times s$

Area

triangle

$$A = \frac{1}{2}(a \times b)$$

$$A = \frac{(a \times b)}{2}$$

parallelogram

$$A = b \times h$$

trapezoid

$$A = \frac{1}{2}h(a + b)$$

$$A = \frac{h(a + b)}{2}$$

circle

$A = \pi r^2$

hexagon

$$A = \frac{a \times p}{4}$$

$$A = \frac{1}{4} (a \times p)$$

Volume

prisms & cylinders

$V = Bh$

pyramids & cones

$V = \frac{1}{3} Bh$

$V = \frac{Bh}{3}$

cube

$V = e^3$

$V = e \times e \times e$

sphere

$V = \frac{4}{3} \pi r^3$

$V = \frac{4 \pi r^3}{3}$

rectangular prism

B = Bh

V = l w h

(8 = l X w)

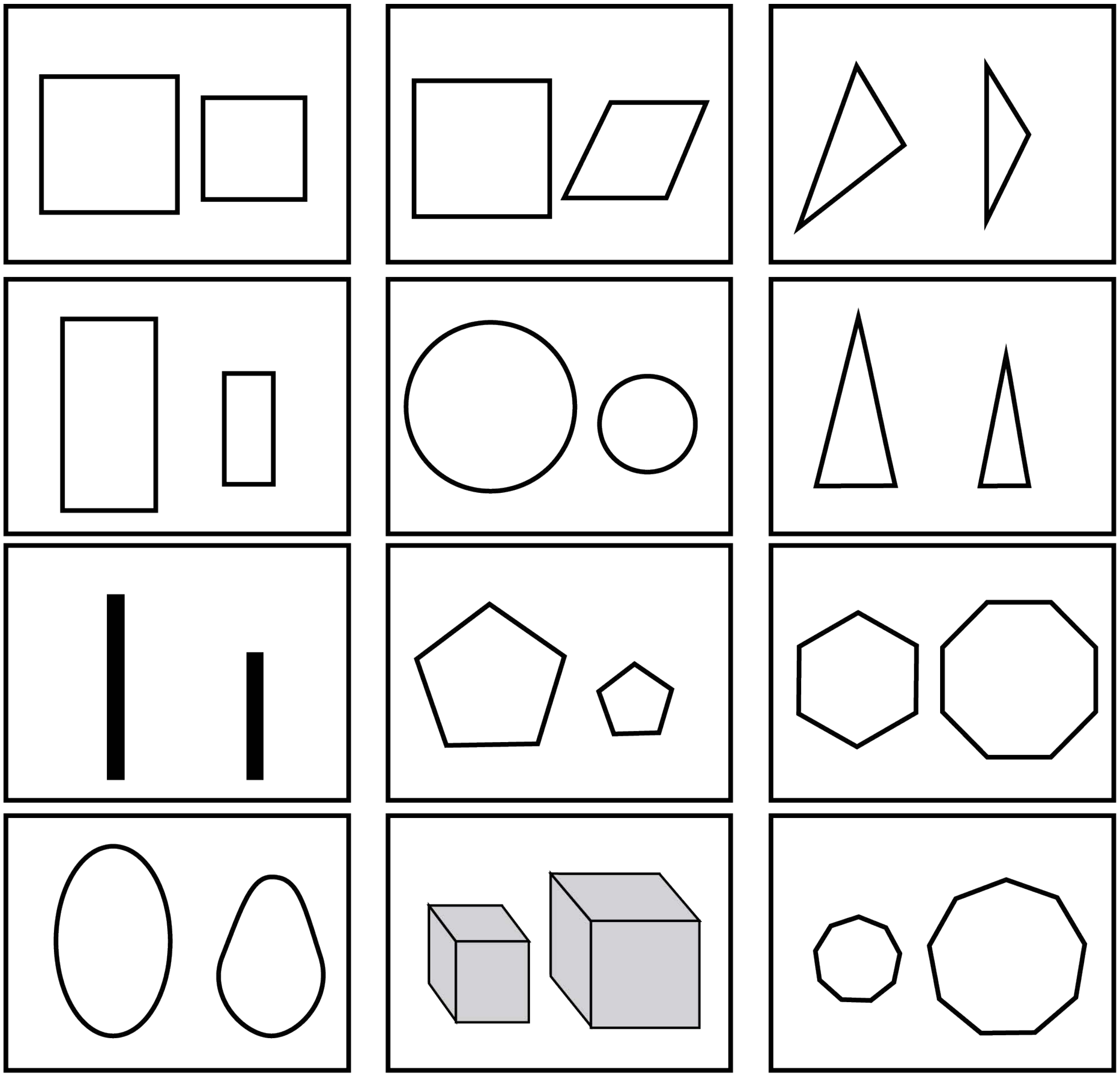
triangular prism	$V = Bh$ $V = \frac{1}{2}\ abh$ $(B = \frac{1}{2}\ ab)$
cylinder	$V = Bh$ $V = \pi r^2 h$ $(B = \pi r^2)$
square pyramid	$V = \frac{1}{3}\ Bh$ $V = \frac{1}{3}\ s^2 h$ $(B = s^2)$
triangular pyramid	$V = \frac{1}{3}\ Bh$ $V = \frac{1}{3}\ (b\ \frac{a}{2}\ )h$ $(B = b\frac{a}{2}\ )$

cone

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3}\pi r^2 h$$

$$(B = \pi r^2)$$



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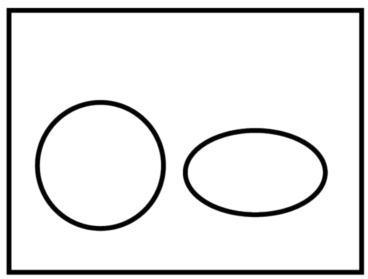

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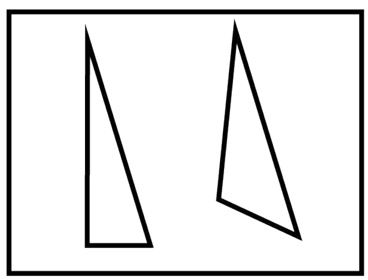
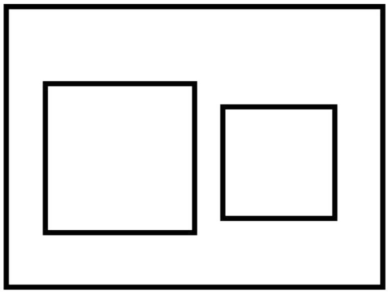
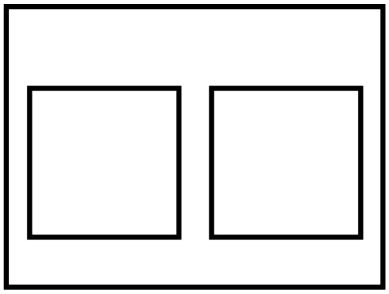
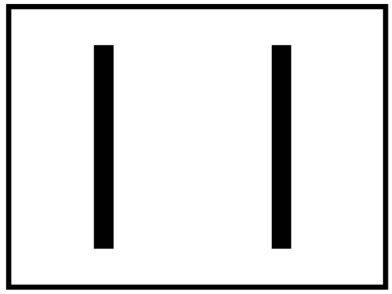
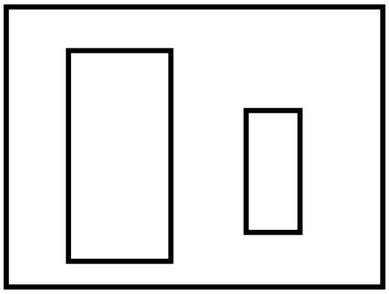
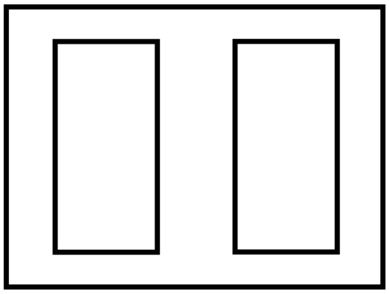
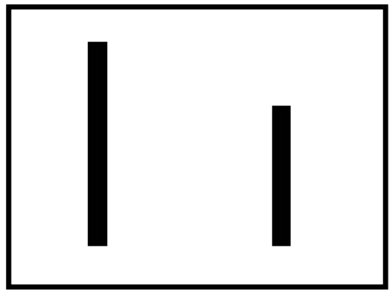
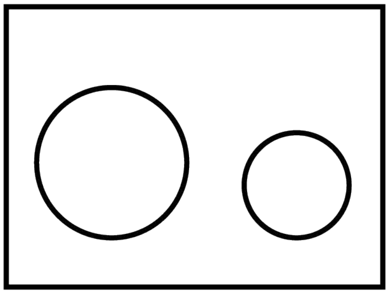
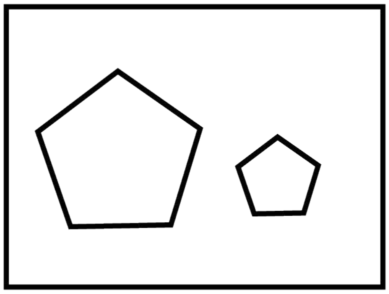
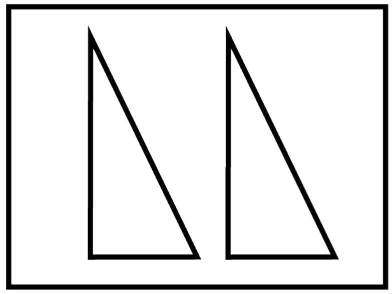
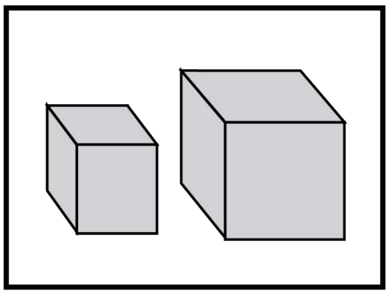
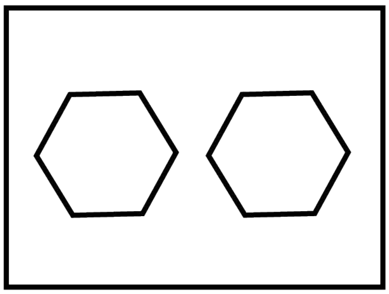
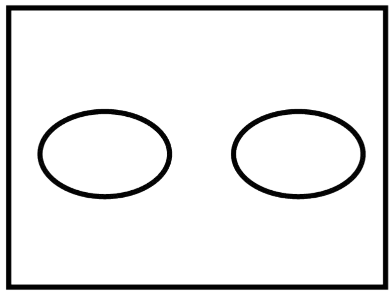
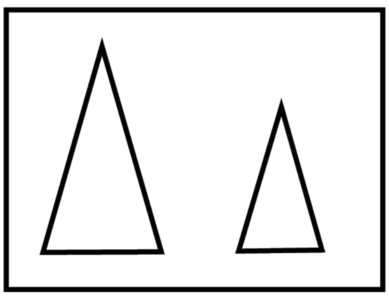
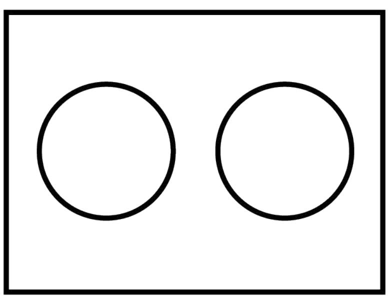


Congruent



Noncongruent

geometric similarity

figures with congruent corresponding angles and proportional linear dimensions

congruent angles

angles which have the same measure

corresponding lengths

those lengths in constant ratio

in proportion

having the same ratio

corresponding parts

parts of geometric figures which are in the same relative position when two figures are oriented the same

linear dimensions

any length of a side or diameter of a circle

scale factor

constant ratio usually applied to scale models or scaled drawings

Geometric Construction

1

Line Segment

1. Mark two points on a piece of paper.

   Label one A and the other B under the points.

1

2. Using a straightedge, draw a line segment connecting the two points.

2

Geometric Construction

2

Perpendlcular to a Line from a Point on the Line

1. Construct a line segment using a straight edge.

1

2. Place the point of a compass at the approximate center of the line segment.

3. Make small arcs which intersect the line at equal distances from the compass point, being careful not to change the angle of the compass arms.

   Label the compass point X and the points where the line is intersected by th arcs A and B.

2

3

4. Increase the space between the point of the compass and the arm holding the pencil, thus forming a larger angle.

4

5. Place the point of the compass on point A and construct an arc above the center of the line segment.

5

6. Move the point of the compass to point B without changing the angle. Construct another arc above the center of the line segment as before. Label the point at which the two arcs intersect C.

6

7. Using a straightedge, construct a line segment which passes through the point on the line (X) and the point above the line where the two arcs intersect (C).

This line will form two adjacent angles of 90° each with the base line.

Therefore, it is perpendicular to the base line.

7

Geometric Construction

3
Perpendicular to a Line from a Point not on the Line

1. Construct a line segment using a straight edge and label it L.

2. Make a point above the line and label it X.

3. Place the point of a compass on point X, set a radius which will intersect the line and construct an arc which intersects line L. Label these points of intersection B and C.

4. Increase the radius of the compass. Place the point of the compass at point 8 and construct an arc. Keeping the same radius, place the point of the compass at C and construct an arc which intersects the previously drawn arc. Label this point of intersection D.

4

5. Draw a line segment through points X and D. This line is perpendicular to line L.

5

Geometric Construction

4
Perpendicular Bisector of a Line Segment

1. Construct a line segment using a straight edge. Label one A and the other B under the points.

1

2. Set the radius of the compass to be greater than half of line A B. Place the point of the compass at point A and construct arcs above and below the line. Keeping the same radius, place the point of the compass at point B and construct arcs above and below the line which intersect the arcs previously drawn. Label these points C and D.

2

3. Draw a line segment through points C and D. This line segment is perpendicular to line A B.

3

Geometric Construction

5
Angle

1. Construct a line segment using a straight edge. Label one A and the other B under the points.

2. Set the angle of the compass at less than 90 °. Place the point of the compass at point A and draw an arc which intersects the line and continues above the point of the compass.

3. Make a point on the arc not directly over point A and label it C.

4. Using a straightedge, connect point A with the point C on the arc to form an angle.

4

Geometric Construction

6
Congruent Angle

1. Construct a line segment using a straight edge. Label one end D and the other E.

   Obtain an angle labeled A, B and C to duplicate.

2. Set the compass at a convenient radius.

   Place the point of the compass on point B, at the vertex of the triangle which is to be duplicated and construct an arc which intersects both sides.

   Label the base line point G and the point on the side F.

3. Keeping the radius of the compass the same, place the point of the compass at D on the line segment.

   Construct an arc which intersects the line segment and label the point of intersection K.

4. Place the point of the compass at G where the arc intersects the base line of the triangle to be duplicated.

   Set the compass radius to equal the distance where the arc intersects the other side at point F.

   Using this radius, place the point of the compass at K and draw an arc which intersects the previously drawn arc.

   Label this point H.

5. Using a straightedge, draw a line from point D through point H.

   This produces an angle equal to the given one, angle ABC.

Geometric Construction

7
Parallel Lines

1. Construct a line segment using a straight edge and label it L.

1

2. Place a point over the approximate center of the line segment and label it X.

2

3. Using a straightedge, draw an oblique line labeled B C through point X to intersect the line segment near its center.

   Label this point D.

3

4. Using X as the vertex and XB as a side, construct an angle congruent to the angle XDL.

4

5. Extend the base of the congruent triangle to the left of point X and label it F.

   This line is parallel to the given line L.

5

Geometric Construction

8
Angle Bisector

1. Construct an angle less than 90 ° and label it A, B and C.

1

2. Set a compass radius less than side AB and side BC.

2

3. Place the point of the compass at the vertex B and construct an arc which insects lines AB and BC. Label the points of intersection D and E.

3

4. Set a compass radius greater than half of the distance between D and E.

4

5. Place the point of the compass at point D and construct an arc within the angle.

5

6. Place the point of the compass at point E and construct an arc which intersects the arc previously drawn. Label this point of intersection F.

6

7. Draw a line segment from point B through point F. This line segment bisects the angle.

7

Geometric Construction

9
Circumscribing a Circle about a Triangle

1. Draw an acute angled scalene triangle and label it A,B and C.

2. Construct perpendicular bisectors of two sides of the triangle, having them extend until they intersect.

   Label the point of intersection 0.

3. Place the point of the compass at point O and set a radius of QA, A being the apex of the triangle.

   Construct a circle which also passes through points B and C on the triangle.

   This circle O is circumscribed about the triangle ABC.

Geometric Construction

10

Inscribing a Circle in a Triangle

1. Draw an acute angled scalene triangle and label it A, B and C.

2. Bisect two angles of the triangle.

   Label the point of intersection of the angle bisectors 0.

3. Construct a line perpendicular to one side of the triangle from point 0.

   Label the point where the perpendicular intersects the side of the triangle F.

4. Place the point of the compass at point O and set a radius of OF.

   Construct a circle which has the sides of the triangle as tangents.

   This circle is inscribed within the triangle ABC.

4

Geometric Construction

11
Equilateral Triangle

1. Construct a line segment using a straight edge and label the end points A and B.

1

2. Place the point of the compass at point A and set the radius to be the distance from A to B.

   Construct an arc above the line AB.

2

3. Keeping the same radius, place the point of the compass at point B and construct another arc which intersects the one previously constructed.

   Label the point of intersection C.

3

4. Using a straightedge, draw a line from point A to point C and from point B to point C.

   This is a triangle with three sides equal or an equilateral triangle.

4

Geometric Construction

12
Hexagon Inscribed in a Circle

1. Accurately draw a circle.

1

2. Do not change the radius which had been set on the compass to draw the circle.

2

3. Place the point of the compass at any point on the circle and construct a small arc which intersects the circle.

3

4. Keeping the same radius, place the point of the compass on the point where the arc intersects the circle and construct another small arc which intersects the circle.

4

5. Follow the same procedure around the circle to the starting point so that there are six small arcs equidistant around the circle.

5

6. Using a straightedge, draw a line between each of the points formed where the arcs intersect the circle. A regular hexagon is inscribed in the circle.

6

Geometric Construction

13
Pentagon

1. Accurately draw a circle and label the center 0.

1

2. Using a straightedge, draw the diameter of the circle through point 0, and label the points where it intersects the circle Band D.

2

3. Construct a diameter perpendicular to BD and label the points where it intersects the circle X and Y.

3

4. Bisect the radius labeled OX and label the point of bisection A.

4

5. Place the point of the compass on point A and set the radius of the compass to equal the distance from A to B, then construct an arc which intersects B and the diameter of the circle **XV.**

5

6. Label the point where the arc intersects the diameter C.

6

7. Place the point of the compass at B and set the radius of the compass the distance from B to C.

Keeping this radius, construct a small arc on the circle, starting at B.

7

8. At the point where the first arc intersects the circle, place the point of the compass and construct another small arc which intersects the circle.

9. Continue to construct small arcs around the circle.

   The fifth arc should be a B.

8

9

10. Using a straightedge, draw a line between each of the points formed where each of the five arcs intersect the circle.

    A regular pentagon results.

10

Geometric Construction

14
Constructing Models of Regular Polyhedra (Platonic Solids)

1. Choose a polyhedron and the appropriate triangle, square or pentagon template.

1

2. Choose a color of construction paper and with a pencil, draw around the template according to the configuration for the polyhedron selected for construction.

2

3. Draw the tabs for folding and pasting according to pattern.

3

4. Carefully cut around the outer edges of the drawing.

5. Fold and crease along the appropriate lines.

6. Using a glue stick or tape, assemble the polyhedron.

Note: A variation using fabric cut, sewn and stuffed with. fiberfill may be used.

Construction of Congruent Angles

1. To the right of the given angle labeled ABC with B and C as the base line, draw a ray (line) with a straightedge (ruler) which is in line with B and C. Label the ray (line) D at the left end and E at the right end. The line DE may be equal to or greater than line BC.

2. Place the point of a compass at B and using a convenient radius, construct an arc to intersect line AB and line BC. At the intersection of the arc with line AB, label that point F. At the intersection of the arc with line BC, label that point G.

3. Using the same radius, place the point of the compass at D and construct an arc which intersects line DE and extends above point D. At the point where the arc intersects the line DE, label that point K.

4. Move the compass to the original figure and place the compass point at G. Set the compass to the length FG.

4

5. Without changing the set length, move the compass to the figure under construction and place the point at K.

5

6. Construct an arc which intersects the previously arc, labeling the point of intersection M.

6

7. Using a straightedge, draw a line extending from point D through M, thus producing an angle congruent to the original angle ABC.

7

Indirect Measurement
Using Similar Triangles

1 . Choose a tall structure to measure such as a flagpole, tree or building.

Example: If the lengths of the shadows are 3 feet for the stick and 21 feet for the structure and the height of the stick is 4 feet, these measurements are substituted in the proportion.

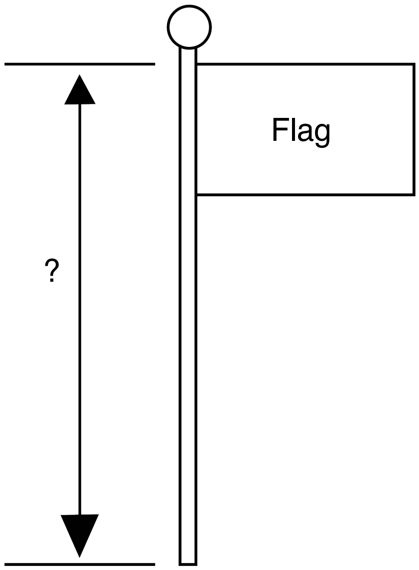
$$\frac{3}{21} = \frac{4}{h}$$

$$3 \times h = 4 \times 21$$

$$3h = 84$$

$$h = 28$$

Therefore the structure is 28 feet in height.

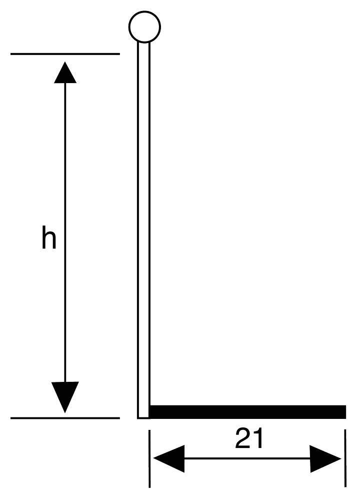


5. substitute the measurements in the following proportion:

$$\frac{\text{length of stick shadow}}{\text{length of structure shadow}} = \frac{\text{height of stick}}{\text{height of structure}}$$

2 9

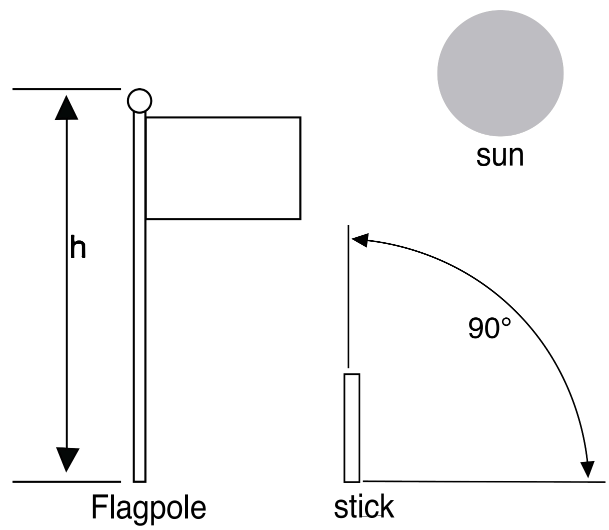
2. Place a straight stick onto the ground to form a right angle, being sure that it is in full sun and near the object to be measured.



3

8

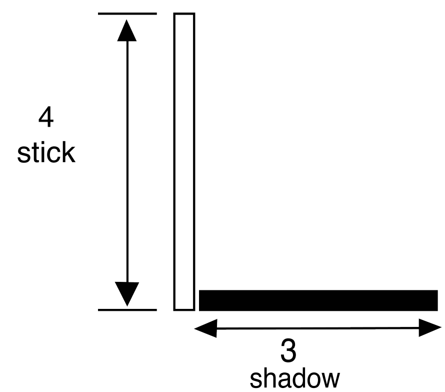
4. Measure and record the length of the shadow of the structure.



7

4

3. Measure and record the height of the stick and the length of the shadow which forms the base.



5

6

Triangle Information

Triangle Inequality

The sum of the measures of any two sides of any triangle always is greater than the measure of the third side.

1

Sum of Angles

The sum of the three angles of a triangle always is 180°.

2

Area of a Triangle

The area of any triangle is determined by multiplying the base by the height and taking one-half of the product.

$A = \frac{1}{2} b h$

3

Building Number Cubes

Name ____________________

1) Choose a number from 3 to 17: ___________

2) List two addends (1 to 9) of your number: **a=**__________ **b=** ___________

3) Get your pieces and arrange them to show the expansion of the binomial cube.

4) Substitute the values of these pieces into the expansion of the binomial equation:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

(____+____)^3 = (____)^3 + 3(____)^2 (____) +3(____)(____)^2 + (____)^3

(_______)^3 = ______ + ______ + ______ + ______

__________ = __________

1. $2x^2 + 6x + 7$
$+ 4x^2 + 2x + 4$

1. $6x^2 + 8x + 11$
2. $5x^2 + 10x + 3$
3. $7x^2 + 6x + 8$
4. $8x^2 + 7x + 8$
5. $4x^2 + 5x + 12$

3. $4x^2 + 2x + 7$
$+ 3x^2 + 4x + 1$

4. $2x^2 + 5x + 1$
$+ 6x^2 + 2x + 7$

5. 1x2 + 2x + 4
+ 3x2 + 3x + 8

2. 3x2 + 5x + 1
+ 2x2 + 5x + 2

1. $6x^2 + 8x + 7$
$\underline{-\ 4x^2 + 2x + 4}$

1. $2x^2 + 6x + 3$
2. $3x^2 + 3x + 6$
3. $x^2 - 2x + 6$
4. $4x^2 + 3x - 6$
5. $4x^2 - x + 1$

3. $4x^2 + 2x + 7$
$\underline{-\ 3x^2 + 4x + 1}$

4. $8x^2 + 5x + 1$
$\underline{-\ 4x^2 + 2x + 7}$

5. $7x^2 + 2x + 9$
$\underline{-\ 3x^2 + 3x + 8}$

2. $5x^2 + 5x + 8$
$\underline{-\ 2x^2 + 2x + 2}$

1. $x^2 + 5x + 4 =$

1. $(x + 4)(x + 1)$
2. $(x + 6)(x + 2)$
3. $(2x + 3)(x + 2)$
4. $(x + 5)(x + 2)$
5. $(x + 3)(x + 7)$

3. $2x^2 + 7x + 6 =$

4. $x^2 + 7x + 10 =$

5. $x^2 + 10x + 21 =$

2. $x^2 + ax + 12 =$

1. $5(2x + 4) =$

1. $10x + 20$
2. $3x^2 + 3x$
3. $2x^2 + 6x$
4. $6x + 12$
5. $12x + 8$

3. $2x(x + 3) =$

4. $6(x + 2) =$

5. $4(3x + 2) =$

2. $X(3x + 3) =$

1. $(x + 1)(x + 4)$

1. $x^2 + 5x + 4$
2. $x^2 + 4x + 3$
3. $x^2 + 5x + 6$
4. $x^2 + 14x + 8$
5. $3x^2 + 14x + 8$

3. $(x + 2)(x + 3) =$

4. $(x + 6)(x + 2) =$

5. $(x + 4)(3x + 2) =$

2. $(x + 1)(x + 3) =$

1. $(x + 1)(x - 4) =$

1. $x^2 - 3x - 4$
2. $x^2 - x - 6$
3. $x^2 - 5x + 6$
4. $x^2 + 4x - 12$
5. $3x^2 - 10x - 8$

3. $(X - 2)(X - 3) =$

4. $(x + 6)(x - 2) =$

5. $(x - 4)(3x + 2) =$

2. $(x + 2)(x - 3) =$

1.(x+1) $\overline{)\ x^2+5x+4}$

1. (x+4)
2. (x + 6)
3. (2x + 3)
4. (x + 2)
5. (x + 3)

.

3. (x + 2) $\overline{)\ 2x^2\ 7x+6}$

4. (x + 5) $\overline{)x^2+7x+10}$

5. (x + 7) $\overline{)x^2+10x+21}$

2. (x + 2) $\overline{)\ x^2+ax+12}$

1. $(2x-2)\overline{)4x^2+2x-6}$

1. (2x + 3)
2. (3x + 4)
3. (2x- 2)
4. (2x + 4)
5. (3x- 2)

3. $(3x+2)\overline{)6x2-2x-4}$

4. $(2x-1)\overline{)4x2+6x-4}$

5. $(x+5)\overline{)3x2}+13x-10$

2. $(x-1)\overline{)3x^2\ x-4}$

decimal fraction- has a denominator which is a power of ten, for example, 0.50

percent- fraction with one hundred as the denominator, for example $\frac{50}{100}$

ratio- compares two similar measures by means of division expressed as a fraction or as two numbers separated by a colon for example, 12:6 or $\frac{12}{6}$

probability- ratio which expresses the chance that a certain event will occur given the number of possible outcomes, for example, the probability of getting heads on a single coin toss is one in two and may be expressed as $P=\frac{1}{2}$ or P= 0.50 or P= 50%

laws of probability- prediction of what might occur when the same event is repeated a large number of times

odds- ratio of probability of the occurrence of an event to the probability of it not occurring

experiment- repetition of a procedure having the possibility of the same outcome each time without being able to predict any single outcome

sample space- represents all possible outcomes in an experiment

event- one outcome or subset of a sample space

independent events- two events in which the outcomes of either are not affected by each other

mutually exclusive events- two events which cannot take place at the same time

conditional probability- the probability that one event will occur providing that another event has already taken place

golden section- the division of a line segment into extreme and mean ratio with the ratio of the whole segment to the long segment the same as the ratio of the long segment to the short segment expressed as Phi or $\frac{1 + 5\surd}{2}$

proportion- mathematical statement of equality between two ratios, for example, $\frac{4}{8} = \frac{2}{4}$ or 4:8::2:4

terms- the four numbers in a proportion

means- the second and third terms in a proportion, for example, in the proportion $\frac{4}{8} = \frac{2}{4}$, 8 is the second term and 2 is the third term

extremes- the first and fourth terms in a proportion, for example, in the proportion $\frac{4}{8} = \frac{2}{4}$, 4 is the first term and 4 is the fourth term mean

proportional- refers to the relationship of equal means to the extremes, for example, in $\frac{2}{4} = \frac{4}{8}$, 4 is the mean proportional to 2 and 8

sequence- the specified ordering of a set of elements, for example, 2, 4, 6, 8 where each term differs from the succeeding term by two finite

sequence- countable number of terms with a last term, for example, in a sequence with four members, {2, 4, 6, 8}, 8 is the last term infinite

sequence- sequence with no last term which can continue forever without end, for example, {1, 3, 5, 7, 9,......} has an infinite number of terms as indicated by ....

Fibonacci sequence- pattern with one for the first two terms and every term thereafter being the sum of the two preceding terms, for example, {1, 1, 2, 3, 5 ,8, 13, 21,......}

arithmetic progression- sequence with the same common difference between successive terms, for example, {3, 6, 9, 12,.....} in which the common difference is 3

geometric progression- sequence with a common ratio between successive terms, for example, {2, 4, 8, 16,....}which has a common ratio of 2

series- sum of terms in a sequence designated by Σ

finite series- sum of terms in a finite sequence expressed as $\sum_{11=1}^{n} dn$

infinite series- sum of terms in a finite sequence expressed as $\sum_{k=}^{\infty} (k + 1)$

arithmetic series- sum of terms in arithmetic sequence expressed as $\frac{n}{2}[2a = (n-1)d]$

geometric series- sum of terms in geometric sequence expressed as $\frac{a - ar^n}{1-r}$

nth term of an arithmetic progression- $a + (n-1)d$

nth term of a geometric progression- $ar^{(n-1)}$

has a denominator which is a power of ten, for example,

0.50

fraction with one hundred as the denominator, for example

$\frac{50}{100}$

compares two similar measures by means of division expressed as a fraction or as two numbers separated by a colon for example,

12:6 or $\frac{12}{6}$

ratio which expresses the chance that a certain event will occur given the number of possible outcomes, for example, the probability of getting heads on a single coin toss is one in two and may be expressed as

$P = \frac{1}{2}$ or $P = 0.50$ or $P = 50\%$

prediction of what might occur when the same event is repeated a large number of times

ratio of probability of the occurrence of an event to the probability of it not occurring

repetition of a procedure having the possibility of the same outcome each time without being able to predict any single outcome

represents all possible outcomes in an experiment one outcome or subset of a sample space

two events in which the outcomes of either are not affected by each other

two events which cannot take place at the same time

the probability that one event will occur providing that another event has already taken place

the division of a line segment into extreme and mean ratio with the ratio of the whole segment to the long segment the same as the ratio of the long segment to the short segment expressed as Phi or

$$\frac{1 + \sqrt{5}}{2}$$

mathematical statement of equality between two ratios, for example,

$\frac{4}{8} = \frac{2}{4}$ or 4:8::2:4

the second and third terms in a proportion, for example, in the proportion $\frac{4}{8} = \frac{2}{4}$, 8 is the second term and 2 is the third term

the first and fourth terms in a proportion, for example, in the proportion 4/8 = 2/4, 4 is the first term and 4 is the fourth term refers to the relationship of equal means to the extremes, for example,

in 2/4 = 4/8, 4 is the mean proportional to 2 and 8

the specified ordering of a set of elements, for example, 2, 4, 6, 8 where each term differs from the succeeding term by two

countable number of terms with a last term,
for example,

in a sequence with four members, {2, 4, 6, 8}, 8 is the last term

sequence with no last term which can continue forever without end,
for example,

{1, 3, 5, 7, 9,......} has an infinite number of terms as indicated by ....co

pattern with one for the first two terms and every term thereafter being the sum of the two preceding terms,
for example,

{1, 1, 2, 3, 5 ,8, 13, 21,......}

sequence with the same common difference between successive terms,
for example,

{3, 6, 9, 12,.....} in which the common difference is 3

sequence with a common ratio between successive terms, for example,

{2, 4, 8, 16,....}
which has a common ratio of 2

sum of terms in a sequence designated by L sum of terms in a finite sequence expressed as

$$\sum_{n=1}^{n} dn$$

sum of terms in a finite sequence expressed as

$$\sum_{k=n}^{\infty} (k+1)$$

sum of terms in arithmetic sequence expressed as

$$\frac{n}{2}\ [2a = (n - 1)d]$$

sum of terms in geometric sequence expressed as

$$\frac{a - ar^{n}}{1 - r}$$

decimal fraction

percent

ratio

probability

laws of probability

odds

experiment

sample space

mutually exclusive events

conditional probability

golden section

proportion

means

extremes

mean proportional

sequence

finite sequence

infinite sequence

Fibonacci sequence

arithmetic progression

geometric progression

series

finite series

infinite series

arithmetic series

geometric series

the four numbers in a proportion

$a + (n-1)d$

$ar^{(n-1)}$

event

independent events

terms

n th term of an arithmetic progression

n th term of a geometric progression

Math Definitions

written symbol which expresses the abstract concept of number

For example: 6

written word which expresses a numeral

For example: six

all the counting numbers except zero

all the natural numbers and zero

set of positive or negative numbers which includes zero those integers which are equally divisible by 2, therefore, have 2 as a factor

For example: 2, 4, 6, 8,

those integers which are not equally divisible by 2, therefore, do not have 2 as a factor

For example: 1, 3, 5, 7,

indicates quantity of members in a set

For example: 10

indicates the order of members in a set

For example: third, fourth, fifth, sixth

identifies an object For

example: license plate

numerals

configurations of sets of numbers into various shapes such as triangles, squares, etc.

even numbers

odd numbers

a number that is either positive (greater than zero), or negative (less than zero)

set of natural numbers which may have a plus sign before the numeral to show that it is greater than zero

set of natural numbers which must have a minus sign before the numeral to show that it is less than zero

integer that indicates no change in size or direction

quantity without end beyond any number symbolized by

∞

sequence of numbers with positive numbers stretching to infinity in one direction from zero and negative numbers stretching to infinity in the opposite direction from zero
For example:

-5 -4 -3 -2 -1 0 1 2 3 4 5

can be located on a number line and has an expansion as a decimal fraction

has a denominator which is some power of ten and is written with a decimal point

For example:

$$0.236 = \frac{236}{1000} = \frac{236}{103}$$

numeral or symbol placed at upper right of another numeral or symbol (also known as base) to indicate that repeated multiplications are to be applied to the base

For example: $2^2 = 4$

numerator of fraction with 100 as the denominator, from the Latin "per centum" which means by the hundred, also written as a decimal fraction

For example: 75% = $\frac{75}{100}$ or **0.75**

a second number that when multiplied by itself gives the original number

For example: √4 = 2

symbol which indicates that a root is to be extracted

For example: √

number word

natural numbers

whole numbers

integers

cardinal number

ordinal number

nominal number

figurate numbers

negative integers

percent

exponent

radical sign

square root

infinity

number line

real number

decimal fraction

numeral

positive integers

signed number

zero

Math definitions

numeral- written symbol which expresses the abstract concept of number

Example: 6

number word- written word which expresses a numeral

Example: six

natural numbers- all the counting numbers except zero

whole numbers- all the natural numbers and zero

integers- set of positive or negative numbers which includes zero

even numbers- those integers which are equally divisible by 2, therefore, have 2 as a factor.

Example:2, 4, 6, 8,

odd numbers- those integers which are not equally divisible by 2, therefore, do not have 2 as a factor

Example:1, 3, 5, 7,

cardinal number- indicates quantity of members in a set

Example:10

ordinal number- indicates the order of members in a set

Example: third, fourth, fifth, sixth

nominal number- identifies an object

Example: license plate numerals

figurate numbers- configurations of sets of numbers into various shapes such as triangles, squares, etc.

signed number- a number that is either positive (greater than zero), or negative (less than zero)

positive integers- set of natural numbers which may have a plus sign before the numeral to show that it is greater than zero negative integers- set of natural numbers which must have a minus sign before the numeral to show that it is less than zero zero- integer that indicates no change in size or direction infinity- quantity without end beyond any number symbolized byoo

number line- sequence of numbers with positive numbers stretching to infinity in one direction from zero and negative numbers stretching to infinity in the opposite direction from zero

Example: -5 -4- -3- 2- 1 0 1 2 3 4 5

real number- can be located on a number line and has an expansion as a decimal fraction

decimal fraction- has a denominator which is some power of ten and is written with a decimal point

Example: $0.236 = \frac{236}{1000} = \frac{236}{10^3}$

exponent- numeral or symbol placed at upper right of another numeral or symbol (also known as base) to indicate that repeated multiplications are to be applied to the base

Example: $2^2 = 4$

percent- numerator of fraction with 100 as the denominator, from the Latin "per centum" which means by the hundred, also written as a decimal fraction

Example: $75\% = \frac{75}{100}$ or 0.75

square root- a second number that when multiplied by itself gives the original number

Example: 4 = 2

radical sign- symbol which indicates that a root is to be extracted numeral located in the upper left corner of a radical sign to indicate the root to be extracted above 2 which is assumed when no numeral is present

For example:

$$\sqrt[3]{}$$

the numeral under the radical

For example:

$$\sqrt{4}$$

square root of a negative number

branch of mathematics using letters to represent unknown quantities or variables in equations

For example:

$$x + y = 16$$

mathematical sentence which implies that there is equality on both sides of the equals sign

For example:

$6 + x = 9$, therefore, x equals 3

involves the performance of calculations by addition, subtraction, multiplication or division used in algebra as a member of an inclusive set of numbers including all real numbers, all imaginary numbers and numbers consisting of parts of both

For example: a+b

a ratio of two integers which can be expressed as a repeating decimal fraction

For example:

$\frac{3}{4}$ =0.7500000000000000..

cannot be expressed as a ratio or non-repeating decimal expansion

For example: 3.141592653589 .. .

two or more numbers which give the original number when multiplied together

For example:

Factors of 12 are, 1, 2, 3, 4, 6, 12

prime numbers which equal the original number when multiplied together

For example:

2 x 2 x 2 x 3 = 24
so 2 and 3 are the prime factors of 24

number which is a factor of two or more numbers
For example:

Factors of 10 are 1, 2, 5, 10

Factors of 15 are 1, 3, 5, 15

Common factors of 10 and 15 are 1 and 5

number greater than one with only two different factors,one and the number itself
For example:

2,3,5,7,11,13,17,19,23,to infinity

(One is not a prime number.)

the process of determining the factors in a number

those prime numbers which differ only by two

For example:

11 and 13,17 and 19

two numbers with only one as a common factor

number with more than two factors

For example:

4,6,8,9,10,12,14

(One is not a composite number.)

number representing the highest factor common to two numbers

For example:
factors of 12 are 1, 2, 3, 4, 6, 12
factors of 18 are 1, 2, 3, 6, 9, 18

greatest or highest common factor is 6

numbers that when multiplied together produce a product

For example: $5X9=45$

5 and 9 are multiples of 45

lowest number which is a common multiple of two or more given numbers
For example:

multiples of 6 are 6, 12, 18, 24, 30, 36, 42,.....

multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, ...

least common multiple is 30

exponent by which a fixed base of a given number is multiplied
For example:

$\log_2 32 = x$ or $2 x = 32$
means what power of 2 gives 32?

The log to base 2 of 32 is 5 ($2^5 = 32$)

an equation stating that two ratios are equal

comparison by division of two similar measures

For example: $25 : 5$ or $\frac{25}{5}$

successive terms in a specified order
For example:

27, 12, 17, 22, 27,.....

(Ordered by adding 5 to the preceding term)

has finite number of terms so that there is a last term
For example:

3, 6, 9, 12

(four members with last term of 12)

has infinite number of terms so that there is no last term

For example: 2, 4, 6, 8, 10,.....

sequence of terms with a common difference between each

For example:

3, 7, 11,15,19,.....

(difference of 4)

sequence of terms with a common ratio between each

For example:

1, 2, 4, 8, 16, 32,.....

(ratio of 2)

sum of terms in the sequence

For example:

1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255

group of elements or members usually enclosed by braces

For example: { 3, 6, 9, 12 }

states how one quantity changes relative to the change in another quantity

Math Definitions 2

index- numeral located in the upper left comer of a radical sign to indicate the root to be extracted above 2 which is assumed when no numeral is present

Example: 3√

radicand- the numeral under the radical

Example: √4

imaginary number- square root of a negative number algebra-

branch of mathematics using letters to represent unknown quantities or variables in equations

Example:x+y=16

equation- mathematical sentence which implies that there is equality on both sides of the equals sign

Example:6+x=9 ,therefore, x equals 3

arithmetic- involves the performance of calculations by addition, subtraction, multiplication or division

complex number - used in algebra as a member of an inclusive set of numbers including all real numbers, all imaginary numbers and numbers consisting of parts of both

Example:a+b

rational number- a ratio of two integers which can be expressed as a repeating decimal fraction

Example:3/4=07500000000000000...

irrational number- cannot be expressed as a ratio or non-repeating decimal expansion

Example:3141592653589 ...

factors- two or more numbers which give the original number when multiplied together

Example:Factors of 12 are, 1, 2, 3, 4, 6, 12

prime factors- prime numbers which equal the original number when multiplied together

Example:2 X 2 X 2 X 3=24 so 2 and 3 are the prime factors of 24

common factors- number which is a factor of two or more numbers

Example: Factors of 10 are 1, 2, 5, 10

Factors of 15 are 1, 3, 5, 15

Common factors of 10 and 15 are 1 and 5

factoring- the process of determining the factors in a number

prime number- number greater than one with only two different factors, one and the number itself

Example:2, 3, 5, 7, 11, 13, 17, 19, 23, to infinity

(One is not a prime number)

twin primes- those prime numbers which differ only by two

Example: 11 and 13, 17 and 19

relatively prime numbers- two numbers with only one as a common factor

composite number- number with more than two factors

Example:4, 6, 8, 9, 10, 12, 14 (One is not a composite number)

greatest common factor (or highest common factor)- number representing the highest factor common to two numbers

Example:factors of 12 are 1, 2, 3, 4, 2, 12

factors of 18 are 1, 2, 3,..§, 9, 18

greatest or highest common factor is 6

multiples- numbers that when multiplied together produce a product

Example:5 x 9=45

5 and 9 are multiples of 45

least common multiple- lowest number which is a common multiple of two or more given numbers

Example- multiples of 6 are 6, 12, 18, 24, 30, 36, 42....

multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40 .

least common multiple is 30

logarithms- exponent by which a fixed base of a given number is multiplied

Example:log $_2$ 32=x or 2^x =32 means what power of 2 gives 32?

The log to base 2 of 32 is 5 (2^5=32)

proportion- an equation stating that two ratios are equal

ratio- comparison by division of two similar measures

Example:25 :5 or 25/5

sequence- successive terms in a specified order

Example:2, 7, 12, 17, 22, 27, .(Ordered by adding 5 to the preceding term)

finite sequence- has finite number of terms so that there is a last term

Example:3, 6, 9, 12 (four members with last term of 12)

infinite sequence- has infinite number of terms so that there is no last term

Example: 2, 4, 6, 8, 10, ...

arithmetic progression- sequence of terms with a common difference between each

Example:3, 7, 11, 15, 19, ...(difference of 4)

geometric progression- sequence of terms with a common ratio between each

Example:1, 2, 4, 8, 16, 32, ...(ratio of 2)

series- sum of terms in the sequence

Example:1+2+4+8+16+32+64+128=255

set- group of elements or members usually enclosed by braces

Example:{3, 6, 9 ,12}

variation- states how one quantity changes relative to the change in another quantity index

radicand

imaginary number

algebra

equation

arithmetic

complex number

rational number

irrational number

factors

prime factors

common factors

factoring

prime number

twin primes

relatively prime numbers

composite number

greatest common factor
(or highest common factor)

multiples

least common multiple

logarithms

proportion

ratio

sequence

finite sequence

infinite sequence

arithmetic progression

geometric progression

series

set

variation

Probability of Tossing Heads with Coln

1. Bring paper, pencil, straight edge and a coin to a table.

2. Write Probability of Tossing Heads at the top of the paper.

3. Draw three horizontal lines across the paper under the heading, the lines to be about one inch apart. Write heads at the left of the top space and tails in the space under it, then draw a vertical line at the right of the words.

4. Predict the number of times heads will occur when the coin is tossed one hundred times and write the prediction to the right of the heading.

5. Toss the coin one hundred times, making a tally mark in the appropriate space for each toss. Add the number of times heads occurred and the number of times tails occurred.

6. Compare the prediction with the actual frequency of the event.

7. Record the frequency of the event as a fraction, a decimal and a percent. For example, if heads occurred fifty times out of one hundred tosses, the fraction is 1/2, the decimal is 0.50 and the percent is 50%.

Probability of Rolling One with a Die

1. Bring paper, pencil, straight edge and a die to a table.

2. Write Probability of Rolling One at the top of the paper.

3. Draw eight horizontal lines across the paper under the heading, the lines to be about one inch apart. Write number rolled at the left of the top space and the numerals 1 through 6 in the spaces under it. Draw a vertical line at the right of the heading and numerals

4. Predict the number of times one will occur when the die is rolled one hundred times and write the prediction to the right of the heading.

5. Roll the die one hundred times, making a tally mark in the appropriate space for each roll. Add the number of times one occurred.
6. Compare the prediction with the actual frequency of the event.

7. Record the frequency of the event as a fraction, a decimal and percent. For example, if one occurred fifteen times out of one hundred rolls, the fraction is 15/100 or 3/20, the decimal is 0.15 and the percent is 15%.

Probability of Selecting a Red Paper Clip from a Bag of Red, Yellow and Blue Clips

1. Bring paper, pencil, straight edge and bag of paper clips to a table.

2. Write Probability of Selecting Red at the top of the paper.

3. Draw five horizontal lines across the paper under the heading, the lines to be about one inch apart. Write color at the left of the top space and the colors red, blue and yellow in the spaces under it. Draw a vertical line at the right of the heading and words.

4. Predict the number of times red will be selected from the bag of paper clips with one hundred selections. Write the prediction to the right of the heading.

5. Select a clip from the bag, tally the color and return the clip to the bag one hundred times. Add the number of times red occurred.
6. Compare the prediction with the actual frequency of the event.

7. Record the frequency of the event as a fraction, a decimal and percent. For example, if red occurred thirty-five times out of one hundred selections, the fraction is 35/100 or 7/20, the decimal is 0.35 and the percent is 35%.

Probability of Card Selection

1. Bring paper, pencil, straight edge and a deck of cards to a table.
2. Write Probability at the top of the paper.
3. Draw two horizontal lines across the paper under the heading, the lines to be about one inch apart. Name a card. Starting in the top left space, write probability of choosing (name of card) then draw two vertical lines about one inch apart at the right of the event.
4. Predict the number of times the named card will be chosen and write the prediction to the right of the heading.
5. Select any card from the deck fifty times, shuffling and cutting the deck three times between each selection. Make a tally mark each time the named card is drawn.
6. Compare the prediction with the actual frequency of the event.
7. Record the frequency of the event as a fraction, a decimal and percent. For example, if the ace of hearts is drawn one time out of fifty draws, the fraction is 1/50 , the decimal is 0.02 and the percent is 2 %.
8. Use the same procedure and investigate probability of choosing a red card or probabllity of choosing an even numbered card or probabllity of choosing a multiple of three.

Sequence Problems

Order a sequence by multiplying each term after the first by three.

1

Order a sequence by adding five to each term after the first.

2

Order a sequence by adding the sum of the first two terms to each term after the first two.

3

Order a sequence by multiplying each term after the first by one half.

4

Order a sequence by subtracting two from each term after the first.

5

Order of Operations

1. Calculate the exponential values.

1

2. Perform the operations inside parentheses.

3. Working from left to right, perform any multiplication and division operations.

2

3

4. Working from left to right, perform any addition and subtraction operations.

History of Mathematics (B.C.E.)

Early humans recorded amounts with knots on cords, pebbles, marks on bone, wood or stone. Probably they counted on their fingers.

1

By 3000 B.C.E., Egyptian mathematicians developed a decimal system, geometry and methods for calculating area and volume. They were able to survey land and to make calculations for construction of the pyramids.

2

By 2100 B.C.E., Babylonians had a sexagesimal system based on sets of 60. Algebra and geometry were developed beyondJhe methods of the Egyptians.

3

About 2000 B.C.E., the Babylonians used π (pi) with a value of 3 1/8 or 3.1250

4

About 800 B.C.E., Queen Dido was given all the land she could surround with the hide of a bull. After cutting the hide into narrow strips.she fastened them together into one long piece. This she placed to form the boundary of a circle which she correctly decided would provide the greatest area. This area became the city of Carthage in North Africa.

5

By 600 B.C.E., a Greek philosopher, Thales, devised geometric proofs by logical deduction.

6

About 550 B.C.E., a Greek philosopher, Pythagoras, proposed that whole numbers or their ratios could be used for the understanding of everything. The Pythagoreans discovered irrational numbers but kept their discovery a secret because it was not in agreement with the philosophy on which their work was based.

7

By 400 B.C.E., it was realized that Pythagorean concepts were not complete. The Greeks had rediscovered irrational numbers which cannot be expressed as a ratio of two whole numbers.

About 370 B.C.E., a Greek astronomer, Eudoxus, found a way to solve problems with irrational numbers by using his theory of proportions. He also established the basis for integral calculus with his method of exhaustion. This provided a way to determine areas of curved figures.

Around 300 B.C.E., one of the most famous Greek mathematicians, Euclid, wrote his book, Elements. By means of abstract definitions and logical deductions, Euclid constructed a complete system of geometry.

About 200 B.C.E., another famous Greek mathematician, Archimedes, derived the value for pi which is the ratio of the circumference of a circle to its diameter. He constructed a 96-sided figure to approximate a circle on which to base his pi calculation. His writings about infinitesimal calculus were lost until 1906.

History of Mathematics
(1st Century to 15th Century).

In the first century in Greece, Hero or Heron developed a theorem for finding the area of a triangle in terms of its three sides.

1

Around 150 C.E., Ptolomy, a Greek astronomer, used geometry and trigonometry to determine motions of planets. His 13-part work was known as Almagest which means greatest

2

By 825 C.E., the Hindu-Arabic decimal system was described by an Arab mathematician, al-Khowarizmi. This system uses zero and place values. He also wrote a book titled *Algebra* which is the source of the name for that branch of mathematics.

3

In the 900's, important contributions were made to trigonometry by Arab astronomers.

4

In the 1000's, geometry was applied to optics by an Arab physician, Alhazen.

5

About 1100, an important book about algebra was written by Omar Khayyam. He was a Persian astronomer and poet.

6

Around 1100, the Hindu-Arabic numeral system came to Europe when a Latin translation of al-Khowarizmi's book was made.

7

In 1202, an algebra book promoting the Hindu-Arabic numeral system was published by Leonardo Fibonacci . He was an Italian mathematician. Eventually Roman numerals were replaced by Hindu-Arabic numerals in Europe.

During the Renaissance in the 1400's and 1500's, mathematics was applied to navigation and to the creation of art.

History of Mathematics
(16th Century to 17th Century)

About 1533, trigonometry was established as separate from astronomy by Regiomontanus, a German mathematician.

1

Around 1591, Francois Viete, a French mathematician, wrote a book about his advancements in the field of algebra.

2

In 1543, Nicholas Copernicus realized that the sun is the center of our universe rather than Earth as had been believed previously. He was a Polish astronomer whose book aroused interest in applying mathematics to the study of celestial motions.

3

In 1545, Girolamo Cardano, an Italian mathematician, wrote about the method of solving equations which included negative numbers in his algebra book, *Ars Magna.*

4

In the early 1600's, Johannes Kepler contributed to calculus through his work in finding the volume of wine barrels and his discovery of three laws of planetary motion.

5

By 1614, logarithms had been discovered by John Napier, a mathematician in Scotland. These simplified complex calculations have been used in astronomy.

6

Around 1600, Galileo applied simple mathematical descriptions to the physics of motion, paving the way fro the mathematics of modern physics. He was an Italian mathematician, astronomer and scientist.

7

In 1637, a French philosopher, Rene Descartes, invented analytic geometry to show that mathematics is the perfect model for exact and certain reasoning. He named numbers involving the square roots of negative numbers *imaginary.* He named rational and irrational numbers that can be found on a number line *real.*

8

During the 1600's, a French mathematician, Pierre de Fermat, laid the basis for calculus through his work with infinitesimals. He was the founder of modern number theory. Archimedes work on infinitesimals was still unknown at this time.

9

In the 1650's in France, Pascal investigated probability theory with de Fermat. Pascal was not only a mathematician but also a philosopher, physicist and Christian morality writer. He invented a calculating machine that was the "computer" of his time.

About 1665, Sir Isaac Newton, an English scientist, developed calculus in his study of motion, gravity and optics. His book, *Principia Mathematica,* is one of the greatest contributions in the history of science.

In the 1670's, Gottfried von Leibniz, a German, developed calculus without knowledge of Newton's work. He proposed an international symbolic language to guide logical reasoning.

12

History of Mathematics

(18th to 19th Centuries)

In the 1700's, a Swiss family of mathematicians made important contributions to mathematics. Jakob Bernoulli worked on probability theory and analytic geometry. His brother, Johann, did work in mathematical astronomy, physics and analytic geometry. Nicolaus, son of Johann, made important contributions to probability theory. Another son, Daniel, applied mathematics to motions in fluids and properties of vibrating strings.

1

In the mid 1700's, Leonhard Euler showed that differential and integral calculus operations were the opposite of each other. He was a Swiss mathematician.

2

In the late 1700's, Joseph Lagrange, a French mathematician, developed calculus in terms of algebra rather than geometry.

3

Around 1800, many French textbooks on mathematics were published for university education. Among the authors who were influential were Legendre, Cauchy, and Fourier. In calculus, Cauchy developed the concept of limit which is never reached in an infinite series.

4

In the early 1800's, a German mathematician, Carl Friedrich Gauss, conducted his work on imaginary numbers and non-Euclidean geometry. He also proved the fundamental theorem of algebra: Every equation has at least one root.

5

About 1830, other non-Euclidian geometries were developed by Bolyai of Hungary, Lobachevsky, of Russia, Riemann of Germany.

6

In the early 1B00's, another German mathematician, August Ferdinand Mobius, established the field of geometry known as topology. This is concerned with properties of geometric figures which are not altered when the figures are deformed.

7

From 1834 until he died in 1871, Charles Babbage of Great Britain worked on a mechanical computing machine. Technology of that time was not adequate for the building of such a precise instrument. It was not until 1944 that the first digital computer could be produced.

8

In 1847, British mathematician, George Boole, developed a system of symbolic logic to be applied to an algebra of sets. This is now known as Boolean algebra.

9

In 1895, Henri Poincare, a French mathematician, made contributions to celestial mechanics, probability theory and electromagnetic wave theory. His most important work was in the area of topology which now is a major branch of mathematics.

Beginning in 1895, an Italian logician, Giuseppe Peano, rewrote all of mathematics in his symbolic language derived from Leibniz's idea. It has not seen much use because of its complexity.

In the late 1800's, Georg
Cantor formulated set theory
and a theory of the infinite. He
was a German mathematician.

History of Mathematics
(20th Century)

About 1910, a philosophy of mathematics called logicism was developed by Alfred North Whitehead and Bertrand Russell, both British. They believed that all mathematical propositions can be derived by logic from a set of axioms which are basic true statements.

1

In the early 1900's a German mathematician, David Hilbert, believed mathematics to be a formal set of rules. He studied imaginary spaces with an infinite number of dimensions.

2

Also in the early 1900's, Brouwer, a Dutch mathematician, stated his belief that laws of mathematics are understood by intuition, not by reason or experience.

3

During the first half of the twentieth century, Albert Einstein, a German-born American, attempted to prove his scientific discoveries mathematically. His most well-known equation is E = me 2.

4

Around 1930, Kurt Godel, an Austrian mathematician, showed that not all theorems can be proven true or false by the axioms of any logical system.

5

In the later 1900's, the study of abstract mathematical structures included the *group,* a collection of items such as numbers with rules for operations. It is applied in areas such as subatomic physics.

6

In the 1970's, computer-based mathematical models were developed to be applied to many systems. This is especially valuable in weather forecasting.

7

In 1981, two Japanese mathematicians calculated rr to two million decimal places.

Branches of Mathematics

Arithmetic

The branch of mathematics concerned with calculations made by counting, measuring, grouping and comparing quantities. It involves the operations of addition, multiplication, division and subtraction with whole numbers, fractions.decimals and square roots. Types of arithmetic are real number, clock or modular, transfinite and arithmetic without numbers.

1

Algebra

The branch of mathematics involving the use of letters to represent variables or unknown quantities in equations to solve problems. Operations also include the use of negative numbers and imaginary numbers. Types of algebra are matrix, vector, sets, real numbers and complex numbers.

2

Geometry

The branch of mathematics which deals with properties of figures such as size and shape and with space. The types of geometry are plane, solid, spherical, Euclidean, non-Euclidean, analytic and transformation.

3

Trigonometry

The branch of mathematics used in calculating distances and angles through study of relationships between angles and sides in triangles.

4

Calculus

The branch of mathematics dealing with variable infinitesimal quantities and with limits which can never be reached. The types of calculus are differential, used to calculate changes in one variable when a related variable changes, and integral calculus, used to calculate sums of infinitesimals after determining a limit.

5

Analysis

The branch of mathematics involving the study of infinite series, sequences of numbers or algebraic expressions that continue indefinitely.

6

Probability

The branch of mathematics dealing with determination of chances for the occurrence of uncertain events.

7

Statistics

The branch of mathematics which analyzes data to determine trends or patterns.

8

Set theory

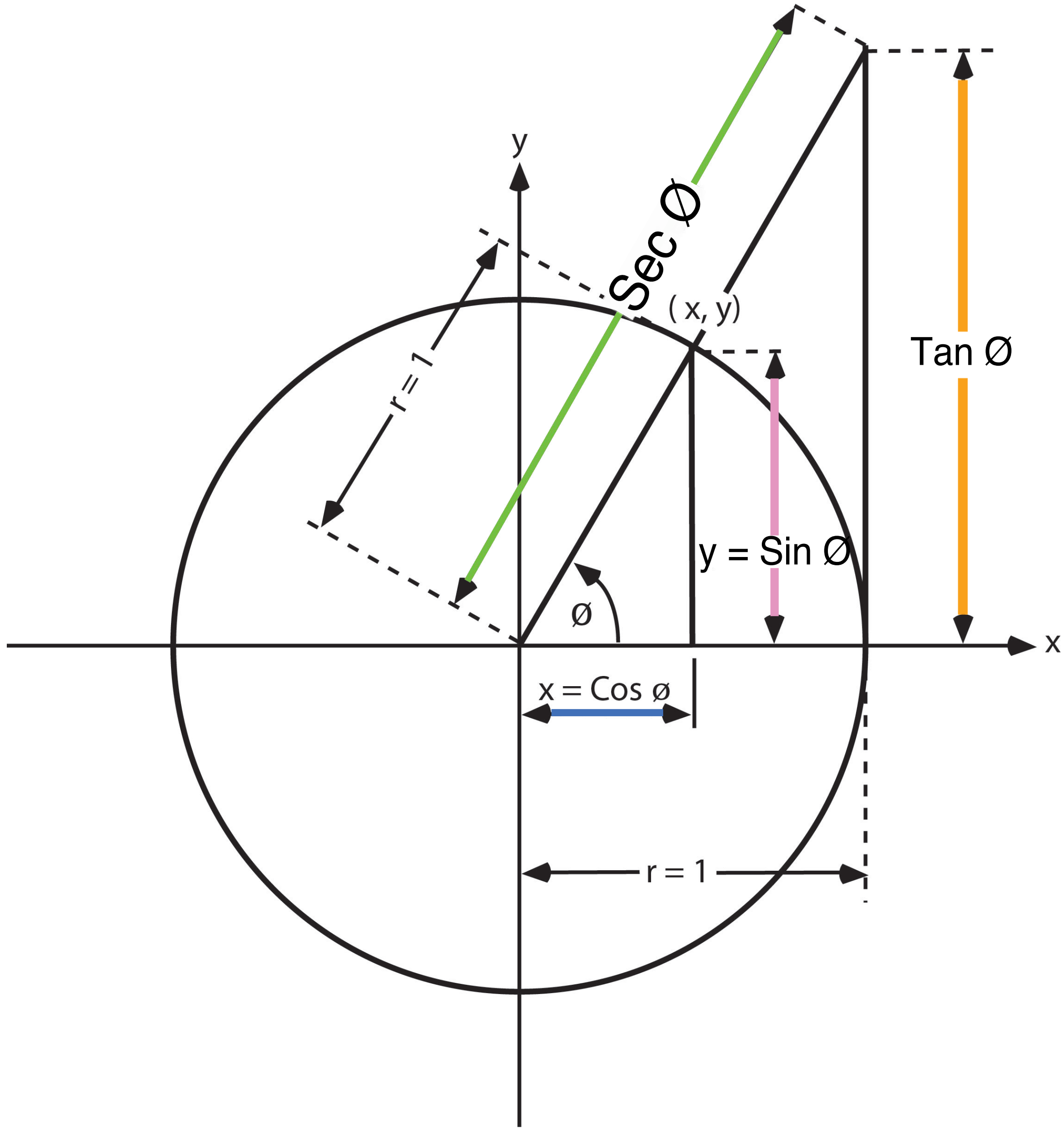
The branch of mathematics concerned with the nature and relations of collections of items such as numbers, objects or ideas.

9

Logic

The branch of mathematics dealing with symbolic logic, a formal system of reasoning that employs mathematical symbols and methods. The development of computers has been made possible by these methods.

10

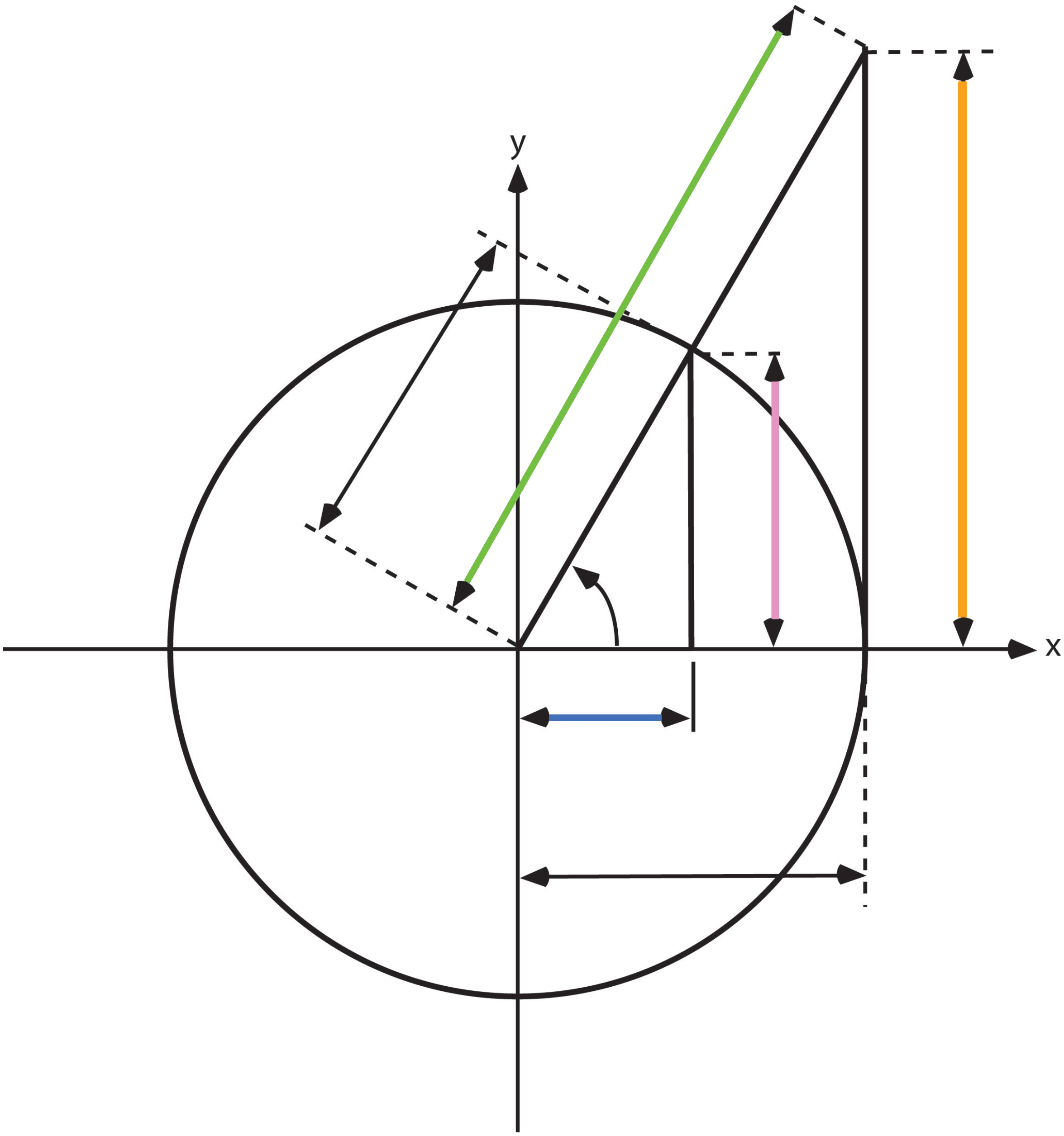


sine function= Sin ø= y

cosine function= Cos ø= x

tangent function = Tan Ø = $\frac{y}{x}$ = $\frac{\text{Sin Ø}}{\text{Cos Ø}}$

secant function = Sec Ø = $\frac{1}{x}$ = $\frac{1}{\text{Cos Ø}}$



sine function= Sin ø = y

cosine function= Cos ø= x

tangent function= Tan ø= $\frac{Y}{x}$= $\frac{\text{Sin ø}}{\text{Cos ø}}$

secant function = Sec ø = $\frac{1}{x}$ = $\frac{1}{\text{Cos ø}}$

r = 1

r = 1

Sec ø

Tan ø

(x,y)

y = Sin ø

x=Cos ø

ø

Sin ø	y	or	$\frac{\text{opposite}}{\text{hypotenuse}}$
Cos ø	x	or	$\frac{\text{adjacent}}{\text{hypotenuse}}$
Tan ø	$\frac{y}{x}$	or	$\frac{\text{opposite}}{\text{adjacent}}$
Sec ø	$\frac{1}{x}$	or	$\frac{\text{hypotenuse}}{\text{adjacent}}$

History of Trigonometry

As early as 1600 B.C., there is evidence that triangles were used to determine distances that could not be measured directly. Egyptian papyri and Babylonian clay tablets recorded that practical problems were solved by measuring triangles.

1

Hipparchus of Alexandria, Egypt, advanced the science of trigonometry about 140 B.C. through its use in astronomical observations. He is said to have calculated a table of chords.

2

Ptolemy developed a "Table of Chords" by the second century A.D. Around the same time, Menelaus wrote about spherical geometry and trigonometry. At this time, no number system had been perfected. With no formulas, each computational step had to be described in words.

3

The love of astronomy by early Hindus and Greeks required a way to solve spherical triangles. Therefore, spherical trigonometry developed before plane trigonometry. Late in the ninth century, Al Battani of Arabia developed the law of cosines and introduced the sine for the chord into Ptolemy's work. He also contributed tangent and cotangent functions and their tables.

4

At the end of the tenth century, Abu'l Wafa improved the accuracy for computing sines and introduced the secant and cosecant.

5

It was not until the thirteenth century that trigonometry was separated from astronomy to become an independent science. The Persian mathematician, Nasiraddin, accomplished this task. In Europe Regiomontanus reproduced Nasiraddin's work independently in the fifteenth century.

6

With the advancement of arithmetic and algebra, the successors of Regiomontanus simplified trigonometry and developed essential formulas for calculations.

7

Early in the sixteenth century Copernicus, a Polish astronomer, proposed that the sun was the center of the universe, not the earth. Because more accurate astronomical calculations and observations were required to prove his theory, it was necessary to extend existing trigonometric tables. These were developed to fifteen places for every angle from zero to ninety degrees in ten-second increments, all without calculators or computers. This task was not completed until about 1700.

8

After the invention of logarithms by John Napier in the early seventeenth century, the development of formulas was stimulated.

9

In the eighteenth century, Moivre and Euler developed analytic trigonometry. By 1807, Joseph Fourier's work on trigonometric series was published.

10