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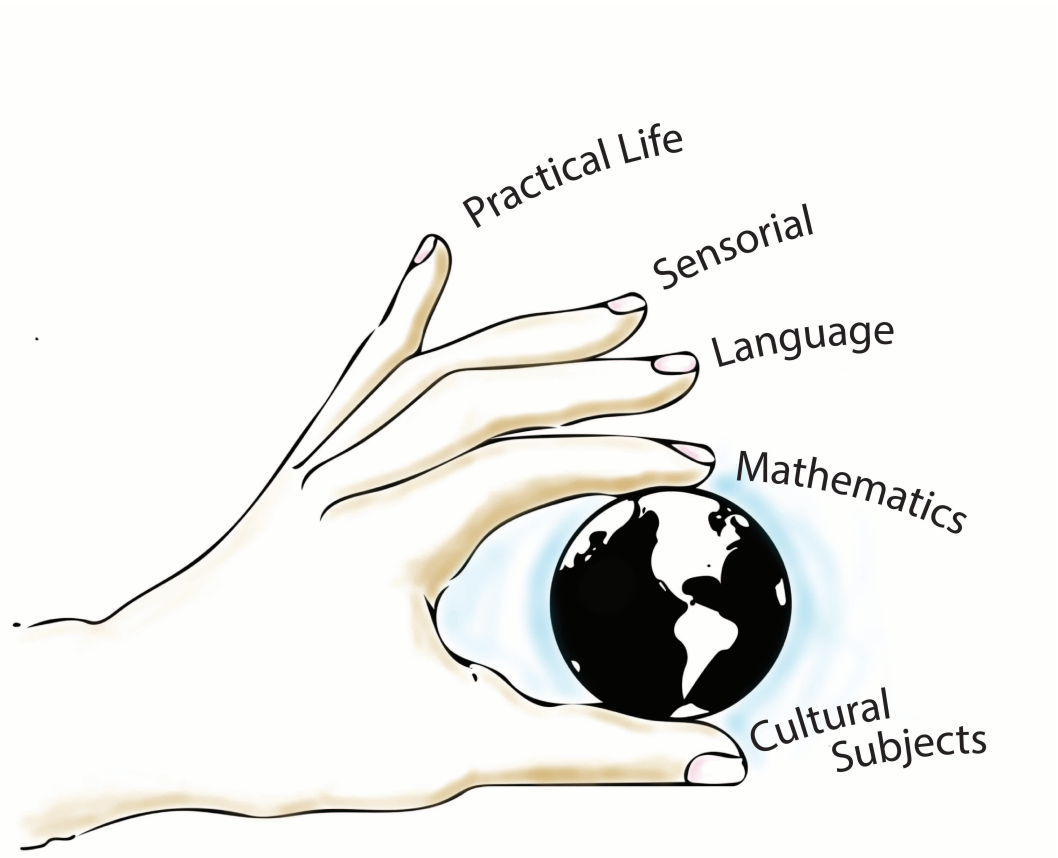

Mathematics

Elementary

Teacher Manual

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MEI, Inc. Mathematics Elementary

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Pegboard Operations

Purposes

To prepare for more abstract operations in mathematics To reinforce concepts

Preliminary Exercises:

Experience with golden bead operations and recording on symbolically colored paper

Materials

Presentation tray with golden beads
Boxes of pegs: two of green, one of blue, one of red
Felt mat
Large pegboard (minimum 28 holes by 28 holes)
Quarter inch graph paper cut in strips 2 3/4 inches wide and 11 inches long
Recording booklets 2 3/4 by 2 3/4 inches with quarter inch graph paper, six pages per book and symbolically colored covers (red for addition, green for subtraction, yellow for multiplication and blue for division)
Problem booklets or cards as used for golden bead operations Small green skittles

Introduction to Counting Pegs

Procedure

1. Invite a child to the lesson and place presentation tray with golden beads near the center of a table between you and the child. Place the boxes of pegs in front of the tray.
2. Arrange the boxes of pegs as follows right to left: green (units), blue (tens), red (hundreds), green (thousands). Open the boxes and place lids in front of them.
   Note: In all operations with pegs, the boxes always remain in the order given in Step 2
3. Spread the felt mat before the child. Remove the unit bead from the presentation tray and place on the mat in front of the green box on the far right. Remove one green peg from its box on the far right and place it to the right of the unit bead. Say that each green peg from the far right box represents one unit.
4. Remove the ten bead bar from the presentation tray and place on the mat in front of the blue box. Remove one blue peg from its box and place it to the right of the ten bar. Say that each blue peg represents one ten.
5. Remove the hundred bead square from the presentation tray and place on the mat in front of the red box. Remove one red peg from its box and place it to the right of the hundred square. Say that each red peg represents one hundred.
6. Remove the cube from the presentation tray and place on the mat in front of the green box on the far left. Remove one green peg from its the box on the far left and place it to the right of the cube. Say that each green peg from the far left box represents one thousand.
7. Replace the golden beads on the presentation tray. Return pegs to boxes. Remove the felt mat.
8. Tell the child to count green unit pegs into the box lid. If necessary, remind the child to stop after ten units have been counted. The ten unit pegs are returned to the box and changed for one blue peg. Have the child count ten blue ten pegs and change for one red hundred peg. Have the child count ten red hundreds pegs and change for one green thousand peg.
9. Ask the child to return the pegs to their boxes. Boxes remain on the table if the next lesson continues, but the presentation tray and mat are returned to the storage place.

Introduction to Recording of Peg Board Exercises

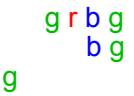
Procedure

1. Ask the child to get a strip of quarter inch graph paper and a pencil.
2. Place any number of pegs up to nine in each of the box lids. Ask the child to place the pegs vertically on the peg board, beginning with units, then to count and record each quantity in the proper place value location on the graph paper strip. The pegs are returned to the boxes and the exercise is repeated several times.
3. Write numerals for quantities no greater than nine for any place value location on the graph paper strip and tell the child to count the designated amounts into the lids, then transfer to the pegboard. Repeat several times.
4. Thank the child and say that the peg board may be chosen. Request that the materials be returned to their proper storage place.

Operations with the Peg Board

Procedure for Addition without Changing

1. Invite the child to the lesson. Ask the child to place the pegboard at the front of the table with the open boxes of pegs at the left.
2. Have the child choose an addition problem booklet without changing, previously used with golden bead addition, then get a recording booklet with a red cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet into the recording booklet, then to count into the box lids the quantity of pegs designated by the first addend of the problem. Tell the child to transfer these to the pegboard, starting at the top right with green unit pegs. For example, in the problem 1123+2342, three green units pegs are placed vertically starting at the top right of the pegboard, two blue tens pegs are placed vertically to the left of the units, one red hundreds peg is place left of tens and one green thousands peg is placed left of hundreds.



4. Have the child count into the box lids the quantity indicated by the second addend and transfer to the peg board. In this example, pegs are placed as follows: two green unit pegs below the other units, four blue tens pegs below the other tens, three red hundreds pegs below the other hundreds and two green thousands pegs below the other thousands.

5. Ask the child to move the unit pegs to the bottom of the board while counting, then record the amount in the recording booklet. Repeat with tens, hundreds and thousands.
6. Have the child clear the board of pegs and repeat the exercise with other addition problems.

Procedure for Addition with Changing

1. Invite the child to the lesson . Ask the child to place the pegboard at the front of the table with the open boxes of pegs at the left.
2. Have the child choose an addition problem booklet with changing, previously used with golden bead addition, then get a recording booklet with a red cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet into the recording booklet, for example, 3656+1755, then to count into the box lids the quantity of pegs designated by the first addend of the problem. Tell the child to transfer these to the pegboard, starting at the top right with green unit pegs.

g r b g
g r b g
g r b g
r b g
r b g
r g

4. Have the child count into the box lids the quantity indicated by the second addend and transfer to the peg board.

g r b g
g r b g
g r b g
r b g
r b g
r g

g r b g
r b g
r b g
r b g
r b g
r
r

5. Ask the child to move the unit pegs to the bottom of the board while counting and changing when ten have been counted. The amount is recorded in the recording booklet. Repeat with tens, hundreds and thousands. In this example, there will be one unit, one ten, four hundreds and five thousands.
6. Have the child clear the board of pegs and repeat the exercise with other addition problems.

Procedure for Multiplication without Changing

1. Invite the child to the lesson . Ask the child to place the pegboard at the front of the table with the open boxes of pegs at the left.
2. Have the child choose a multiplication problem booklet without changing, previously used with golden bead multiplication, then get a recording booklet with a yellow cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet onto the recording paper, for example, 2341 X 2, then to count into the box lids the quantity of pegs designated by the multiplicand of the problem. In this example, there will be one green unit peg, four blue tens pegs, three red hundreds pegs and two green thousands pegs. Tell the child to transfer these to the pegboard, starting at the top right with green unit pegs.

4. Have the child count pegs into the box lids, using the same quantity for the multiplicand as above and transfer to the peg board. Pegs representing the multiplicand are repeated as many times as indicated by the multiplier.

   In this example, the multiplier is two, so pegs representing the multiplicand are placed on the board two times

.

5. Ask the child to move the unit pegs to the bottom of the board while counting, then record the amount in the recording booklet. Repeat with tens, hundreds and thousands.
6. Have the child clear the board of pegs and repeat the exercise with other multiplication problems.

Procedure for Multiplication with Changing

1. Invite the child to the lesson . Ask the child to place the peg board at the front of the table with the open boxes of pegs at the left.

2. Have the child choose a multiplication problem booklet with changing, previously used with golden bead multiplication, then get a recording booklet with a yellow cover and a pencil which are placed on the dominant side.

3. Ask the child to write a problem from the booklet onto the recording paper, for example, 1645 x 3, then to count into the box lids the quantity of pegs designated by the multiplicand. Tell the child to transfer these to the pegboard, starting at the top right with green unit pegs.

4. Have the child count pegs into the box lids for the quantity of the multiplicand and place on the peg board as many times as indicated by the multiplier.

   In this example, the multiplier is three, so the multiplicand, 1645, is placed on the pegboard three times.

5. Ask the child to move the unit pegs to the bottom of the board while counting and changing when ten have been counted. The amount is recorded in the recording booklet. Repeat with tens, hundreds and thousands.

6. Have the child clear the board of pegs, then do other multiplication problems.

Procedure for Subtraction without Changing

1. Invite the child to the lesson . Ask the child to place the pegboard at the front of the table with the open boxes of pegs at the left.
2. Have the child choose a subtraction problem booklet without changing, previously used with golden bead subtraction, then get a recording booklet with a green cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet into the recording booklet, then to count into the box lids the quantity of pegs designated by the minuend of the problem. Tell the child to transfer these to the pegboard, starting at the top right with green unit pegs.

   For example, in the problem 4563- 1231, the minuend is 4563, so three green units pegs, six blue tens pegs, five red hundreds pegs and four green thousand pegs are placed on the pegboard.

   g r b g
   g r b g
   g r b g
   g r b
   r b
   b

4. Have the child count and transfer the quantity of pegs indicated by the subtrahend to a location three rows below the minuend.

   In this example, the subtrahend which consists of one green units peg, three blue tens pegs, two red hundreds pegs and one green thousands pegs, is removed from the minuend and placed three rows below it

   g r b g
   g r b g
   g r b

   g r b g
   r b
   b

5. Ask the child to count and transfer the pegs at the top of the pegboard to a location three rows below the subtrahend. This represents the difference which is recorded in the recording booklet.

   In this example, the difference is 3332.

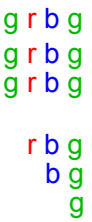
   g r b g
   r b
   b

   g r b g
   g r b g
   g r b

6. Have the child clear the board of pegs and repeat the exercise with other subtraction problems.

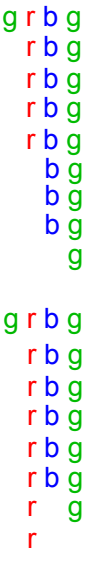
Procedure for Subtraction with Changing

1. Invite the child to the lesson . Ask the child to place the peg board at the front of the table with the open boxes of pegs at the left.
2. Have the child choose a subtraction problem booklet with changing, previously used with golden bead subtraction, then get a recording booklet with a green cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet into the recording booklet, then to count into the box lids the quantity of pegs designated by the minuend of the problem. Tell the child to transfer these to the pegboard, starting at the top right with green unit pegs. For example, in the problem 3456-1589, the minuend is 3456, so six green units pegs, five blue tens pegs, four red hundreds pegs and three green thousands pegs are placed at the top of the pegboard.



4. Beginning with units, have the child count and transfer the quantity of pegs indicated by the subtrahend to a location three rows below the minuend. Changing may be required in more than one place, but the child is familiar with this procedure from previous work.

Drawing for item 4



Drawing for item 5

5. Ask the child to count and transfer the pegs from the top of the pegboard to a location three rows below the subtrahend. This represents the difference which is recorded in the recording booklet. In this example, the difference is 1867.
6. Have the child clear the board of pegs and repeat the exercise with other subtraction problems.

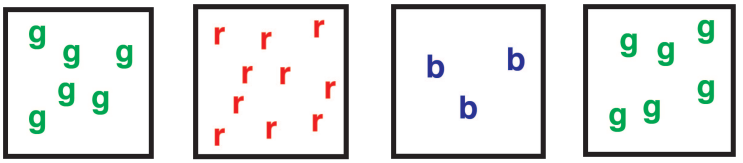
Variation:

Show the child how to record changing

Procedure for Division without Changing

1. Invite the child to the lesson. Ask the child to place the peg board at the front of the table with the open boxes of pegs at the left.
2. Have the child choose a division problem booklet without changing, previously used with golden bead division, then get a recording booklet with a blue cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet into the recording booklet, then to count into the box lids the quantity of pegs designated by the dividend of the problem.

   For example, in the problem 6936 divided by 3, pegs representing the dividend, 6936, are placed in the appropriate box lids.

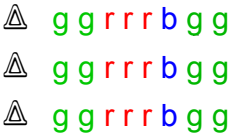


4. Tell the child to place small green skittles in a column on the left of the pegboard to represent the divisor.

   In this example, there will be three green skittles.

5. Starting with thousands, have the child give each skittle one green thousand peg in turn. The child continues to distribute the thousands pegs so that the pegs are in rows to the right of each skittle.

6. Ask the child to repeat division with red hundred pegs, placing them to the right of the green thousand pegs, then to divide blue ten pegs, placing them to the right of the red pegs and finally to divide green unit pegs, placing them to the right of the blue ten pegs.



7. To find what one skittle has received, tell the child to count the green thousand pegs on the bottom row and record that number, to count the red hundred pegs on the bottom row and record that number, to count the blue ten pegs and record, and lastly count the green unit pegs and record.
8. Have the child clear the board of pegs and repeat the exercise with other division problems.

Procedure for Division with Changing

1. Invite the child to the lesson. Ask the child to place the peg board at the front of the table with the open boxes of pegs at the left.
2. Have the child choose a division problem booklet with changing, previously used with golden bead division, then get a recording booklet with a blue cover and a pencil which are placed on the dominant side.
3. Ask the child to write a problem from the booklet into the recording booklet, then to count into the box lids the quantity of pegs designated by the dividend of the problem.

   For example, in the problem 7312 divided by 4, the pegs for the dividend, 7312, are placed in the box lids.
4. Tell the child to place small green skittles in a column at the left of the peg board to represent the divisor. In this example, there will be four skittles.
5. Starting with thousands, have the child give each skittle one green thousand peg in turn, for equal distribution. The pegs are placed in a row to the right of each skittle. Each peg remaining after equal distribution is changed for ten red hundred pegs which are counted into the hand before being added to the red pegs previously counted into the lid of the box as part of the dividend.
6. Ask the child to repeat division with red hundred pegs, placing them to the right of the green thousand pegs, then to divide blue ten pegs placing them to the right of the red pegs and finally to divide green unit pegs placing them to the right of the blue ten pegs. Tell the child that any pegs remaining in the box lid after equal division of green unit pegs are known as the remainder.
7. To find what one skittle has received, tell the child to count the green thousand pegs on the bottom row and record that number, to count the red hundred pegs on the bottom row and record that number, to count the blue ten pegs and record, to count the green unit pegs and record and finally, to count the pegs remaining in the unit box lid and record to the right of the product with R preceding the remainder. In this example, the quotient is 1828 and there is no remainder.
8. Have the child clear the board of pegs and repeat the exercise with other division problems.

Control of Error

Symbolic colors of pegs
Problem booklets or cards for operations with and without changing with answers on back

Compound Multiplication with Golden Beads

Purposes

To establish a basis for understanding compound multiplication
To prepare for exercises involving squares and square roots To provide a foundation for long division

Preliminary exercises

Simple multiplication with golden beads

Materials

Tray of golden bead material with nine thousand cubes, thirty hundred squares, box of tens, box of units
Large felt mat
Yellow ribbon 1/2 inch wide and as long as the width of the mat
Yellow ribbon 1/4 inch wide and as long as the width of the mat
Paper lined with symbolic colors, pencil
Problem booklets or cards for multiplying by ten; by multiples of ten; by one hundred; by multiples of one hundred; by units and tens; by units, tens and hundreds
White labels 1 inch by 4 1/4 inches for first partial product, second partial product and third partial product

Procedure for multiplication by ten

1. Invite a child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right with the yellow ribbon near the bottom so that there will be room for the combined beads below it.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for multiplying by ten and place on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 23 X 10.
4. Ask the child to lay out the the tens vertically and units horizontally ten times beginning at the top of the mat. In this example two ten bars and three units are laid out are laid out ten times. See illustration at right.
5. Beginning with the top right unit, have the child count the unit beads in the far right column, changing every ten units for a ten bar which is placed below the yellow ribbon in the column of tens.
6. Have the child count the ten bars in their far right column, exchanging every ten ten bars for a one hundred square, stacking the hundred squares one on top of the other below the yellow ribbon at the left of tens place.
7. Tell the child to count and record the amount.
   In this example, there are two hundreds, three tens, or 230.

8. The child may continue to do additional problems or put the materials away.

Variations

Invite the child to do problems involving multiplication by ten with multiplicands containing hundreds.

When the child realizes that multiplication by ten involves changing units to tens, tens to hundreds, hundreds to thousands, tell the child to lay out the quantity for the multiplicand one time. Say that one multiplied by ten is ten, so each unit bead in the multiplicand can be exchanged for a ten bar. Say that ten multiplied by ten is one hundred, so each ten can be exchanged for a hundred square. Likewise, each hundred can be changed for a thousand. Therefore, multiplication is possible without laying out the multiplicand ten times.

Procedure for multiplication by multiples of ten

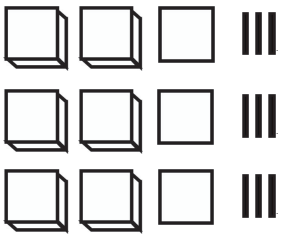
1. Invite a child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right with the yellow ribbon near the bottom so that there will be room for the combined beads below it.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for multiplying by multiples of ten and place on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 213 X 30.
4. Ask the child to lay out the units, tens and hundreds , according to the quantity specified by the multiplicand, near the top of the mat.

   In this example two hundreds, one ten and three units are laid out with hundreds side by side, tens placed vertically and units horizontally.

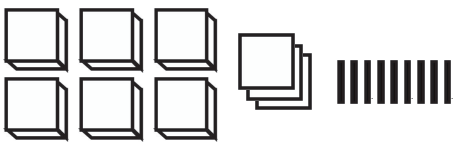
5. Remind the child that each unit bead in the multiplicand can be exchanged for a ten bar, each ten in the multiplicand can be exchanged for a hundred square, each hundred in the multiplicand can be exchanged for a thousand cube. The child proceeds to change the multiplicand, each unit for a ten, each ten for a hundred, each hundred for a thousand.

6. Tell the child that the multiplicand has now been multiplied by ten, but the multiplier indicates more than one ten. Ask how many tens are in the multiplier. Have the child lay out the quantity of beads derived from step 5 the number of times indicated by the multiple of ten in the multiplier.

   In this example, 2130 is laid out three times.



7. Tell the child to count vertically and exchange bead quantities, placing them below the yellow ribbon. All beads below the yellow ribbon are counted and the amount is recorded. In this example, there are six thousands, three hundreds and nine tens or 6390.



8. The child may continue to do additional problems or put the materials away.

Procedure for multiplication by one hundred

1. Invite a child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right with the yellow ribbon near the bottom so that there will be room for the combined beads below it.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for multiplying by one hundred and place on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 32 X 100.
4. Ask the child to lay out the quantity specified by the multiplicand, near the top of the mat.

   In this example three tens are laid out vertically and two units horizontally.

5. Tell the child that when multiplying by one hundred, units are changed for hundreds and tens are changed for thousands..

   In this example, there are three cubes of a thousand and two hundred squares.

6. The beads above the yellow ribbon are moved below it. All beads below the yellow ribbon are counted and the amount is recorded.

   In this example, are be three thousand cubes, two hundred squares or 3200.

7. The child may continue to do additional problems or put the materials away.

Procedure for multiplication by multiples of one hundred

1. Invite a child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right with the yellow ribbon near the bottom so that there will be room for the combined beads below it.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for multiplying by multiples of one hundred and place on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 24 X 200.
4. Ask the child to lay out the tens vertically and units horizontally, according to the quantity specified by the multiplicand, near top of the mat.

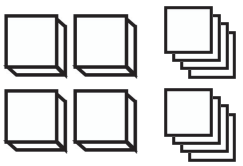
   In this example two tens are placed vertically and four units horizontally.

5. Remind the child that each unit bead in the multiplicand can be exchanged for a hundred square, each ten in the multiplicand can be exchanged for a thousand cube when multiplying by one hundred. The child proceeds to change each unit for a hundred square, each ten for a thousand cube.

   In this example, there are two thousands, four hundreds.

6. Tell the child that the multiplicand has now been multiplied by one hundred, but the multiplier is more than one hundred. Ask how many hundreds are in the multiplier. In this example, two. Say that the amount derived from step 5 must be laid out the number of times indicated by the number of hundreds in the multiplier.

   In this example, two thousands and four hundreds are laid out two times..



7. After laying out the beads, have the child count and exchange beads when necessary, placing them below the yellow ribbon according to place value. Tell the child to record the answer.
   In this example, 4800 is recorded.

8. The child may continue to do additional problems or put the materials away.

Control of Error

Problem booklets or cards with answers on the back Yellow ribbon

Procedure for multiplication by tens and units

1. Invite a child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right with the wide yellow ribbon near the bottom so that there will be room for the combined beads below it.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for multiplying by tens and units and place on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 23 X 22.
4. Remind the child that the multiplicand is first multiplied by units so the quantity for the multiplicand is laid out with tens placed vertically and units horizontally according to the number of times designated by the units in the multiplier.

   In this example, two tens and three units are laid out two times, near the top of the mat.

5. Have the child place the narrow yellow ribbon below the beads, move the beads below the narrow ribbon as they are counted and place the first partial product label to the right of the sum.

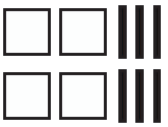
   In this example, the first partial product is 23 X 2 or 46.

```
  23
x 22
  46
```

(first partial product)

first partial product

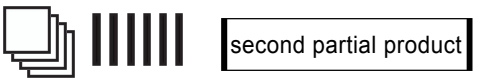
6. Tell the child to lay out the multiplicand again and multiply the multiplicand by ten as previously practiced, changing units to tens and tens to hundreds. This amount is laid out the number of times indicated by the multiple of ten in the multiplier. In this example, three units are changed for three tens and two tens are changed for two hundreds. Since the multiplier indicates two tens, this amount is laid out again.



7. Tell the child to place the narrow yellow ribbon below these beads, and to move the beads below the ribbon as they are counted to give the second partial product. The second partial product label is placed to the right of this sum.

   In this example, the second partial product is 23 X 20 or 460.

```
  23
x 22
  46
 460   (second partial product)
```



8. Have the child add the first and second partial products, exchanging if necessary, by counting and moving the beads below the wide yellow ribbon. In this example, there are six units, no tens and four hundreds below the wide yellow ribbon.

9. Tell the child to record the amount.

   In this example, 46+ 460 = 506.

```
  23
x 22
  46
 460
 506
```

10. The child may continue to do additional problems or put the materials away.

Variations

Invite the child to do problems involving multiplication by tens and units with multiplicands containing hundreds.

Invite the child to record partial products on the recording paper before writing the final product.

Procedure for multiplication by hundreds, tens and units

1. Invite a child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right with the yellow ribbon near the bottom so that there will be room for the combined beads below it.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for multiplying by hundreds, tens and units and place on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 23 X 212.
4. Remind the child that the multiplicand is first multiplied by units so the multiplicand is laid out with units and tens placed the number of times designated by the units in the

   multiplier. In this example, two tens and three units are laid out two times.

5. Have the child place the narrow yellow ribbon below the beads, move the beads below the narrow ribbon as they are counted and place the first partial product label to the right of the sum.

   In this example, the first partial product is 23 X 2 or 46.

```
  23
x 212
  46   (first partial product)
```

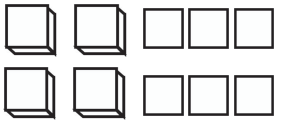
6. Tell the child to lay out the multiplicand again and multiply the multiplicand by ten as previously practiced, changing units to tens and tens to hundreds. This amount is laid out the number of times indicated by the multiple of ten in the multiplier. In this example, three units are changed for three tens and two tens are changed for two hundreds. Since the multiplier indicates one ten, this amount is not laid out again.

7. Tell the child to place the narrow yellow ribbon below these beads, and to move the beads below the ribbon as they are counted to give the second partial product. The second partial product label is placed to the right of this sum..

   In this example, the second partial product is 23 X 10 or 230.

```
  23
x 212
  46
 230   (second partial product)
```

8. Tell the child to lay out the multiplicand again and multiply the multiplicand by one hundred. This amount is laid out the number of times indicated by the multiple of a hundred in the multiplier. In this example, three units are changed for three hundreds and two tens are changed for two thousands. Since the multiplier indicates two hundreds, this amount is laid out again.



9. Tell the child to place the narrow yellow ribbon below these beads, and to move the beads below the ribbon as they are counted to give the third partial product. The third partial product label is placed to the right of this sum.

   In this example, the third partial product is 23 X 200 or 4600.

```
  23
x 212
  46
 230
4600  (third partial product)
```

10. Have the child add the three partial products by counting and moving the beads below the wide yellow ribbon.

    In this example,

    23 x 2 or 46 (1st partial product) plus 23 x 10 or 230 (2nd partial product) plus 23 x 200 or 4600 (3rd partial product) = 4876

```
  23
x 212
  46
 230
4600
4876
```

11. The child may continue to do additional problems or put the materials away.

Variations

Invite the child to do problems involving multiplication by hundreds, tens and units with recording of partial products.

Compound Multiplication with Stamps

Purposes:

To prepare for more abstract operations in mathematics To reinforce mathematical concepts

Preliminary Exercises:

Individual compound multiplication with golden beads Simple multiplication with changing with stamps

Materials:

Container with green stamps marked 1000, red stamps marked 100, blue stamps marked 10 and green stamps marked 1

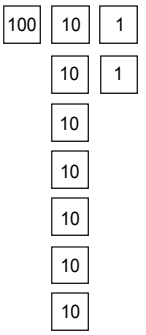
Paper with symbolically colored lines for recording problems, pencil

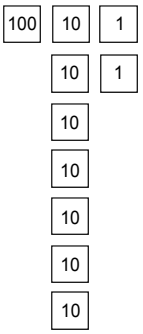
Problem cards or booklets with compound multiplication problems

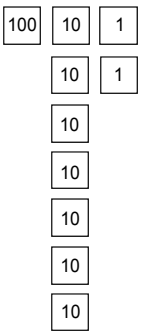
Procedure

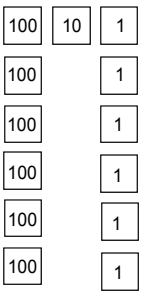
1. Invite the child to bring the container of stamps and place at the top of a table with the stamps in place value order.
2. Have the child get recording paper, a pencil and a problem booklet or card for multiplying by units, tens and hundreds which are placed on the child's dominant side.
3. After the child copies a problem, for example,172 X 23, remind the child that the multiplicand is first multiplied by units. Tell the child to count the specified number of stamps, then place them at the center of the table in proper place value location. In this example, one red hundred stamp, seven blue ten stamps and two green units stamps are placed on the table This is repeated the number of times indicated by the units multiplier, in this example, three. Have the child count, change and record this amount as the first partial product.

   In this example, it is 516.



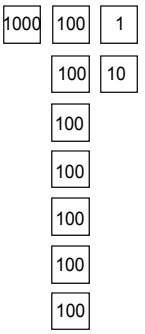






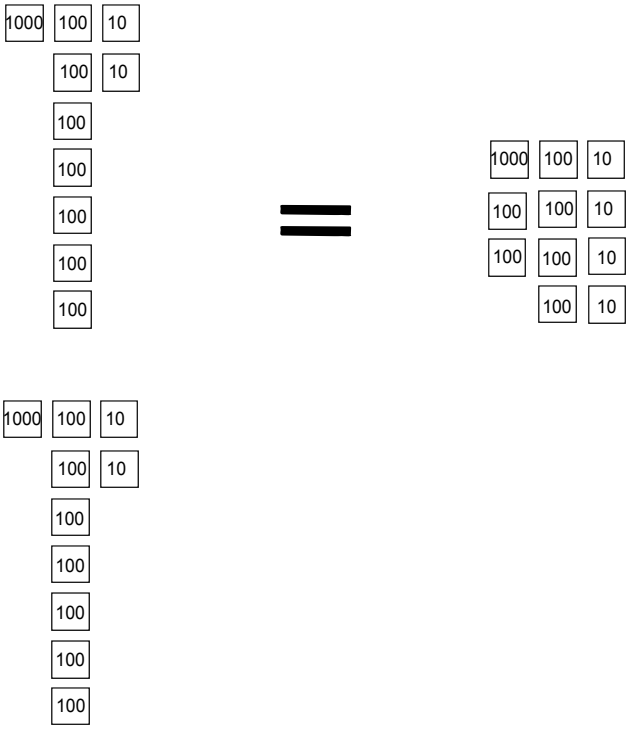
4. Tell the child to place the amount of the multiplicand on the table. In this example, it is 172. Remind the child that multiplication by ten involves changing units in the multiplicand to tens, tens in the multiplicand to hundreds, hundreds in the multiplicand to thousands. Have the child change the stamps to multiply by ten.

   In this example, the two green unit stamps are changed for two blue ten stamps, the seven blue ten stamps are changed for seven red hundred stamps, the one red hundred stamp is changed for one green thousand stamp.



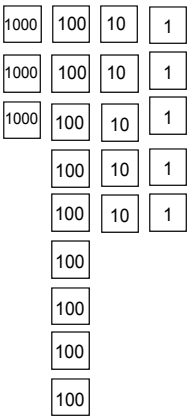
5. Ask the child to determine how many tens are in the multiplier and to place the stamps from the previous step according to the number of times indicated. In this example, it is two times, so two blue ten stamps, seven red hundred stamps,and one green thousand stamp are placed on the table below the previous amount. Have the child add these amounts to obtain the second partial product, changing as necessary.

   In this example, it is 3440.



6. Tell the child to count, change when necessary and record the amount represented by the first and second partial products stamps.

   In this example, it is 3956.



7. Invite the child to continue to do additional problems or put the materials away.

Control of Error:
Answers on back of cards or booklets.

Expanded Notation, Place Value and Periods

Purposes

To further develop understanding of place value To facilitate the reading of large numbers
To aid mathematical operations

Preliminary Exercises

Practice with golden bead problems

Materials

Small numeral cards previously used with golden beads
Container with controls for periods and mute cards for matching
Container of small cards on which are drawn large commas
Container of color coded small numeral cards from simple to billion periods with place value indicated on the reverse Recording paper and pencil

Procedure for Expanded Notation

1 '. Invite a child to bring paper, pencil and numeral cards and place at the center of a table with paper and pencil at the dominant side.

2. Tell the child to build any four place number with the cards and write it.

   For example, 2376.

   The child is familiar with place value represented by symbolic colors from previous golden bead problems stated with the numeral cards.

3. Have the child separate the cards so that they continue to be in the proper sequence. Ask the child to name the place value of each card, beginning with units, then write the expanded notation.

   For example, 2376 = 2000 + 300 + 70 + 6

4. Invite the child to continue building and expanding other numbers or to put the materials away.

Procedure for Periods

1. Invite a child to bring paper, pencil, container of commas numeral cards and place at the center of a table with paper and pencil at the dominant side.
2. Tell the child to build any four place number with the numeral cards, then w rite it.

   For example, 2376 .

3. Explain that to make it easier to read large numbers, the numerals are divided into groups of three, called periods. Show the child how to count three places, beginning at the right, then to place a comma between the third and fourth numerals of the small numeral cards.

   In this example, the movable comma is placed between two and three and the child writes the comma in its proper place 2,376.

4. Have the child return the small numeral cards and bring those for simple to billions periods. Ask the child to read the numerals on the cards, then read the place value on the reverse. State that periods are always counted from right to left. Tell the child to arrange the cards according to periods.
5. Write a large number, show how to count the three places and insert a comma in several periods, then read it aloud.

   For example, 1,356,487,951.

6. Have the child write large numbers, count periods of three from right to left and place commas accordingly, then read the number aloud.
7. Dictate large numbers and have the child write them, placing the commas appropriately. Ask the child to read aloud what has been written.
8. Continue until it is apparent that the child has grasped the concept.
   Note: It may be necessary to remind the child that, when reading numbers aloud, "and" is used to denote a decimal point.

Control of Error

The teacher
Symbolic colors of numeral cards
Name of periods on back of numeral cards

Rounding Numbers

Purposes

To develop an understanding of rounding numbers
To learn the procedure for rounding to the nearest units, tens and hundreds

Preliminary Exercises

Practice with materials involving place value Experience using bead chains

Materials

Bead chains for hundred and thousand
Strips on which to place chains
Container of arrows with numerals for bead chain counting
Container of rounding problems for tens
Container of rounding problems for tens, hundreds and thousands

Procedure

1. Invite the child to lay out the hundred bead chain on its strip, then to count and place the appropriate numeral arrows.

2. Have the child bring the container of rounding problems for tens and choose a problem. Ask the child to locate the problem number on the bead chain, then determine the nearest ten and record it. For example, "Round 37 to the nearest ten," the child locates the thirty-seventh bead and determines that it is closest to forty, so the child records the answer, 37 is rounded to 40.

3. Invite the child to continue with rounding problems or to put away the materials.

Control of Error

Arrows with numerals placed beside bead chain Answer on reverse of problem

Variations

Invite the child to use the thousand chain and problems for rounding to hundreds, following the procedure previously learned with the hundred chain.

Invite the child to do the rounding problems without using the chains.

Invite the child to estimate to the nearest number on any problems.

Compound Multiplication with the Pegboard

Purposes:

To prepare for more abstract operations in mathematics To reinforce concepts

Preliminary Exercises:

Simple multiplication with changing on the pegboard Compound multiplication with stamps

Materials:

Large pegboard (minimum 28 holes by 28 holes)
Boxes of pegs: two of green, two of blue, two of red
Recording booklet with quarter inch graph paper pages and yellow covers, pencil Problem cards or booklets with compound multiplication problems

Procedure for multiplication by ten

1. Invite the child to bring the pegs and pegboard and place the pegboard at the front of a table with the open boxes of pegs at the left in the usual order. Review the value of the pegs: green for units and units of thousands, blue for tens and tens of thousands, red for hundreds and hundreds of thousands.
2. Have the child get a recording booklet, a pencil and a problem booklet or card of problems for multiplying by ten which are placed on the child's dominant side.
3. After the child copies a problem, for example, 17 X 10, tell the child to count the specified number of pegs representing the multiplicand into the box lids. In this example, seven green unit pegs and one blue ten peg are counted into their lids, then placed on the board.

b g g g g g g g

4. Remind the child that multiplication by ten involves changing units to ten, tens to hundreds, hundreds to thousands. Have the child change the pegs to multiply by ten. In this example, the seven green unit pegs are changed for seven blue ten pegs and the blue ten peg is changed for one red hundreds peg.

r b b b b b b b

5. Tell the child to count and record the amount represented by the pegs. In this example, 170.
6. Invite the child to continue to do additional problems or put the materials away.

Procedure for multiplication by multiples of ten

1. Invite the child to bring the pegs and pegboard and place the pegboard at the front of a table with the open boxes of pegs at the left in the usual order.
2. Have the child get a recording booklet, a pencil and a problem booklet or card of problems for multiplying by multiples of ten to be placed on the child's dominant side.
3. After the child copies a problem, for example, 113 X 40, tell the child to count the specified number of pegs representing the multiplicand into the box lids.

   In this example, three green unit pegs, one blue ten peg and one red hundred peg are counted into their lids. The pegs are placed on the board.

   r b g g g

4. Remind the child that each green unit peg in the multiplicand can be exchanged for a blue ten peg, each blue ten peg in the multiplicand can be exchanged for a red hundred peg, each red hundred peg in the multiplicand can be exchanged for a green thousand peg. The child proceeds to make the changes.

   In this example, there are one green thousand peg, one red hundred peg, three blue ten pegs and no unit pegs, to make 1130.

   g r b b b

5. Tell the child that the multiplicand has now been multiplied by ten, but ask how many tens are in the multiplier. This is the number of times the pegs representing the multiplicand are placed on the board.

   In the example, it is four, so the child lays out the changed multiplicand,1130, four times.

   G r b b b
   G r b b b
   G r b b b
   G r b b b

6. Ask the child to move the pegs to the bottom of the board while counting, changing as required, and recording the number.

   In this example, there are no unit pegs, two blue ten pegs, five red hundred pegs and four green thousand pegs or 4520.

   G G G G r r r r r b b

7. Invite the child to continue to do additional problems or put the materials away.

Procedure for multiplication by one hundred

1. Invite the child to bring the pegs and pegboard and place the pegboard at the front of a table with the open boxes of pegs at the left in the usual order.
2. Have the child get a recording booklet, a pencil and a problem booklet or card of problems for multiplying by 100 which are placed on the child's dominant side.
3. After the child copies a problem, for example, 24 X 100, tell the child to count the specified number of pegs representing the multiplicand into the box lids.

   In this example, four green unit pegs and two blue ten pegs are counted into their lids. The pegs are placed on the board.

   b b g g g g

4. Remind the child that when multiplying by ten, each green unit peg in the multiplicand can be exchanged for a blue ten peg, each blue ten peg in the multiplicand can be exchanged for a red hundred peg, each red hundred peg in the multiplicand can be exchanged for a green thousand peg. The child proceeds to make the changes.

   In this example, there are two red hundred pegs, four blue ten pegs and no unit pegs, to make 240.

   r r b b b b

5. Tell the child that the multiplicand has now been multiplied by ten, but ask how many tens are in one hundred, the multiplier. The child knows that there are ten, so the the tens are changed for hundreds, hundreds are changed for thousands, and the amount recorded.

   In this example, the child changes four blue ten pegs for four red hundred pegs and two red hundred pegs for two green thousand pegs to make 2400 which the child records.

   G G r r r r

6. Invite the child to continue to do additional problems or put the materials away.

Variation:

Once it is understood that multiplying by one hundred involves changing units for hundreds and tens for thousands, the child may multiply by making such changes in the multiplicand once.

Procedure for multiplication by multiples of a hundred

1. Invite the child to bring the pegs and pegboard and place the pegboard at the front of a table with the open boxes of pegs at the left in the usual order.
2. Have the child get recording paper, a pencil and a problem booklet or card of problems for multiplying by multiples of 100 which are placed on the child's dominant side.
3. After the child copies a problem, for example, 29 X 300, tell the child to count the specified number of pegs representing the multiplicand into the box lids.

   In this example, nine green unit pegs and two blue ten pegs are counted into their lids. The pegs are placed on the board.

   b b g g g g g g g g g

4. Remind the child that when multiplying by one hundred, each green unit peg can be exchanged for a red hundred peg, each ten in the multiplicand can be exchanged for a green thousand peg. The child proceeds to make the changes.

   In this example, there are two green thousand pegs, nine red hundred pegs and no ten or unit pegs, to make 2900.

   G G r r r r r r r r r

5. Tell the child that the multiplicand has now been multiplied by one hundred, but ask how many hundreds are in the multiplier. In the example, there are three.

6. Have the child place the quantity derived in step 4 on the pegboard the number of times indicated by the number of hundreds in the multiplier. In this example, two green thousand pegs and nine red hundred pegs are placed on the board three times.

   G G r r r r r r r r r
   G G r r r r r r r r r
   G G r r r r r r r r r
   G G r r r r r r r r r

7. Tell the child to count the pegs, moving them to the bottom of the board, changing as necessary, then recording the answer. In this example, the child records 8700.

   G G G G G G G G r r r r r r r

8. Invite the child to continue to do additional problems or put the materials away.

Variation:

Provide problems with products greater than 9999.

Procedure for multiplication by units and tens

1. Invite the child to bring the pegs and pegboard and place the pegboard at the front of a table with the open boxes of pegs at the left in the usual order. Review the value of the pegs: green for units and units of thousands, blue for tens and tens of thousands, red for hundreds and hundreds of thousands.
2. Have the child get recording booklet, a pencil and a problem booklet or card for multiplying by tens and units which are placed on the child's dominant side.
3. After the child copies a problem, for example, 17 X 23, remind the child that the multiplicand is first multiplied by units. Tell the child to count the specified number of pegs representing the multiplicand into the box lids, then place them on the board. In this example, seven green unit pegs and one blue ten peg are counted into their lids, then placed on the board.

   This is repeated the number of times indicated by the unit multiplier, in this example, three.

   b g g g g g g g
   b g g g g g g g
   b g g g g g g g

4. Have the child count and record the first partial product, in this example, 51.

   b b b b b g

5. Tell the child to count the specified number of pegs representing the multiplicand into the box lids again , then to place them on the board, skipping a row below the pegs already on the board. Remind the child that multiplication by ten involves changing units in the multiplicand to ten, tens in the multiplicand to hundreds, hundreds in the multiplicand to thousands. Have the child change the pegs to multiply by ten.

   In this example, the seven green unit pegs are changed for seven blue ten pegs and the blue ten peg is changed for one red hundred peg.

   r b b b b b b b

6. The multiplicand has now been multiplied by ten. Ask the child how many tens there are in the multiplier. Tell the child to lay out the quantity derived from multiplying by ten according to the number of tens in the multiplier.

   In this example, there are two tens in the multiplier, so 170 is laid out again.

   r b b b b b b b
   r b b b b b b b

7. Tell the child to count, change when necessary and record the amount for the second partial product, in this example, 340.

   r r r b b b b

8. Have the child add the partial products and record the amount, in this example, **51 + 340 = 391.**

9. Invite the child to continue to do additional problems or put the materials away.

Procedure for multiplication by units, tens and hundreds

1. Invite the child to bring the pegs and pegboard and place the pegboard at the front of a table with the open boxes of pegs at the left in the usual order. Review the value of the pegs: green for units and units of thousands, blue for tens and tens of thousands, red for hundreds and hundreds of thousands.
2. Have the child get recording booklet, a pencil and a problem booklet or card for multiplying by units, tens and hundreds which are placed on the child's dominant side.
3. After the child copies a problem, for example,172 X 213, remind the child that the multiplicand is first multiplied by units. Tell the child to count the specified number of pegs representing the multiplicand into the box lids, then place the pegs on the board. In this example, two green unit pegs, seven blue ten pegs and one red hundred peg are counted into their lids, then placed on the board. This is repeated the number of times indicated by the unit multiplier, in this example, three. Have the child count and record the first partial product, in this example, 516.

r b b b b b b g g
r b b b b b b g g
r b b b b b b g g

= r r r r r b g g g g g g

4. Tell the child to count the specified number of pegs representing the multiplicand into the box lids again , then to place them on the board, skipping a row below the pegs already on the board. Remind the child that multiplication by ten involves changing units in the multiplicand to tens, tens in the multiplicand to hundreds, hundreds in the multiplicand to thousands. Have the child change the pegs to multiply by ten. In this example, the two green unit pegs are changed for two blue ten pegs, the seven blue ten pegs are changed for seven red hundred pegs, the one red hundred peg is changed for one green thousand peg. Since the multiplier for tens is one, this amount is put out only one time. Have the child count and record the second partial product, in this example, 1720.

G r r r r r r r b b

5. Tell the child to count the specified number of pegs representing the multiplicand into the box lids again , then to place them on the board, skipping a row below the pegs already on the board. Remind the child that it is multiplication by one hundred, each green unit peg in the multiplicand is exchanged for a red hundred peg, each blue ten peg in the multiplicand is exchanged for a green thousand peg and each red hundred peg in the multiplicand is changed for a blue tens of thousands peg. The child proceeds to make the changes. In this example, the two green unit pegs are changed for two red hundred pegs, the seven blue ten pegs are changed for seven green thousand pegs and the one red hundred peg is changed for one blue tens of thousands peg.

B G G G G G G G r r

6. This amount is put out the number of times for the hundreds multiplier, in this example two.

B G G G G G G G r r

B G G G G G G G r r

7. Tell the child to count, change when necessary and record the amount represented by the pegs for the third partial product, in this example, 34,400.

B B B G G G G r r r r

8. Have the child add the partial products and record the amount, in this example, 516+1,720+34,400 = 36,636.
9. Invite the child to continue to do additional problems or put the materials away.

Geometrical Multiplication with Golden Beads

Purpose

To prepare for the exercises involving squares and square roots

Preliminary Exercises

Practice with compound multiplication with golden beads

Materials

Tray of golden bead material with nine thousand cubes, thirty hundred squares, box of tens, box of units
Large felt mat
Quarter inch graph paper and pencil for recording
Problem booklets or cards

Procedure for Geometric Multiplication with Multiplier in Units

1. Invite the child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right.
2. Tell the child to get recording paper, a pencil and a problem booklet or card which contains problems for geometrical multiplication with multipliers in units only, placing these items on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 24 X 6.
4. Tell the child to lay out tens and units in a row to form the multiplicand as many times as indicated by the multiplier.

   In this example, two ten bars and four unit beads are placed horizontally six times.

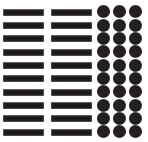
5. Ask the child to count the units beginning at the top right and moving down the column, changing ten units for a ten bar as necessary. The ten bar is placed vertically to the left of units place below the horizontal ten bars. After all units are counted and changed for ten bars, the child counts ten bars, changing for hundred squares as necessary.

   In this example, there are one hundred square, four ten bars, four units.

6. Have the child count the beads and record the answer. In this example, 24 X 6=144
7. Invite the child to continue to do additional problems or put the materials away.

Procedure for Geometric Multiplication with Multiplier of Ten

1. Invite the child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right.
2. Ask the child to get recording paper, a pencil and a problem booklet or card which contains problems for geometrical multiplication with only tens for multipliers, placing these items on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 23 X 10.
4. Tell the child to lay out units and tens as indicated by the multiplicand. In this example, two ten bars bars and three unit beads are placed horizontally ten times.



5. Ask the child to count the beads, beginning with the unit at top right and counting down the column, changing ten units for a ten bar as necessary. The ten bar is placed vertically in place of the changed units.

6. Have the child count the horizontal ten bars, beginning at the top right and changing ten ten bars for a hundred square as necessary. The hundred squares replace the changed tens bars.

   In this example, there are two hundred squares, three tens.

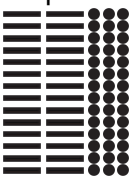
7. Have the child count the beads and record the answer. In this example, 230.

8. Invite the child to continue to do additional problems or put the materials away.

Preliminary Procedure for Golden Bead Geometric Multiplication with Tens and Units

1. Invite the child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right.
2. Ask the child to get recording paper, a pencil and a problem booklet or card which contains problems for geometrical multiplication with tens and units, placing these items on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 23 X 14.
4. Tell the child to lay out tens and units in a row to form the multiplicand as many times as indicated by the multiplier.

   In this example, two ten bars and three unit beads are placed horizontally. This is repeated fourteen times, working from bottom to top.



5. Ask the child to count the beads, beginning with the unit at top right and counting down the column, changing ten units for a ten bar as necessary. The ten bar is placed vertically where the unit beads had been located. In this example, four unit beads remain below the vertical ten bar. Have the child count the unit beads in the column to the left of the vertical ten bar, changing as before, so that there is now another vertical ten bar. This continues until all units starting from the top have been counted and changed for vertically placed ten bars. There is a rectangle of uncounted unit beads below the vertical ten bars.
   In this example, there are three vertical ten bars, each having four unit beads below them.

6. Have the child count the horizontal tens bars which are at the left of the vertical tens bars, beginning at the top and changing ten tens bars for a hundred square. The hundred square is placed in the space where the counted ten bars had been located.
   In this example, there are four horizontal ten bars below the hundred square. Have the child count the ten bars to the left of the hundred square, changing as before.
   In this example, there are two hundred squares in the upper left with three vertical ten bars to their right. Under each of the hundred squares there are four horizontal tens bars. To the right of the horizontal tens will be twelve unit beads.

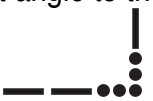
7. Show the child that the multiplicand is represented by the horizontal beads along the bottom of the rectangle and the multiplier is represented by the vertical beads along the right of the rectangle.
8. Have the child count and change each ten units at the lower right for a ten bar which is placed below the horizontal ten bars. Tell the child to count the horizontal tens to the left of the units, any tens bars resulting from changing units and the vertical tens at the upper right, changing when necessary. Ask the child to count the hundreds in the upper left and record the sum. In this example, there are three hundreds, two tens and two units or 322.

9. Invite the child to continue to do additional problems or put the materials away.

Procedure for Geometric Multiplication with Multiplier of Tens and Units

1. Invite the child to bring the tray of golden beads and a large felt mat to a table, placing the tray at the left and the felt mat at the right.
2. Ask the child to get recording paper, a pencil and a problem booklet or card which contains problems for geometrical multiplication with tens and units, placing these items on the child's dominant side.
3. Have the child copy a problem onto the recording paper, for example, 23 X 13.
4. Tell the child to lay out the multiplicand, with ten bars and units horizontally near the bottom of the mat with units to the right of the ten bars, then to lay out for the multiplier with ten bars and units vertically at the right with the unit at the angle shared by both the horizontal and vertical bead arrangements.

   In this example, there are two tens and three units placed horizontally at the bottom and one ten and three units vertically at a right angle to the horizontal bead arrangement.



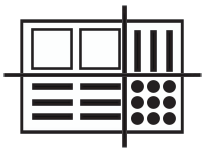
5. Have the child form a rectangle of unit beads within the area of the horizontal and vertical beads.
   In this example, there is a rectangle of nine beads arranged in three groups of three.

6. Tell the child to form a rectangle with ten bars above those placed horizontally at the bottom.
   In the example, there is a rectangle of six ten bars placed horizontally.

7. Ask the child to form a rectangle with ten bars to the left of those placed vertically at the right side.
   In the example, there is a rectangle of three ten bars placed vertically.

8. Have the child complete the entire rectangle with hundred squares.
   In this example, there are two hundred squares placed in the upper left.

9. Show the child the four rectangles which compose the large rectangle. In this example, there are two hundreds which make one part, three vertical tens which make another part, six horizontal tens which make another part and nine units which make the last part of the rectangle.



Second Partial Product
First Partial Product

10. Tell the child to count the unit beads and horizontal ten bars first to obtain the first partial product and record; next to count the vertical ten bars and hundreds squares to obtain the second partial product and record. Beads are moved to the top right of the mat after being counted.The two partial products are added together. In this example, there are nine units and six tens in the first partial product (69), three tens and two hundreds in the second partial product (230) for a total of 299.
11. Invite the child to continue to do additional problems or put the materials away.

Geometric Multiplication with the Stamp Exercise

Purpose

To prepare for the exercises involving squares and square roots

Preliminary Exercises

Practice with geometrical multiplication with golden beads
Work with compound multiplication using the stamp exercise

Materials

Box containing wooden squares (stamps) with green unit stamps, blue ten stamps, red hundred stamps, green thousand stamps
Quarter inch graph paper, pencil
Problem booklets or cards

Procedure for Geometric Multiplication with Multiplier in Units

1. Invite the child to bring the stamp exercise to a table, then to get recording paper, a pencil and a problem booklet or card which contains problems for geometrical multiplication with units. The box of stamps is at the back of the table and the recording paper is on the child's dominant side.

2. Have the child copy a problem onto the recording paper, for example, 325 X 3.

3. Ask the child to place stamps on the table to represent the multiplicand. In this example, there are three red hundred stamps, two blue ten stamps, five green unit stamps are

100	100	100	10	10	1	1	1	1	1

4. Tell the child to place stamps on the table as many times as the multiplier indicates. In this example, the multiplier is three, so the arrangement is

100	100	100	10	10	1	1	1	1	1
100	100	100	10	10	1	1	1	1	1
100	100	100	10	10	1	1	1	1	1

5. Have the child count the stamps beginning with green unit stamps and changing ten green unit stamps for one blue ten stamp as necessary. If there are fewer than ten stamps, these are placed in a row below unit's place. In this example, there are five green unit stamps placed horizontally at the far right of the table. The green stamps for which the blue ten was exchanged are returned to the box. The child places the exchanged stamp at the bottom of its place.

100	100	100	10	10
100	100	100	10	10
100	100	100	10	10

10	1	1	1	1

6. Have the child count the blue ten stamps, changing ten blue ten stamps for one red hundred stamp as necessary. If there are fewer than ten blue stamps, these are placed in a row to the left of the units. The blue stamps for which a red hundred was exchanged are returned to the box. The child places the exchanged stamp at the bottom of its place. In this example, there will be seven blue ten stamps to the left of the five green unit stamps. No blue ten stamps are exchanged for red hundred stamps in this example.

100	100	100
100	100	100
100	100	100

10	10	10	10	10	10	10	1	1	1	1	1

7. Have the child count the red hundreds stamps, changing ten red hundred stamps for one green thousand stamp as necessary. If there are fewer than ten red stamps, these are placed in a row to the left of the tens. The red stamps for which a green thousand stamp was exchanged are returned to the box. The child places the exchanged stamp at the bottom of its place. In this example, there are nine red hundred stamps to the left of the seven blue ten stamps. No red hundred stamps are exchanged for green thousand stamps in this example.

100	100	100	10	10	10	1	1	1
100	100	100	10	10	10	1	1	
100	100	100	10					

8. Tell the child to count and record the number of stamps in each place. In this example, the answer is 975.

9. Invite the child to continue to do additional problems or put the materials away.

Procedure for Geometric Multiplication with Multiplier of Ten

1. Invite the child to bring the box of stamps to a table, then to get recording paper, a pencil and a problem booklet or card which contains problems for geometrical multiplication with tens. The box of stamps is at the back of the table and the recording paper is on the child's dominant side.

2. Have the child copy a problem onto the recording paper, for example, 3 X 10.

3. Tell the child to place green unit stamps as indicated by the multiplicand on the table ten times. In this example, there are ten rows of three green unit stamps.

1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1

4. Ask the child to count downward the green unit stamps starting at the top of the right column, and to exchange ten green unit stamps for one blue ten stamp. In this example, there are three blue ten stamps.

10	10	10

5. Have the child count and record answer. In this example, 3 X 10 = 30.

6. Invite the child to continue to do additional problems or put the materials away.

Procedure for Multiplication by Units, Tens and Hundreds

1. Invite the child to bring the box of stamps to a table, then to get a recording booklet, a pencil and a problem booklet or card which contains problems for geometrical multiplication with units, tens and hundreds in the multiplier. The box of stamps is at the back of the table and the recording paper is on the child's dominant side.

2. Have the child copy a problem onto the recording paper, for example, 243 X 34.

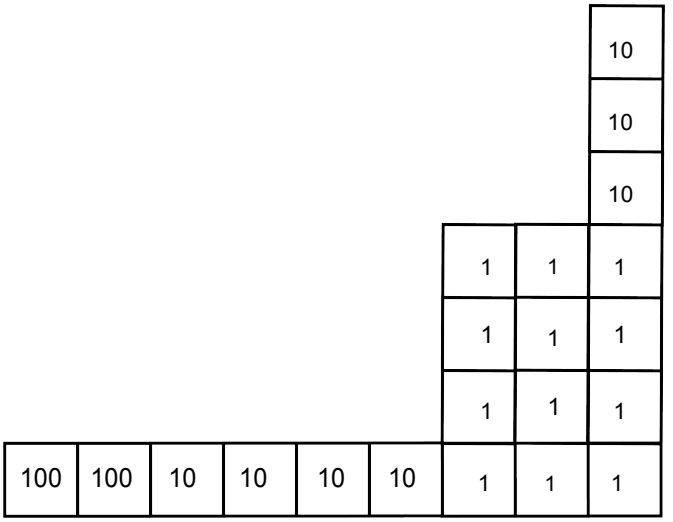
3. Tell the child to place horizontally on the table green unit stamps, blue ten stamps and red hundred stamps as indicated by the multiplicand . In this example, there are two red hundred stamps, four blue ten stamps and three green unit stamps.

100	100	10	10	10	10	1	1	1

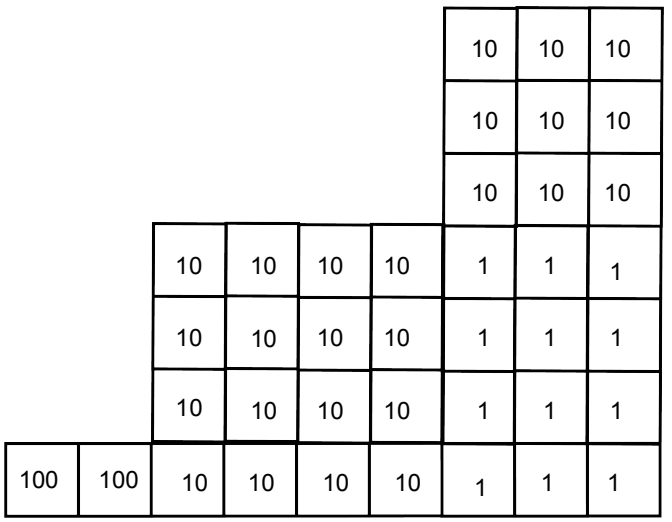
4. Ask the child to place stamps representing the multiplier vertically with the first unit on the right shared. In this example, three ten stamps and three unit stamps are placed so that the fourth unit stamp is shared with the unit stamp of the multiplicand.

								10
								10
								10
								1
								1
								1
100	100	10	10	10	10	1	1	1

5. Tell the child to form a rectangle with unit stamps using those on the bottom and on the right side as guides. In this example, there are twelve unit stamps.



6. Have the child form rectangles with the ten stamps to the left and above units. In this example, there are sixteen ten stamps to the left of the rectangle formed from unit stamps and nine ten stamps above rectangle formed from units.



7. Tell the child to form rectangles with the hundred stamps to the left of each of the rectangles formed from tens stamps. In this example, there are eight hundred stamps at the lower left and twelve hundred stamps at the upper left of the tens.

		100	100	100	100	10	10	10
		100	100	100	100	10	10	10
		100	100	100	100	10	10	10
100	100	10	10	10	10	1	1	1
100	100	10	10	10	10	1	1	1
100	100	10	10	10	10	1	1	1
100	100	10	10	10	10	1	1	1

8. Have the child complete a rectangle bounded by the hundred stamps with thousands stamps. In this example, there are six thousands stamps.

1000	1000	100	100	100	100	10	10	10
1000	1000	100	100	100	100	10	10	10
1000	1000	100	100	100	100	10	10	10
100	100	10	10	10	10	1	1	1
100	100	10	10	10	10	1	1	1
100	100	10	10	10	10	1	1	1
100	100	10	10	10	10	1	1	1

9. Ask the child to count and record the amount of units, tens and hundreds in the lower rectangle formed by units, tens and hundreds. This is the first partial product.
   In this example, the first partial product consists of twelve units, sixteen tens and eight hundreds or 972.
10. Have the child count and record the amount of tens in the upper right corner, the hundreds to the left of these tens and the thousands to the left of these hundreds This is the second partial product. In this example, the second partial product consists of nine tens, twelve hundreds and six thousands or 7290.
11. Tell the child to add the first and second partial products to obtain the final product.

    In this example, 972 (first partial product)+ 7290 (second partial product)= 8262.

12. Invite the child to continue to do additional problems or put the materials away.

Geometrical Multiplication with the Pegboard

Purpose

To prepare for the exercises involving squares and square roots

Preliminary Exercises

Practice with geometrical multiplication with golden beads and stamps

Materials

Large pegboard (minimum 28 holes by 28 holes)
Boxes of pegs: two of green, two of blue, two of red
Booklet with six pages of quarter inch graph paper and a yellow cover,
pencil Problem booklets or cards

Procedure for Geometric Multiplication with Multiplier of Tens and Units

1. Invite the child to bring the pegboard and pegs to a table, then to get the recording booklet, a pencil and a problem booklet or card which contains problems for geometrical multiplication with units. The boxes of pegs are at the left of the pegboard and the recording booklet and problems are on the child's dominant side.

2. Have the child copy a problem onto the recording paper, for example, 243 X 43.

3. Tell the child to place horizontally on the board green unit pegs, blue ten pegs and red hundred pegs as indicated by the multiplicand . In this example, there are two red hundred pegs, four blue ten pegs and three green unit pegs.

r r b b b b g g g

4. Ask the child to place pegs vertically to represent the multiplier, sharing the first unit peg with the multiplicand. In this example, there are four blue ten pegs and three green unit pegs.

b
b
b
b
g g g
r r b b b b g g g

5. Tell the child to place green unit pegs to form a rectangle, starting at the bottom with the unit pegs along the bottom and side as guides.

6. Have the child form a rectangle with blue ten pegs to the left of the units with the tens along the bottom and units at the side as guides. Tell the child to form a rectangle with blue ten pegs above the units with the units along the bottom and the tens at the side as guides.

7. Ask the child to form rectangles with red hundred pegs to the left of tens using the hundreds along the bottom and the tens at the side as guides.

8. Tell the child to place green thousands pegs in the remaining area above and to the left of the hundreds.

9. Beginning with units, have the child count the pegs and record the amount, changing when necessary.

10. The child may continue to do additional problems or put the materials away.

Variation: Have the child record partial products first, then add for the final product.

Checker Board

Purposes
To reinforce the concept of place value To prepare for abstract multiplication

Preliminary Exercises
Compound multiplication with golden beads

Materials
Checker board with decimal system hierarchies up to one hundred million
Container of twenty of each of the colored bead bars one through nine Container of white tiles and gray tiles with numerals one to nine in symbolic colors
Recording paper with symbolically colored lines
Graph paper
Red, blue and green pencils
Compound multiplication problems

Procedure for Introduction to the Checker Board

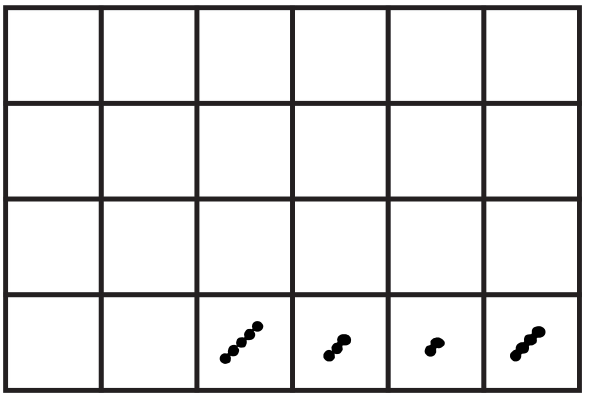
1. Invite a child to bring the checker board and container of bead bars to a table, placing them beside the checker board on the child's dominant side.
2. Ask the child to read the numerals across the bottom of the checker board beginning at the lower right corner with units, then to read the numerals at the right side from bottom to top, starting with units. Call the child's attention to the symbolic color code of the board.
3. Have the child name the values of the colored squares in the second row from right to left, then in the third and fourth rows, always from right to left.
4. Remove a bead bar from the container and place it on the green unit square at the lower right corner. Tell the child that unit are indicated according to the number of beads on the bead bar. For example, if a three bead bar is placed on the green unit square, it represents three units. Move the bead bar to the left so that it is in the blue ten square. Explain that the value is now ten times the number of beads on the bead bar. For example, if a three bar is placed in the blue ten square, it represents thirty units or three tens.
5. Continue to move the bead bar to the left, explaining the change in value in each square, then repeat by moving the bead bar vertically to each square, starting at the green unit square.
6. Place a bead bar in any square on the checker board and ask the child to name the value. Repeat until the child understands the concept.
7. Give the child a list of amounts to form on the checker board, for example,

20,000
600,000
9,000
50,000
65,473
182,526

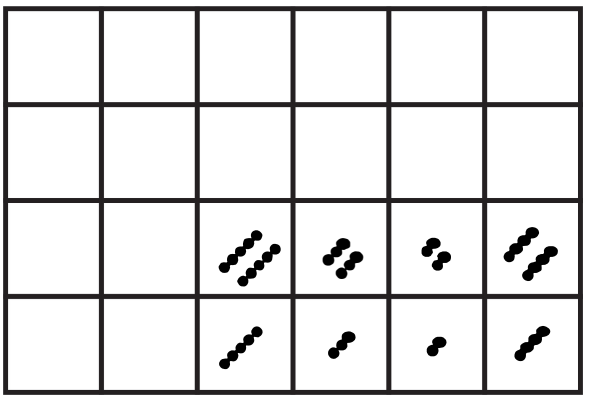
8. Invite the child to continue as long as desired.

Procedure for Multiplication on the Checker Board

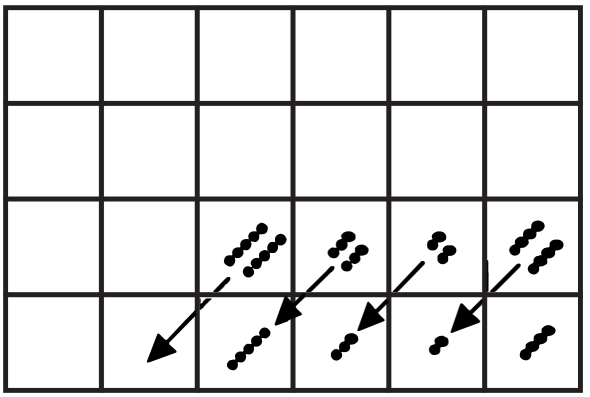
1. Invite a child to bring the checker board, container of bead bars, container of numeral tiles, written problems and recording paper to a table, placing them beside the checker board on the child's dominant side.
2. Choose a problem and have the child copy it on the recording paper. Ask the child to form the multiplicand with the white numeral tiles at the bottom of the checker board in the appropriate place value position, then to form the multiplier with the gray numeral tiles along the right edge of the checker board in the appropriate place value position. For example, in the problem 5324 X 21, the white tile with green numeral four is placed at the bottom right below the green unit square, the white tile with blue numeral two is placed below the blue ten square and left of the unit tile, the white tile with red numeral three is placed below the red hundred square left of the ten tile and the white tile with green numeral five is placed below the green thousand square left of the hundred tile. For the multiplier, the gray tile with green numeral one is placed to the right of the unit square, the gray tile with blue numeral two is placed to the right of the ten square and above the one gray tile.
3. Tell the child that the multiplicand is multiplied by units by placing bead bars on the squares at the bottom of the checker board. Have the child turn face down all numerals in the multiplier except units. For this example, the gray tile for ten (with the numeral two) in the multiplier is turned face down. On the bottom row have the child place a four bead bar on the unit square (4 X 1), a two bead bar on the ten square (2 X 10), a three bead bar on the hundred square (3 X 100) and a five bead bar on the thousand square (5 X 1000).



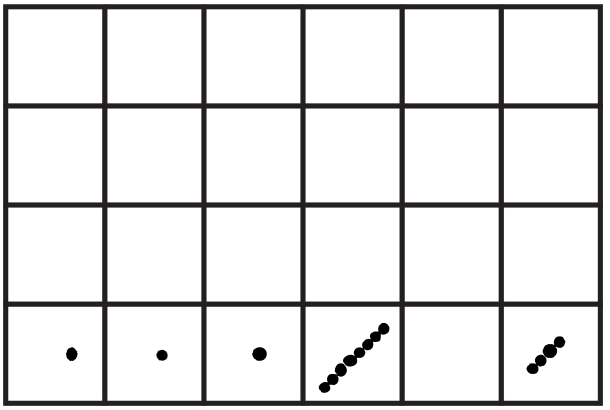
4. Tell the child to turn the unit tile face down and the ten tile face up in preparation for multiplying by tens. Have the child place appropriate bead bars in the second row of squares above the beads placed when units were used as the multiplier. In this example, two four bead bars are placed in the ten square on the second row, two two bead bars are placed in the hundred square, two three bead bars are placed in the thousand square and two five bead bars are placed in ten thousand square.



5. To obtain the product, ask the child to move all the bead bars from the second row to the first row by moving them diagonally to the same color square. The unit square is on the bottom so it stays as is and no bead bars are moved into unit square. In this example, have the child move the two four bead bars from the second row to the ten square on the first row, the two two bead bars to the hundred square on the first row, two three bead bars to the thousand square on the first row and two five bead bars to the tens of thousands square on the first row.



6. Tell the child to count and change the beads on the bottom row except for units which remains the same. There is only one bead bar in each square. In this example, the four bead bar remains in the unit square. Have the child count the bead bars in the ten square. There are ten so the beads are removed but a one bead bar is placed in the hundred square to show that the value of the beads in the ten square was one hundred. The ten square is empty. Have the child count the beads in the hundred square and change them for one eight bead bar. Tell the child to count the beads in the thousandth square. There are eleven so the child changes them for a one bead bar to be moved into tens of thousands square and a one bead bar remaining in the thousand square. The child counts the bead bars in the tens of thousands square. There are eleven so the child changes them for a one bead bar which remains in the tens of thousands square and moves a one bead bar into the hundreds of thousands square.



7. Ask the child to read the product from the beads on the bottom row of the checker board.

   In this example, it is one hundred eleven thousand, eight hundred four. Have the child read the entire equation: 5324 X 21 =111,804.

8. Invite the child to continue with other problems or to put the materials away.

Procedure for Recording Partial Products

1. After the child has had practice with multiplication on the checker board with three and four digit multipliers, demonstrate the method for recording partial products.
2. Have the child write the problem in vertical form.
   For example,

```
  5324
x  21
```

3. After the bead bars are placed on the bottom row of the checker board according to the problem, tell the child to count them and record as the first partial product. For example,

```
  5324
x  21
-----
  5324   first partial product
```

4. After the bead bars are placed on the second row from the bottom of the checker board according to the problem, tell the child to count them and record as second partial product. For example,

```
  5324
x  21
-----
  5324   first partial
106480   product second
         partial product
```

5. Have the child add the partial products to arrive at the final product.
   For example,

```
  5324
x  21
-----
  5324   first partial
106480   product second
         partial product
111804   final product
```

Procedure for Checker Board Multiplication by Graphing

1. Invite a child who has had practice with checker board multiplication to bring graph paper, colored pencils and compound multiplication problems to a table.
2. Ask the child to choose a problem and to write it at the upper right of the graph paper which is used with the longer side parallel to the front of the table edge.
3. Tell the child to count the squares on the graph paper and to draw a horizontal green line representing the number of units in the multiplicand.. Counting begins at the lower right corner at least two squares from the right side and ten squares from the bottom of the paper. Counting is done from right to left. Under the colored line, have the child write the numeral representing the number of units in the multiplicand.
   For example, in the problem 3425 X 43, a green line is drawn under five squares.

4. Tell the child to count the squares on the graph paper to the left of the green line and draw a blue line representing the number of tens in the multiplicand. Under the colored line, have the child write the numeral representing the number of tens in the multiplicand. In this example, a blue line is drawn under two squares.

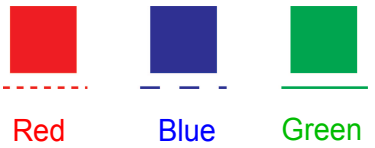
5. Tell the child to count the squares on the graph paper to the left of the blue line and draw a red line representing the number of hundreds in the multiplicand. Under the colored line, have the child write the numeral representing the number of hundreds in the multiplicand. In this example, a red line is drawn under four squares.

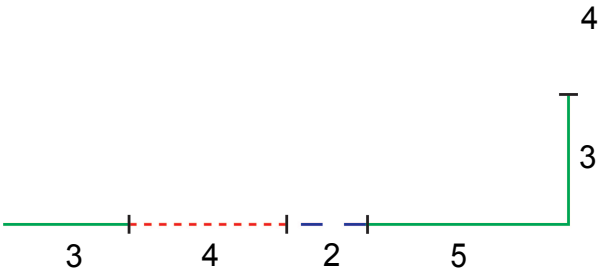
6. Tell the child to count the squares on the graph paper to the left of the red line and draw a green line representing the number of thousands in the multiplicand. Under the colored line, have the child write the numeral representing the number of thousands in the multiplicand. In this example, a green line is drawn under three squares.

7. To form the multiplier, tell the child to count squares vertically and to draw a green line representing the number of units in the multiplier. Counting and the line begin at the point where the first green units line for the multiplicand originated. The numeral indicating units in the multiplier is written to the right of this green line. In the example, the vertical green line is drawn to the right of three squares and the numeral three is written to the right

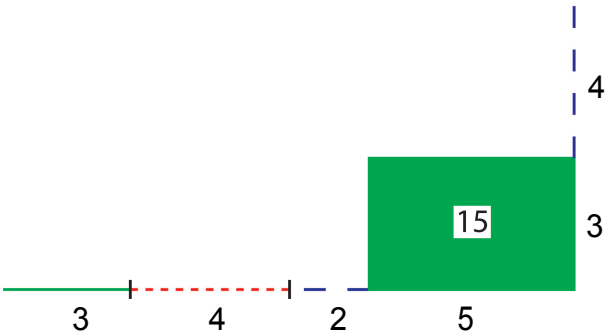
8. To continue forming the multiplier, tell the child to count squares vertically and to draw a blue line representing the number of tens in the multiplier. The blue line originates where the previous green line ends. The numerai indicating tens in the multipiier is written to the right of the biue iine. In this exampie, the vertical blue line is drawn to the right of four squares directly above the vertical green iine and the numeral four is written to its right.

9. Have the child continue forming the multiplier by drawing lines in appropriate colors to represent the entire multiplier.

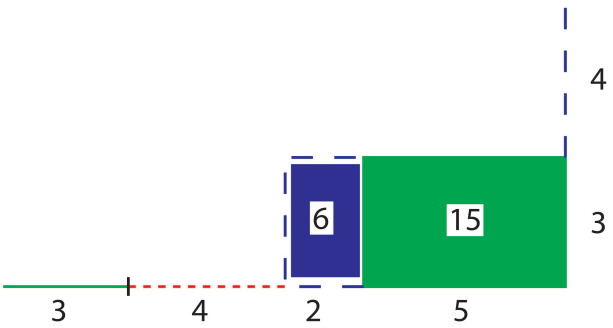




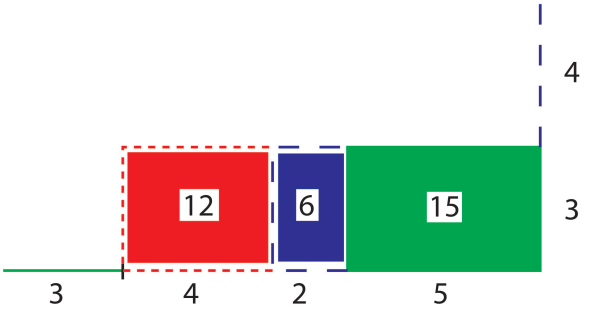
10. Using the green unit lines as boundaries, have the child color the squares green and write the number of squares in that area. In this example, there are fifteen squares colored green and the numeral fifteen is written inside that grid.



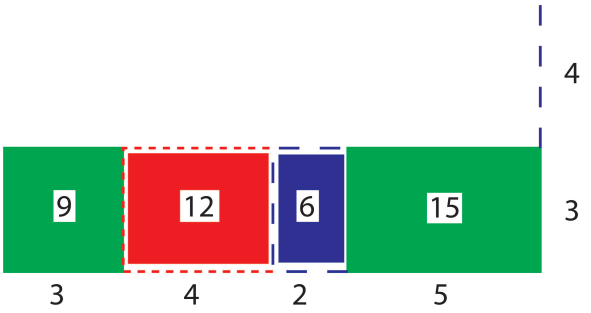
11. Using the blue ten lines as boundaries, have the child color the squares blue at the left of the green unit grid and write the number of squares in that area. In this example, there are six blue squares to the left of the green unit grid.



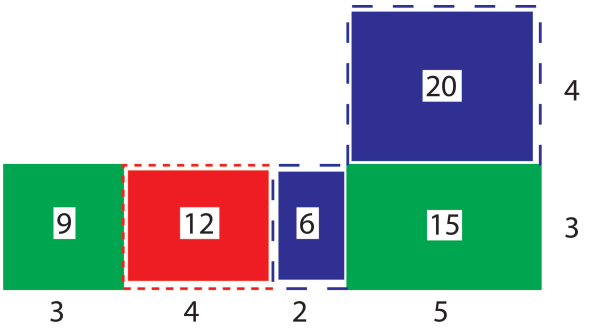
12. Using the red hundred lines as boundaries, have the child color the squares red at the left of the blue ten grid and write the number of squares in that area. In this example, there are twelve red squares to the left of the blue ten grid.



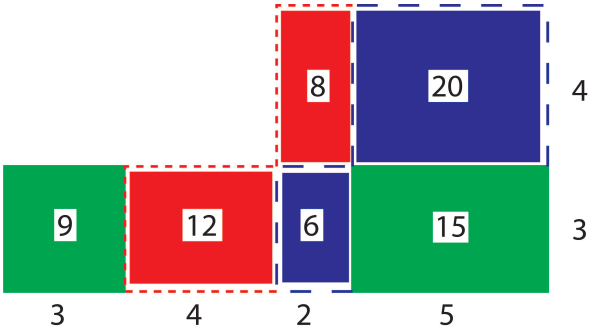
13. Using the green thousand lines as boundaries, have the child color the squares green and write the number of squares in that area. In this example, there are nine green squares to the left of the red hundred grid.



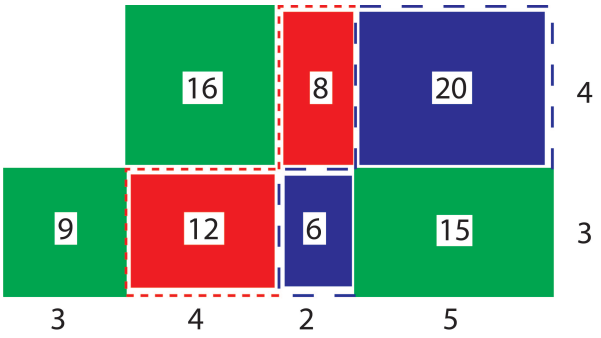
14. Using the blue ten line of the multiplier and the green unit grid as boundaries, have the child color the squares blue above the green unit grid and write the number of squares in that area. In this example, there are twenty blue ten squares above the green unit grid.



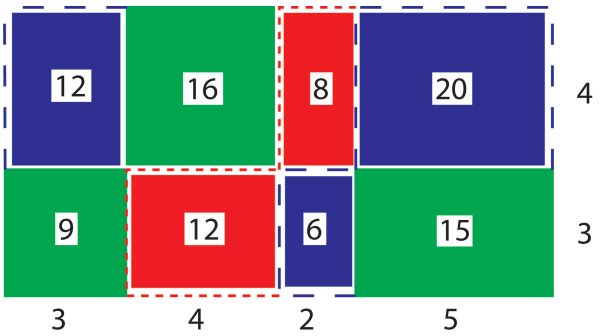
15. Using the two blue ten grids as boundaries, have the child color the squares red and write the number of squares in that area. In this example, there are eight red hundred squares.



16. Using the two red grids as boundaries, have the child color the squares green and write the number of squares in that area. In this example, there are sixteen green thousand squares.



17. Using the two green grids as boundaries, have the child color the squares blue and write the number of squares in that area. In this example, there are twelve blue tens of thousands squares.



18. To obtain the final product, remind the child of the procedure used with the bead bars whereby the partial products were added diagonally. Tell the child to record the number of units in the lower right green grid below the completed graph, then to add the number of tens in the tens grids, the number of hundreds in the hundreds grids and so on, writing these amounts in a right to left sequence. Have the child cross out the numerals in tens places and write those amount above the next numeral to the left. These amounts are added to obtain the final product.
In this example:

	12	25	20	26	15
	2	2	2	1	
1	2	25	20	26	5
1	4	7	2	7	5

Control of Error:
Symbolic colors on the checkerboard Answers in the problem booklets

Long Division with Golden Beads

Group Exercise

Purpose

To introduce the concept of long division To prepare for abstract long division

Preliminary Exercises

Practice with group and individual short division with golden beads

Materials

Tray of golden bead material with nine thousand cubes, thirty hundred squares, box of ten bars, box of units
Large felt mat
Trays for each participating child
Nine green bows with safety pins
Nine blue bows with safety pins
Nine red bows with safety pins
Large and small numeral cards
Written problem cards for long division

Procedure with Divisor of Tens and Units

1. Invite children to participate in group long division, the number according to the amount of the divisor. For example, the problem 42 divided into 8946 requires four children to collect for ten each and two to collect for one each. Pin blue bows on the tens children and green bows on the units children.

2. Ask one child to spread the felt mat on a table and others to arrange the large and small numeral cards on another table.

3. Have a child place the large numeral cards representing the dividend to the right of the mat according to the problem card chosen by the teacher. In the example, the cards are 8946.

4. Tell another child to collect and place to the left of the large numeral cards the amount of golden beads in the dividend as indicated. In this example, there will be eight thousand cubes, nine hundred squares, four ten bars and six units.

5. Ask the children to bring their trays to the table on which the golden bead dividend has been placed and put one thousand on each of the trays of those wearing blue bows. Those wearing green bows get ten times less or one hundred on each of their trays. Note: The children stand together according to the colors of their bows

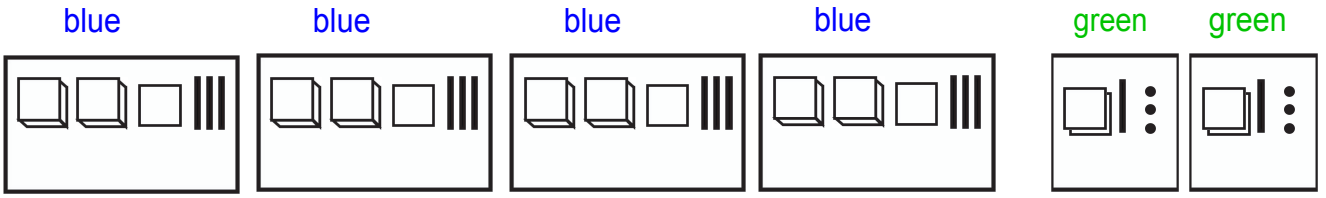
6. Continue placing thousands on the trays of children with blue bows and hundreds on the trays of children wearing green bows until all thousands have been divided. In this example, those with blue bows have two thousands cubes on each tray, those with green bows have two hundreds squares on each tray. There are five hundreds, four tens and six units remaining on the table.

7. Now begin dividing the remaining hundreds among those wearing blue bows, giving those with green bows ten time less or ten bars. In this example, those with blue bows each receive one hundred square, those with green bows each receive one ten bar.

8. Divide the remaining tens among those wearing blue bows, giving those with green bows ten times less or units. In this example, those with blue bows receive three tens and those with green bows receive three units.

9. The answer is what one gets, that is, what one person wearing a green bow has on the tray, so small numeral cards are placed on the tray of one child wearing a green bow to indicate the quotient. In this example, the small numeral cards on the tray will be 213.

10. Continue the activity as long as interest is shown.



Procedure with Hundreds in the Divisor

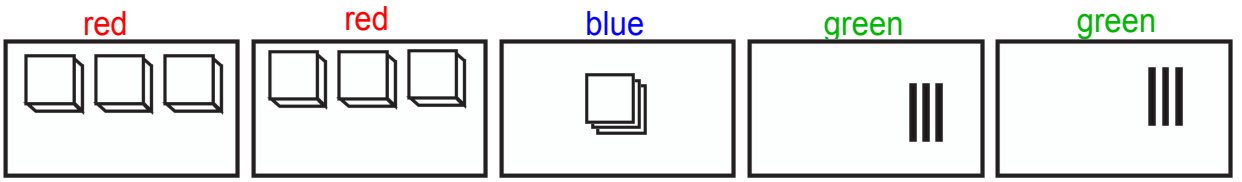
1. Invite those to the lesson who have participated in the previous group long division, the number according to the amount of the divisor. Pin the appropriate number and color of bows on the children. For example, the problem 212 divided into 6784 requires two children with red bows, one with a blue bow and two with green bows.

2. Ask one child to spread the felt mat on a table and others to arrange the large and small numeral cards on another table.

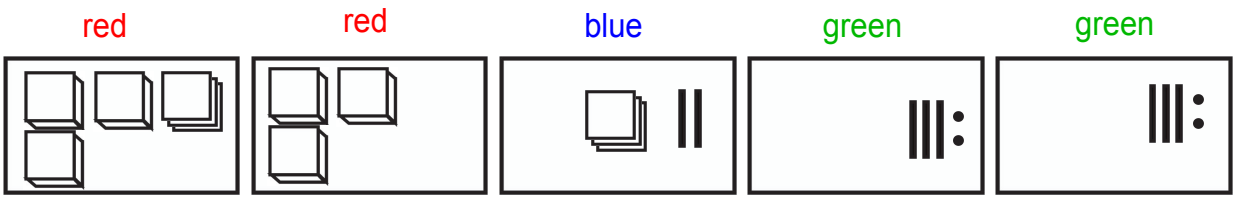
3. Have a child place the large numeral cards representing the dividend to the right of the mat according to the problem card chosen by the teacher. In this example, the cards will be 6784.

4. Tell another child to collect and place beside the large numeral cards the amount of golden beads in the dividend as indicated. In this example, there will be six thousand cubes,seven hundred squares, eight ten bars and four units.

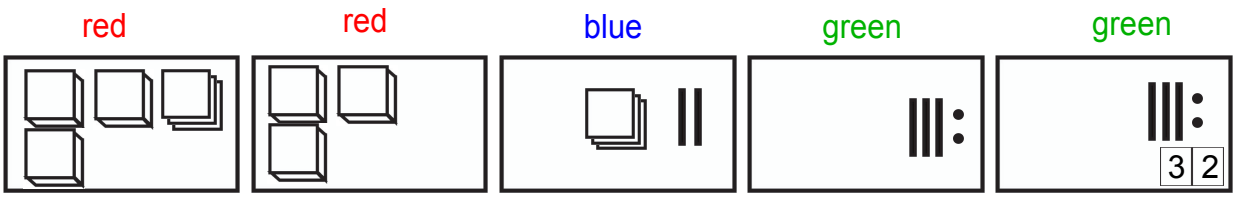
5. Ask the children to bring their trays to the table on which the golden bead dividend has been placed and put one thousand on each of the trays of those wearing red bows. Those wearing blue bows get ten times less or one hundred on each of their trays. Those wearing green bows get ten times less or one ten on each of their trays. Continue placing thousands on the trays of children with red bows, hundreds on the trays of children wearing blue bows, tens on the trays of those with green bows until all thousands have been divided. In this example, those with red bows have three thousand cubes on each tray, those with blue bows have three hundred squares on each tray, those with green bows have three ten bars on each tray. There are four hundreds, two tens and four units remaining on the table.



6. Now begin dividing the remaining hundreds among those wearing red bows, giving those with blue bows ten time less or a ten bar each time the red bow children receive a hundred and giving those with green bows ten times less or a unit. In this example, those with red bows each receive two hundred squares, those with blue bows each receive two ten bars and those with green bows each receive two units.



7. The answer is what one gets, that is, what one person wearing a green bow has on the tray, so small numeral cards are placed on the tray of one child wearing a green bow to indicate the quotient. In this example, the small numeral cards on the tray will be 32.



8. Continue the activity as long as interest is shown.

Individual Exercise

Purpose

To prepare for abstract long division

Preliminary Exercises

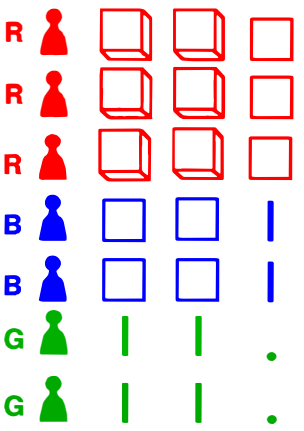
Practice with group long division with golden beads

Materials

Tray of golden bead material with nine thousand cubes, thirty hundred squares, box of ten bars, box of units
Large felt mat
Nine large green skittles
Nine large blue skittles
Nine large red skittles
Recording paper with symbolically colored lines
Written problem cards for long division

Procedure

1. Invite a child who has practiced the group exercises in long division to bring the materials to a table, to place the felt mat near the front of it with the recording paper on the child's dominant side, then to copy a problem on the paper.
2. Tell the child that the large skittles will represent people, with red skittles for hundreds, blue for tens and green for units and that these will be arranged vertically at the left side of the mat according to the divisor, starting with hundreds at the top.
3. Have the child place at the top of the mat the quantity of beads indicated by the dividend of the problem previously copied, then arrange the skittles at the left of the mat according to the divisor. For example, in the problem 322 divided into 6762, six thousands, seven hundreds, six tens and two units are placed at the top of the mat and at the left side are three red skittles, two blue skittles and two green skittles placed vertically starting with the red hundred skittles at the top and ending with green unit skittles at the bottom.
4. Ask the child to give each red hundred skittle a thousand cube, each blue ten skittle ten times less or a hundred square and each green unit skittle ten times less or a ten bar and to continue until all thousands have been divided. In this example, each red hundred skittle will receive two thousand cubes, each blue tens skittle will receive two hundred squares and each green unit skittle will receive two ten bars, leaving three hundreds, two tens and two units to be divided.
5. Tell the child to divide the remaining hundreds among the red skittles, the remaining tens among the blue skittles and the remaining units among the green skittles.
6. Upon completion of division, the answer is what one green skittle receives. In this example it is two ten bars and one unit or 21.
7. Have the child count and record the number of beads which one green skittle receives.
8. Continue the activity as long as interest is shown.



Long Division with Stamps

Purpose

To prepare for abstract long division

Preliminary Exercises

Practice with individual long division with golden beads

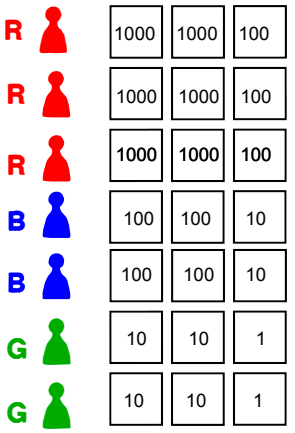
Materials

Container with red stamps marked 100, blue stamps marked 10, green stamps marked 1 or 1000 and small skittles in red, blue and green
Recording paper
Long division problems

Procedure

1. Invite a child who has practiced individual long division to bring the materials to a table, to place the recording paper on the dominant side, then to copy a problem on the paper.
2. Remind the child that skittles represent people with red skittles for hundreds, blue skittles for tens and green for units, and that these will be arranged vertically at the left side according to the divisor, starting at the top.
3. Have the child place the quantity of stamps indicated by the dividend of the problem at the top of the work area, then arrange the skittles at the left of the area according to the divisor. For example, in the problem 6762+322, six thousands, seven hundreds, six tens and two units stamps are placed at the top of the work area and at the left side are three red skittles, two blue skittles and two green skittles placed vertically starting with the red hundred skittles at the top and ending with green unit skittles at the bottom.
4. Ask the child to give each red hundred skittle a thousand stamp, each blue tens skittle ten times less or a hundred stamp and each green units skittle ten times less or a ten stamp and to continue until all thousands have been divided. In this example each red hundreds skittle gets two green thousands stamps, each blue tens skittle gets two red hundreds stamps, and each green units skittle gets two blue tens stamps, leaving three hundreds, two tens and two units to be divided.
5. Tell the child to divide the remaining hundreds stamps among the red skittles, the remaining tens among the blue skittles and the remaining units among the green skittles. In this example each red hundreds skittle gets one unit stamp.
6. Upon completion of division, the answer is what one green skittle receives.

7. Have the child count and record the number of stamps which one green unit skittle receives. In this example it is two tens and one unit.



8. Invite the child to continue the activity.

Long Division with Pegboard

Purpose

To prepare for abstract long division

Preliminary Exercises

Practice with long division with stamps

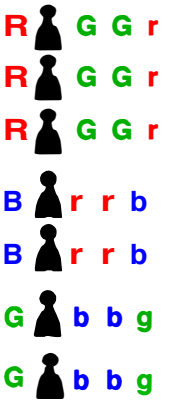
Materials

Large pegboard
Small skittles in red, blue and green
Red, blue and green pegs, each in a separate container Recording paper
Long division problems

Procedure

1. Invite a child who has practiced long division with stamps to bring the materials to a table, to place the recording paper on the dominant side, then to copy a problem on the paper.
2. Remind the child that skittles represent people with red skittles for hundreds, blue skittles for tens and green for units, and that these will be arranged vertically at the left side according to the divisor, starting at the top.
3. Have the child place the quantity of pegs indicated by the dividend of the problem in the box lids at the top of the pegboard, then arrange the skittles at the left of the pegboard according to the divisor. For example, in the problem 6762+322, six thousands, seven hundreds, six tens and two units pegs are placed in the box lids at the top of the pegboard and at the left side are three red skittles, two blue skittles and two green skittles placed vertically starting with the red hundred skittles at the top and ending with green unit skittles at the bottom.
4. **Ask** the child to give each red hundred skittle a thousand peg, each blue tens skittle ten times less or a hundred peg and each green units skittle ten times less or a ten peg and to continue until all thousands have been divided. In this example each red hundreds skittle gets two green thousands pegs, each blue tens skittle gets two red hundreds pegs, and each green units skittle gets two blue tens pegs, leaving three hundreds, two tens and two units to be divided.
5. Tell the child to divide the remaining hundreds pegs among the red skittles, the remaining tens among the blue skittles and the remaining units among the green skittles. In this example each hundreds skittle gets one hundreds peg, each tens skittle gets one tens peg and each unit skittle gets one units peg.
6. Upon completion of division, the answer is what one green skittle receives.

7. Have the child count and record the number of pegs which one green unit skittle receives. In this example it is two tens and one unit.



8. Invite the child to continue the activity.

Variation:

Provide problems with dividends greater than thousands and divisors greater than hundreds.

Division with Three Division Pegboards

Purpose
To prepare for abstract long division

Preliminary exercises
Practice with short division on the division boards
Practice with golden bead individual division

Materials
Three division boards, one with green band at top (units board), one with blue band (tens board) and one with red band (hundreds board)
Two containers of green beads, one of red beads and one of blue beads
Nine small green skittles, nine small blue skittles and nine small red skittles
Recording paper with symbolically colored lines
Division problems suitable for division boards
Pencil

Preliminary Procedure

1. Invite a child to the lesson who has practiced with the preliminary exercises. Ask the child to bring the materials to a table, placing them so that the green board is at the right and the blue board is left of the green. Remind the child of the value of the skittles: each green skittle represents one unit, each blue skittle represents one ten.
2. Have the child copy a problem with a divisor in units and tens that requires no changing, for example; 288/24. Ask the child to place skittles on the boards to represent the divisor. In this example, there will be two blue skittles on the blue board and four green skittles on the green board.
3. Tell the child to count out the beads representing the dividend. In this example, there are two red hundred beads, eight blue ten beads and eight green unit beads in the container lids.
4. Ask the child to place the red beads above the blue board, the blue beads above the green board and the green beads to the right of that. Remind the child that when each ten skittle receives a red hundred bead, each unit skittle must receive ten times less or one blue ten bead. Have the child divide the beads, beginning with giving a red hundred bead to a blue skittle, then a blue ten bead to a green skittle and continuing until all red hundred beads have been divided. In this example, each blue skittle received one red hundred bead and each green skittle received one blue ten bead.

$$\begin{array}{r} 1 \\ 24\overline{)288} \\ \underline{240} \\ 48 \end{array}$$

5. Have the child count the beads on the boards to determine the number of beads which have been divided. The number of beads remaining to be divided are determined by counting the beads remaining in the container lids. In this example, the quantity of two hundred forty was divided and forty-eight is left to be divided.
   The child records the amounts.
6. Tell the child to clear the boards of the beads just divided and recorded, placing those beads back into the containers, not the lids. Have the child move the red container to the left of the blue board since all hundreds have been divided, then move the blue beads above the blue board and the green beads above the green board. Remind the child that when each ten skittle receives one blue ten bead, each unit skittle must receive ten times less or one green unit bead.

7. Ask the child to divide the remaining beads in the container lids among the skittles, giving a green unit bead to the units skittle each time a blue ten bead is given to a ten skittle. In this example, each blue skittle will receive two blue beads and each green skittle will receive two green beads. Have the child record the amount that one green skittle received. In this example, it is two. Have the child record the amount divided, in this example forty eight.

$$\begin{array}{r} 12 \\ 24\overline{)\,288} \\ \underline{240} \\ 48 \\ \underline{48} \\ 00 \end{array}$$

8. Invite the child to continue with other problems after clearing the boards of beads and skittles.

Procedure with Divisors of Hundreds, Tens and Units with Changing and Remainders

1. Invite a child to the lesson who has practiced with the preliminary exercises. Ask the child to bring the materials to a table, placing them so that the green board is at the right, the blue board is left of the green and the red board is left of the blue.
2. Have the child copy a problem with a divisor in hundreds, tens and units, for example, 2841/125. Ask the child to place skittles on the boards to represent the divisor. In this example, there will be one red skittle on the red board, two blue skittles on the blue board and five green skittles on the green board.
3. Tell the child to count out the beads representing the dividend. In this example, there will be two green thousand beads, eight red hundred beads, four blue ten beads and one green unit beads in the container lids.
4. Ask the child to place the green thousand beads above the red board , the red hundred beads above the blue board and the blue ten beads above the green board with the green unit beads to the right of that. Have the child divide the beads, beginning by giving a green thousand beads to the red skittle, a red hundred bead to each blue skittle, then a blue ten bead to each green skittle. After all the thousand beads have been divided, the child records the work, then removes those beads from the board, placing them back into the containers. In this example, the answer is what one unit skittle received which is two ten beads.

$$\begin{array}{r} 2 \\ 125\overline{)2841} \\ \underline{250} \\ 341 \end{array}$$

5. Tell the child to move the container of green thousand beads to the left of the boards, the red hundred beads above the red board, the blue ten beads above the blue board and the green unit beads above the green board. Have the child give the red skittle a red bead, each blue skittle a blue bead and each green skittle a green bead, continuing until all have been divided. Changing may be necessary.

6. After all beads have been divided, the child records the work. In this example, the answer is what one unit skittle received which is two. Nine ten beads and one unit bead remain undivided.

$$\begin{array}{r} 22\text{r } 91 \\ 125\overline{)2841} \\ \underline{250} \\ 341 \\ \underline{250} \\ 91 \end{array}$$

7. Invite the child to continue with other problems after clearing the boards of beads.

Quotition with Golden Beads

Purpose

To introduce the concept of quotition by dividing into groups
To prepare for abstract long division

Preliminary Exercises

Practice with individual long division with golden beads

Materials

Tray of golden bead material with nine thousand cubes, nine hundred squares, box of ten bars, box of units
Large felt mat
Recording paper with symbolically colored lines
Written problem cards for long division by quotition

Preliminary Procedure

1. Invite a child who has practiced with individual golden bead division to bring the tray of beads, felt mat, recording paper and problem cards to a table.
2. Spread the mat at the front of the table and place an even quantity of unit beads before the child. Ask the child to arrange the beads into groups with equal amounts. For example, show the child eight beads and ask how many groups of two can be made. (8/2 = 4 groups).

3. After it is apparent that the child understands the procedure with unit beads, place ten bars on the mat and ask the child to arrange these into groups of units. For example, three tens can be arranged into ten groups of three units each by changing ten bars for ten units each. Count and change the beads back into ten bars.
4. Show that counting across the ten bars will be the same as changing for units, that is, three ten bars are the same as ten groups of three units.

5. Lay out a quantity of beads with both tens and units and show the child how to divide these into groups. For example, to divide seventy two into three groups (72/3), the child gets seven tens and two units. Remind the child that the tens can be grouped. In this example, two groups of three ten bars can be made. By counting down, the child can see that there are twenty rows of three beads each or a total of sixty beads.

6. Tell the child to change any remaining ten bars outside the group, then to combine and group all the units. In this example, the ten bar outside the group is changed for ten units which are combined with the two units of the original quantity and arranged in groups of three, giving four groups. The answer is read down the right side, in this example 72+3=24.

7. Invite the child to continue the activity with other problems.

Procedure for Long Division with Tens and Units

1. Invite a child who has practiced with preliminary quotition exercises to bring the tray of beads, felt mat, recording paper and problem cards to a table.
2. Have the child place the beads on the mat according to the dividend of the problem. For example, for the problem 72/24, seven tens and two units are placed on the mat.

3. Tell the child to make groups of tens and units according to the divisor. In this example, two tens and four units are arranged in groups after changing a ten bar for ten units.

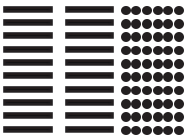
4. Have the child record the work, after reading the answer down the right side, in this example 72÷24=3

5. Invite the child to continue the activity with other problems.

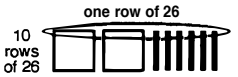
Preliminary Procedure for Long Division with Hundreds, Tens and Units

1. Invite a child who has practiced with tens and units quotition exercises to bring the tray of beads, felt mat, recording paper and problem cards to a table.
2. Have the child place the beads on the mat according to the dividend of the problem. For example, for the problem 260/26, two hundreds, six tens are placed on the mat.

3. Tell the child to change the hundreds for tens to be able to make groups of tens and to change a ten for units. Group according to the divisor. In this example, there are twenty-five tens and ten units to arrange in groups.Keep changing as necessary to make groups according to the divisor. In this example there are 10 rows (groups) of 26.



4. Change the beads back to two hundreds and six tens by counting vertically. Show the child that counting across the top row reveals one row of twenty-six beads. Remind the child that two hundreds and six tens are the same as ten rows of twenty-six.

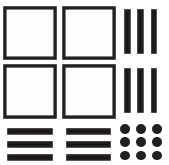


5. Invite the child to continue the activity with other problems.

Procedure for Long Division with Hundreds, Tens and Units

1. Invite a child who has practiced the preliminary exercises to bring the tray of beads, felt mat, recording paper and problem cards to a table.
2. Ask the child to place the beads on the mat according to the dividend of the problem. For example, for the problem 529/23, five hundreds, two tens and nine units are placed on the mat.

3. Tell the child to group the hundreds and tens according the divisor and to change as necessary to obtain tens and units. Group according to the divisor. In this example, there are four hundred squares, twelve ten bars and nine units to arrange in groups according to the illustration. The child should recognize that this is the same pattern as in geometrical multiplication.



4. Have the child record the work. The answer can be read down the right side. In this example there are 23 rows or groups of 23.

5. Invite the child to continue the activity with other problems.

Quotition on the Pegboard

Purpose

To prepare for abstract long division

Preliminary Exercises

Practice with golden bead quotition
Practice with geometrical multiplication on the large pegboard

Materials

Large pegboard
Red, blue and green pegs, each color in a separate container
Recording paper

Preliminary Procedure for Quotition

1. Invite the child to bring the materials to a table, open the containers and place at the left of the board. Give the child a short division problem, for example, 8/2.
2. Tell the child to take out the number of green unit pegs indicated by the dividend and determine the number of groups which can be formed according to the divisor. In this example, four groups of two are formed. 8/2 = 4. The child records the work.
3. Give the child a problem involving changing tens for units, for example 20/2. Ask the child to take out the number of blue ten pegs indicated by the dividend, then to change the ten pegs for unit pegs. In this example, there are twenty green unit pegs. Tell the child to count the number of rows. In this example there are 1O rows. Have the child count the beads vertically and change them back into blue ten pegs. Ask the child to "read" the answer down the right side. In this example there is one blue ten peg.
4. Tell the child to determine the number of groups to be formed according to the divisor. In this example, one group is formed of twenty each, so 20/2 = 10. The child records the work.
5. Give the child a problem involving quotition with tens, for example, 80/2. Ask the child to take out the number of blue ten pegs indicated by the dividend and determine the number of groups to be formed according to the divisor. In this example eight blue ten pegs are grouped in two's and there are four groups. The child records the work by reading the answer down the right side.
6. Once the child grasps the procedure with units and with tens, give a division problem involving hundreds, for example, 800/4.
7. Have the child take out the number of red hundred pegs indicated by the dividend and group the pegs according to the divisor. In this example the eight hundred pegs will be divided into two groups of four each. The child records the work.
8. Invite the child to continue the activity with other problems.

Procedure for Short Division by Quotition

1. Invite a child to bring the materials to a table, and to place the opened containers of pegs to the left of the pegboard. Provide a problem involving division of units, tens, hundreds and thousands by units. The child records the work at each stage.

2. Ask the child to take out the quantity of pegs indicated by the dividend and to determine the number of groups to be made according to the divisor. For example, in the problem 5852/4, there are five green thousand pegs, eight red hundred pegs, five blue ten pegs and two green unit pegs. The child forms one group of four thousand pegs.

```
   1          GGGG
4)5852
  4000
  1852
```

The thousand peg not grouped is changed for ten red hundred pegs so that there are now eighteen red hundred pegs which the child groups by fours to form four groups of four hundred pegs each.

```
   14         GGGG
4)5852        rrrr
  4000        rrrr
  1852        rrrr
  1600        rrrr
   252
```

The two red hundred pegs are changed for twenty blue ten pegs so that now there are twenty-five blue ten pegs to group. There are six groups of four ten pegs each.

```
   146        GGGG
4)5825        rrrr
  4000        rrrr
  1852        rrrr
  1600        rrrr
   252         bbbb
   240         bbbb
    12         bbbb
               bbbb
               bbbb
               bbbb
```

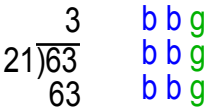
The blue ten peg not grouped is changed for ten green unit pegs so that now there are twelve green unit pegs to group. There are three groups of four unit pegs each.

3. Invite the child to continue the activity with other problems.

Procedure for Long Division by Quotition (units and tens)

1. Invite a child to bring the materials to a table and to place the opened containers of pegs to the left of the pegboard. Provide a problem involving division of units and tens by units and tens. The child records the work at each stage.

2. Ask the child to take out the quantity of pegs indicated by the dividend and to determine the number of groups to be made according to the divisor. For example, in the problem 63 ÷ 21, there are six blue ten pegs and three green unit pegs. The child makes three groups of twenty-one.

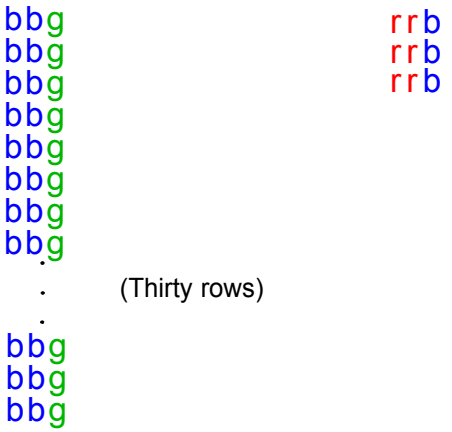


3. Invite the child to continue the activity with other problems.

Procedure for Long Division by Quotition (units, tens, hundreds)

1. Invite a child to bring the materials to a table and to place the opened containers of pegs to the left of the pegboard. Provide a problem involving division of units, tens and hundreds by units and tens. The child records the work at each stage.

2. Ask the child to take out the quantity of pegs indicated by the dividend and to determine the number of groups to be made according to the divisor. For example, in the problem 630+21, there are six red hundred pegs, three blue ten pegs and no unit pegs. The child determines that thirty groups of twenty-one can be formed by changing the six hundreds to sixty tens and three tens to thirty units. Then change back into 6 hundreds and 3 tens by counting vertically. Three tens or thirty can be read down the right side as the answer.



3. After practicing with many problems, the child will discover that grouping may be considered in several ways.
   For example,
   one group of 630 is the same as ten groups of 63;
   one group of 6,300 is the same as one hundred groups of 63;
   one group of 63,000 is the same as one thousand groups of 63.

4. Invite the child to continue the activity with other problems.

Procedure for Long Division by Quotition (units, tens, hundreds, thousands)

1. Invite a child to bring the materials to a table and to place the opened containers of pegs to the left of the pegboard. Provide a problem involving division of units, tens, hundreds and thousands by units and tens. The child records the work at each stage.

2. Ask the child to take out the quantity of pegs indicated by the dividend and to determine the number of groups to be made according to the divisor. For example, in the problem 48,829+23, there are four blue tens of thousand pegs, eight green thousand pegs, eight red hundred pegs, two blue ten pegs, nine green unit pegs.

Two groups of twenty-three thousand each are formed.

```
    2
23)48829
   46000
    2829
```

BBGGG
BBGGG

From the amount remaining, one group of two thousand, three hundred are formed.

```
    21
23)48829
   46000
    2829
    2300
     529
```

BBGGG
BBGGG
GGrrr

From the remaining amount, two groups of two hundred thirty each are formed.

```
    212
23)48829
   46000
    2829
    2300
     529
     460
      69
```

BBGGG
BBGGG
GGrrr
rrbbb
rrbbb

From the remaining amount, three groups of twenty-three each are formed.

```
    2123
23)48829
   46000
    2829
    2300
     529
     460
      69
      69
       0
```

BBGGG
BBGGG
GGrrr
rrbbb
rrbbb
bbggg
bbggg
bbggg

Two thousand one hundred twenty-three groups of twenty three are formed from forty-eight thousand eight hundred twenty-nine.

3. The answer is read vertically on the right side of the pegs. In this example, there are two green thousand pegs, one red hundred peg, two blue ten pegs and three green unit pegs reading from top to bottom on the right side.
4. Invite the child to continue with other problems.

Division with Division Material Set

Purposes

To give additional preparation for the decimal system exercise of division
To provide further practice leading to abstract division work

Preliminary Exercises

Practice with quotition exercises
Work with three division pegboards

Materials

Four color-coded division boards(two green, one red, one blue) each with 81 holes for beads, 9 depressions across the top for skittles, vertical numerals along left side
Tray with seven racks containing ten tubes per rack with ten color-coded beads per tube (thirty tubes of green beads, twenty tubes of red beads, twenty tubes of blue beads)
Note: There are three white racks at the right of the tray, three gray racks to their left and one black rack at the far left. The black rack is unnecessary and may be removed from the tray.
Seven color-coded cups (one red,one blue, one green with a white exterior; one red,one blue, one green with a gray exterior; one green with a black exterior)
Container with nine small red skittles; container with nine small blue skittles; two containers with nine green skittles each
Container of green beads; container of blue beads; container of red beads
Recording paper with symbolically colored lines and a pencil
Division problems without changing with divisors of two or more digits
Division problems with changing with divisors of two or more digits

Procedure

1. Invite a child to bring the division material set, recording paper, pencil and problems to a table, placing the division material set toward the back of the table with the recording paper and problems on the child's dominant side.
2. Tell the child to place the cups with white exteriors in front of the white racks with green at the far right, blue to its left and red left of the blue, then to place the cups with gray exteriors in front of the gray racks in hierarchical order as with the white cups and lastly to place the black cup to the left of the gray cups. Tell the child to place the division boards in the front of the tray with the green unit board to the far right, the blue ten board to its left, the red hundred board to the left of the tens board and the green thousand board at the far left. The individual boxes containing skittles and beads remain in the left side of the tray.
3. Have the child choose a problem without changing and copy it on the recording paper, for example, 4,958,892 + 12 =
4. Ask the child to form the divisor with appropriate color-coded skittles by placing them on the appropriate division boards. In this example, one blue skittle is placed in the first depression at the top left of the blue ten division board and two green skittles are placed at the top left of the unit division board.
5. Tell the child to form the dividend with appropriate color-coded beads from the boxes by counting the appropriate number and color of beads into the cups. In this example, 2 green unit beads are placed in the green cup at the right, 9 blue ten beads in the blue cup, 8 red hundred beads in the red cup, 8 green thousand beads in the green cup to the left of hundreds, five blue tens of thousands beads in the blue cup to the left of thousands, 9 red hundreds of thousands beads in the red cup to the left of tens of thousands, and 4 green millions beads in the cup at the far left.

6. Ask the child to divide the beads beginning with the beads at the far left by placing one under each skittle on the left-most board, then one from the next lower place value under the skittles of the next lower place value board, continuing the process until the unit skittles have received beads and all of the highest place value beads have been divided.. In this example one of the million beads is placed under the ten skittle, one of the hundred thousand beads is placed under each unit skittle, then the process is repeated until all the millions are divided. Have the child record the amount that one unit skittle receives

$$12\overline{)\,4,958,892}\quad 4$$

7. Tell the child to return the beads on the division boards to their boxes. Have the child divide the beads beginning with the quantity at the far left, placing one under each skittle on the left-most board, then one from the next lower place value under the skittles of the next lower place value board, continuing the process until the unit skittles have received beads and all of the highest place value beads have been divided. In this this example, the remaining red hundreds of thousands bead is placed under the skittle on the tens board, one blue tens of thousands bead is placed under each unit skittle. Have the child record the amount that one skittle receives.

$$12\overline{)\,4,958,892}\quad 41$$

8. Tell the child to return the beads on the division boards to their boxes. Have the child divide the remaining beads beginning with the quantity at the far left, placing one under each skittle on the left-most board, then one from the next lower place value under each skittle of the next lower place value board, continuing the process until the unit skittles have received beads and all of the highest place value beads have been divided.. In this this example, a blue tens of thousands bead is placed under the ten skittle, a green thousands bead is placed under each unit skittle. Have the child record the amount that one skittle receives.

$$12\overline{)\,4,958,892}\quad 413$$

9. Tell the child to return the beads on the division boards to their boxes. Have the child divide the beads beginning with the quantity at the far left, placing one under each skittle on the left-most board, then one from the next lower place value under each skittle of the next lower place value board, continuing the process until the unit skittles have received beads and all of the highest place value beads have been divided.. In this this example, a green thousands bead is placed under the tens skittle and a red hundreds bead is placed under each unit skittle. This is repeated and the child records the amount that one skittle receives.

$$12\overline{)\,4,958,892}\quad 413\ 2$$

10. Tell the child to return the beads on the division boards to their boxes. Have the child divide the remaining beads beginning with the quantity at the far left, placing one under each skittle on the left-most board, then one from the next lower place value under each skittle of the next lower place value board, continuing the process until the unit skittles have received beads and all of the highest place value beads have been divided. In this this example, a red hundred bead is placed under the blue ten skittle and a blue ten bead is placed under each green unit skittle. This is repeated until all of the hundred beads have been divided. Have the child record the amount that one skittle receives.

$$12\overline{)\,4,958,892}\quad 413\ 24$$

11. Tell the child to return the beads on the division boards to their boxes. Have the child divide the remaining beads beginning with the quantity at the far left, placing one under each skittle on the left-most board, then one from the next lower place value under each skittle of the next lower place value board, continuing the process until the unit skittles have received beads and all of the highest place value beads have been divided. In this this example, a blue ten bead is placed under the blue ten skittle and a green unit bead is placed under each unit skittle. Have the child record the amount that one skittle receives.

$$12\overline{)\,4{,}958{,}892}\quad 413\ 241$$

12. Tell the child to clear the boards of beads and skittles and choose another problem.

Control of Error

Symbolically colored beads, cups, skittles and division boards
Tubes containing ten beads each for ease of changing
Answers on reverse of cards or in back of problem booklet

Variations

Invite the child to do problems involving changing using the tubes. Invite the child to do problems with larger divisors.

Prime Numbers

Purposes

To prepare for exercises in finding least common multiples and greatest common factors
To establish a basis for exercises with fractions

Preliminary Exercises

Finding multiples on the pegboard
Finding least common multiple on the pegboard
Finding multiples on the "Multiples of Numbers" paper
Factoring numbers on the peg board
Finding highest common factors on the peg board
Operations with the pegboard

Materials

"Multiples of Numbers" paper
Pencil
Container of one inch by one inch cards with one prime number numeral per card
Definition cards with control

Procedure for Definition Cards

1. Invite the child to use the definition cards by matching definitions to words in preparation for determining prime numbers.

Procedure for Determining Prime Numbers by Sieve of Eratosthenes

1. Invite the child to bring "Multiples of Numbers" paper. Explain that the "sieve of Eratosthenes" may be used to find prime numbers rather than circling all numbers multiple times as in determining multiples and common multiples.
2. Tell the child to cross out 1 because it is neither a prime number nor a composite number.
3. Have the child circle the first prime number,2,then cross out the multiples of 2 by marking through every second number after 2.
4. Ask the child to circle the next prime number,3,then cross out the multiples of 3 by marking through every third number after 3.
5. The same procedure is used for the subsequent numbers which have not been crossed out. The prime numbers will be 2,3,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67, 71,73, 79,83,89,97

Control of Error

Prime number numeral cards

Variation

Invite the child make a set of prime number numeral cards for personal use.

Factor Tree for Finding Prime Factors

Purposes

To prepare for exercises in finding least common factor and greatest common factor
To establish a basis for exercises with fractions

Preliminary Exercises

Finding prime factors on the pegboard

Materials

Problem booklet
Lined paper and pencil
Booklet or cards with statements about factors

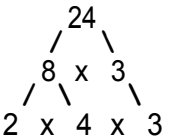
Procedure

1. Invite a child to bring paper, pencil and the factor tree problem booklet to a table.
2. Have the child choose and copy a problem at the top of the paper, for example, 24.
3. Ask the child to write on the first line under the numeral at the top of the page any two numbers that when multiplied together, give a product the same as the problem number. For example,

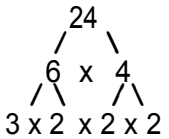
or

4. Have the child look at the factor tree to decide if further factoring is possible or if numbers are prime. In this example, the child recognizes

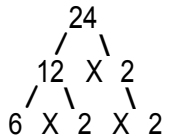
three as a prime number and factors the eight



or factors the six and four

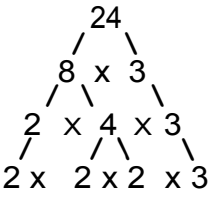


or factors the twelve and two

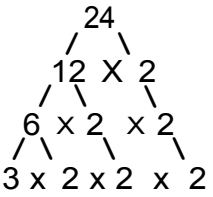


5. Ask the child to look at the factor tree and decide if further factoring is possible. In this example,

the four can be factored.

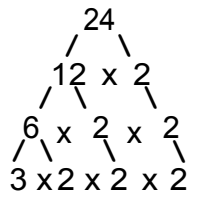


or

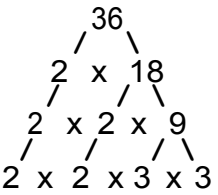


6. Show the child the booklet or card which states:
The **set of prime factors** for any number are the bottom line of its factor tree.

Factor tree for 24



Factor Tree for 36



The **greatest common factor** for a given set of numbers can be found by underlining the common factors at the bottom line of the factor trees and computing the product.

prime factors of 24 are 3x2x2x2
prime factors of 36 are 2x2x3x3
The Greatest common factor of 24 and 36 is 2x2x3=12

The **least common multiple** for a given set of numbers can be found by underlining the common factors of both sets, eliminating any numbers in the second set which are in the first, then multiplying the factors from the first set by the remaining factors in the second set.

Control of Error

Prime number booklet produced from work with pegboard

prime factors of 24 are 2x2x2x3

prime factors of 36 are 2x2x3x3

The least common multiple of 24 and 36 is (2x2x2x3) x 3 = 72

Greatest Common Factors of Numbers Over One Hundred

Purpose

To prepare for further work with fractions

Preliminary Exercises

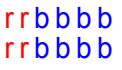
Practice with exercises for finding greatest common factors and least common multiples of numbers up to one hundred

Materials

Large pegboard
Boxes of pegs: two of green, two of blue, two of red
Container of prime number numeral cards
Problem cards or booklets for numbers over one hundred

Procedure for finding prime factors of one number over one hundred

1. Invite the child to bring container of prime number numeral cards, problems, pegs and pegboard and place the pegboard at the front of a table with open boxes of pegs at the left in the usual order with the remainder of the materials on the right.
2. Have the child choose a problem and place the appropriate number of pegs in the box lids. For example, the problem 480 requires four red pegs and eight blue pegs to be placed in the lids.
3. Tell the child to place the pegs on the pegboard, grouping vertically by two's, for the first prime number changing if necessary. If the pegs cannot be grouped by two's, have the child try grouping by the next appropriate prime number. In the example, there will be two rows with two red pegs and four blue pegs.

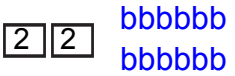


4. Ask the child to remove all but the top row and place a prime number numeral card to the left of the remaining row of pegs. In the example, a 2 card will be placed.

5. Have the child group the pegs vertically by two's, changing if necessary. If the pegs cannot be grouped by two's, have the child try grouping by the next appropriate prime number. In the example, there will be one red peg and two blue pegs in each of two rows.

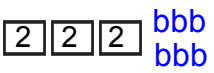
6. Ask the child to remove all but the top row and place a prime number numeral card to the right of the numeral card by the remaining row of pegs. In the example, a 2 card will be placed.

7. Tell the child to group the pegs vertically by two's, changing if necessary. If the pegs cannot be grouped by two's, have the child try grouping by the next appropriate prime number. In the example, the red hundred peg will be changed for ten blue tens pegs.and there will be two rows of six blue pegs each

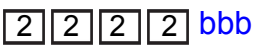


8. Have the child remove all but the top row and place an appropriate card to the right of the cards on the remaining row.

9. Tell the child to group the pegs vertically by two's, changing if necessary. If the pegs cannot be grouped by two's, have the child try grouping by the next appropriate prime number. In the example, there will be three blue tens pegs in each of two rows.

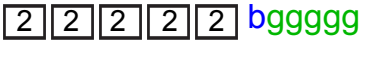


10. Have the child remove all but the top row and place an appropriate card to the right of the cards on the remaining row.

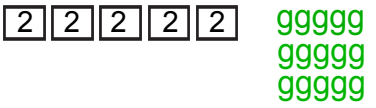


11. Tell the child to group the pegs vertically by two's, changing if necessary. If the pegs cannot be grouped by two's, have the child try grouping by the next appropriate prime number. In the example, one blue peg must be changed for ten green units pegs, so there will be one blue peg and five green pegs in each of two rows.

12. Have the child remove all but the top row and place an appropriate card to the right of the cards on the remaining row.



13. Tell the child to group the pegs vertically by two's, changing if necessary. If the pegs cannot be grouped by two's, have the child try grouping by the next appropriate prime number. In the example, one blue peg must be changed for ten green units pegs, so the fifteen green units pegs will be divided into three rows of five green pegs each.



14. Have the child remove all but the top row and place an appropriate card to the right of the cards on the remaining row.

15. Tell the child to continue grouping until it is not possible to further divide the pegs. At that point, have the child count and remove the remaining pegs and place the appropriate card to the left of the other cards. In the example, the numeral cards will be

16. Invite the child to continue with other problems or to put the materials away.

Control of Error

Multiplication of prime factors to obtain the original number
Answers in problem booklets or cards

Finding Greatest Common Factors of Two or More Numbers

Purpose

To prepare for further work with fractions

Preliminary Exercises

Practice with exercises for finding greatest common factors and least common multiples of numbers up to one hundred

Practice with exercises for finding prime factors of numbers over one hundred

Materials

Large pegboard

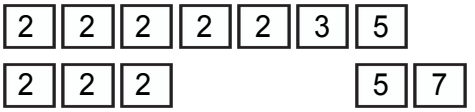
Boxes of pegs: two of green, two of blue, two of red

Container of prime number numeral cards

Greatest common factor problem cards or booklets with two or more of numbers over one hundred

Procedure

1. Invite the child to bring container of prime number numeral cards, problems, pegs and pegboard and place the pegboard at the front of a table with open boxes of pegs at the left in the usual order with the remainder of the materials on the right.
2. Have the child choose a problem and place the appropriate number of pegs in the box lids for the first number. After finding the prime factors of this number, tell the child to save its prime number numeral cards at the top left of the board.
3. Ask the child to place the appropriate number of pegs in the box lids for the second number and find its prime factors. After finding the prime factors of this number, tell the child to move the numeral cards representing the prime factors of the first number below the second set of cards with the numerals aligned in columns. Have the child write the original problem and the numerals representing the common factors. For example, prime factors of 480 are 2, 2, 2, 2, 2, 3, 5 and prime factors of 280 are 2, 2, 2, 5, 7. The cards are aligned as follows:



   The common factors of 480 and 280 are 2, 2, 2 and 5. The preceding statement is written by the child.
4. Have the child stack the common factor numeral cards and move them below the row previously formed, then multiply to find the greatest common factor which is written below the common factor statement.

   In this example,

   2 X 2 X 2 X 5 = 40

   The greatest common factor of 480 and 280 is 40.

   This statement is written below the statement about the common factors.

   The common factors of 480 and 280 are 2, 2, 2 and 5. The greatest common factor of 480 and 280 is 40.
5. Invite the child to continue with other problems or to put the work away.

Control of Error

Answers in problem booklets

Variation

Invite the child to do problems involving more than two numbers for finding common factors and greatest common factor.

Finding Least Common Multiples of Numbers Over One Hundred

Purpose

To prepare for further work with fractions

Preliminary Exercises

Practice with exercises for finding greatest common factors and least common multiples of numbers up to one hundred

Practice with exercises for finding prime factors of numbers over one hundred

Materials

Large pegboard

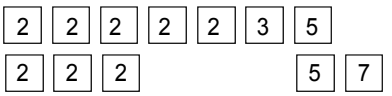
Boxes of pegs: two of green, two of blue, two of red

Container of prime number numeral cards

Least common multiple problem cards or booklets with problems of two or more of numbers

Procedure

1. Invite the child to bring container of prime number numeral cards, problems, pegs and pegboard and place the pegboard at the front of a table with open boxes of pegs at the left in the usual order with the remainder of the materials on the right.
2. Have the child choose a problem and place the appropriate number of pegs in the box lids for the first number. After finding the prime factors of this number, tell the child to save its prime number numeral cards at the top left of the board.
3. Ask the child to place the appropriate number of pegs in the box lids for the second number and find its prime factors. After finding the prime factors of this number, tell the child to move the numeral cards representing the prime factors of the first number above the second set of cards with the numerals aligned in columns. Have the child write the original problem and the numerals representing the common factors. For example, prime factors of 480 are 2, 2, 2, 2, 2, 3, 5 and prime factors of 280 are 2, 2, 2, 5, 7. The cards are aligned as follows:



4. Invite the child to stack the cards in each column, to move these below the previous location and to move any single cards down to form a row with the stacked cards. In the example, the cards are arranged as follows:

5. Tell the child to multiply the prime factors to find the least common multiple. In the example, 2 X 2 X 2 X 2 X 2 X 3 X 5 X 7 = 3360.
6. Have the child copy the problem and answer. In the example, the statement is
   The least common multiple of 480 and 280 is 3360.
7. Invite the child to continue with other problems or to put the work away.

Control of Error

Answers in problem booklets

Figurate Numbers

Purposes

To develop understanding of squares and square roots of numbers
To learn the sequence of squared numbers
To discover that each squared number is the sum of a sequence of odd numbers
To recognize patterns in number sequences

Preliminary exercises

Work with bead chains for squares of numbers
Practice with pegboard operations

Material

Large pegboard
Boxes of pegs in two colors other than red, green and blue Recording paper with grid
Pencil
Colored pencils
Graph paper

Forming Perfect Squares of Numbers

Procedure for Square Pattern

1. Invite a child to bring the pegboard and pegs to a table and to place the grid paper and pencil on the dominant side.

2. Tell the child to form the square of one by placing one peg in the upper left corner of the pegboard, then to record in the upper left corner of the paper $1^2 = 1$.

3. Have the child form the square of two by adding pegs of the other color to the right and below the one peg. Ask the child to record $2^2 = 1 + 3 = 4$ as shown in the table below.

4. Tell the child to continue building squares with alternating colors of pegs and recording the results up to ten.

$1^2 = 1$	= 1
$2^2 = 1+3$	= 4
$3^2 = 1+3+5$	= 9
$4^2 = 1+3+5+7$	= 16
$5^2 = 1+3+5+7+9$	= 25
$6^2 = 1+3+5+7+9+11$	= 36
$7^2 = 1+3+5+7+9+11+13$	= 49
$8^2 = 1+3+5+7+9+11+13+15$	= 64
$9^2 = 1+3+5+7+9+11+13+15+17$	= 81
$10^2 = 1+3+5+7+9+11+13+15+17+19$	= 100

5. Call attention to the sequence of square numbers and the sequence of odd numbers whose sums result in the square number.

Procedure for Diagonal Pattern

1. Invite a child to bring the pegboard and pegs to a table and to place the grid paper and pencil on the dominant side.

2. Tell the child to form the square of one by placing one peg in the upper left corner of the pegboard, then to record in the upper left corner of the paper $1^2 = 1$.

3. Show the child how to place on a diagonal two pegs of a different color, that is , one different colored peg to the right and another peg below . For the fourth peg, return to the first color to complete the square of two. Have the child record this result as

$$2^2 = 1 + \underline{2} + 1.$$

4. For the square of three, add two pegs the same color as the fourth peg to form a diagonal, then place diagonally two pegs of the other color followed by one peg of the first color. Have the child record the result as

$$3^2 = 1 + 2 + \underline{3} + 2 + 1$$

5. Tell the child to continue building squares on the diagonal and recording up to

$$10^2 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + \underline{10} + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

Forming Triangular Number Patterns

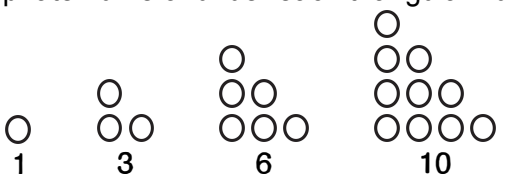
Procedure

1. Invite the child to bring graph paper and colored pencils to a table and to write the heading Triangular Numbers at the top.

2. Tell the child to fill in a graph paper square near the bottom left of the paper with any color and under it to write the numeral one.

3. Have the child choose another color and fill in two squares to the right of the one square on the same line. Call attention to the configuration not being a triangle. Ask the child to color the square above the left colored square so that there is a triangle of three and to write the numeral three under it.

4. Tell the child to continue coloring squares with different colors to form triangles and to write the appropriate numeral under each triangular number pattern.



5. Call attention to the combination of any two consecutive triangles to form a square. For example, by combining the triangular numbers three and six, a square of nine is formed which is 3^2.

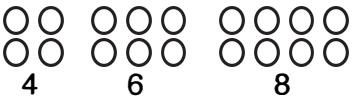
6. Ask the child to discover an easy way of knowing what the next number in the sequence will be.

Forming Rectangular Number Patterns

Procedure

1. Invite the child to bring pegboard and pegs to a table.

2. Near the bottom of the pegboard, have the child arrange pegs of numbers to form rectangles. Remind the child that squares are also rectangles.



3. Tell the child to list those numbers which do not form rectangles. The child will recognize that these are prime numbers.

Variations

Invite the child to investigate other number sequences such as the Fibonacci numbers

Mathematical Properties

Purposes

To develop understanding of mathematical concepts
To acquire the ability to recognize names and examples of mathematical properties
To develop application skills

Preliminary Exercises

Practice with addition, multiplication, subtraction and division
Work with variables

Materials

Container of definition cards for each mathematical property with identifying label on the reverse and separate labels for each of the above
Container of example cards with the name of the property on the reverse
Note: Definition cards are 2 1/2 by 4 1/4 inches mounted on white or neutral cover stock which is 3 1/2 by 5 1/4 inches. Labels and example cards are 1 by 4 1/4 mounted on 2 by 5 1/4 cover stock to match definition cards.

Introductory Procedure

1. Invite a child to bring the container of definition cards and labels to a table and to place them on the child's dominant side. Have the child place the definition cards in a row in the center of the table and arrange the labels near the bottom of the table.
2. Ask the child to read the definition card, then to read the label on the reverse and locate the corresponding label to place above it.
3. Invite the child to continue as long as desired, then to return the materials to their storage place.

Procedure for Identification of Mathematical Properties

1. Invite a child to bring the container of definition cards, labels and example cards to a table and to place them on the child's dominant side. Have the child place the definition cards in a row in the center of the table with the appropriate label above each.
2. Ask the child to read the reverse of the example card, match it with the appropriate labeled definition and place the example upward at the lower edge of the definition card. Tell the child to read the example before continuing with the other cards.
3. Invite the child to continue as long as desired, then to return the materials to their storage place.

Control of Error

Labels on reverse of definition cards Labels on reverse of example cards
Same number of example for each property

Variations

Invite the child to place the labels above the appropriate definition without referring to the reverse of the card.
Invite the child to match the examples of properties with the labels without referring to the reverse of the example.

FRACTIONS

Operations with Fractions of Unlike Denominators

Purpose

To develop understanding of calculations involving fractions with unlike denominators

Preliminary Exercises

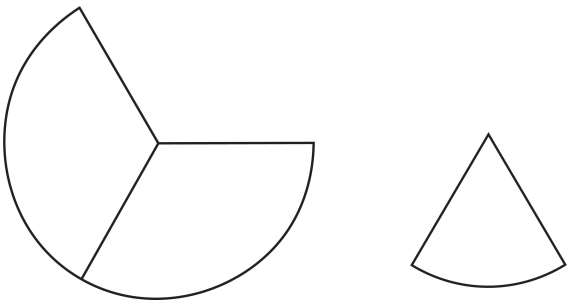
Practice in operations with common denominator fractions
Work with factors and multiples

Materials

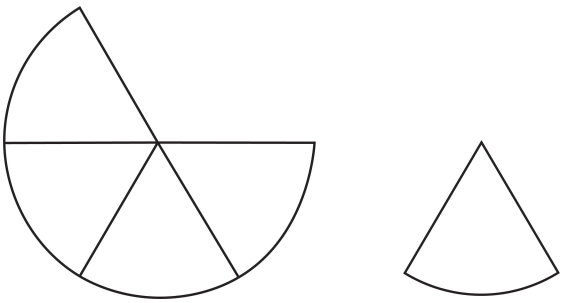
Five sets of cut-out labeled fraction parts for one half through one tenth and ten complete circles
(Note: These are not in frames as were the beginning materials.)
Color coded booklets or cards with problems having unlike denominators
(Note: Be sure the problems can be worked with the material, that is, nothing can add up to be over ten tenths.)
Recording paper and pencil
Rule cards for operations with fractions

Procedure for Addition of Fractions with Unlike Denominators

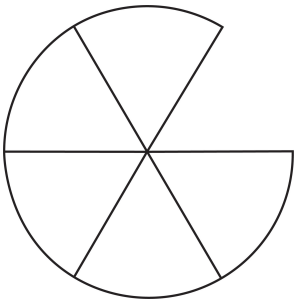
1. Invite a child to bring the fraction materials and place on a table so that there is space in front of the divided circles which are arranged in order from one to ten tenths. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fraction parts indicated at the front of the table. Tell the child that the fractions must be changed so that the denominators are the same. For example, in the problem 2/3 + 1/6 =, two 1/3 parts and one 1/6 part are placed at the front of the table.



3. Ask the child to determine the lowest common multiple of the denominator and to change the fraction parts so that there is a common denominator. In this example, each one third part is changed to two sixths.



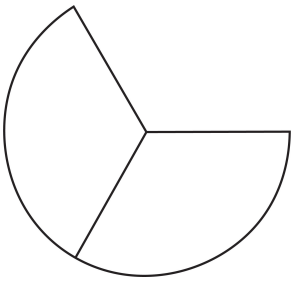
4. Tell the child to rewrite the problem showing the changed common denominators, then to add the numerators. In this example, $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$



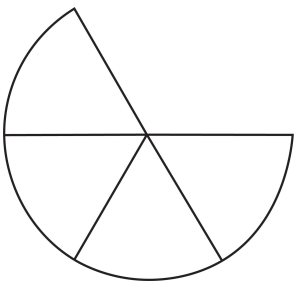
5. The child may continue to do additional problems or put the materials away.

Procedure for Subtraction of Fractions with Unlike Denominators

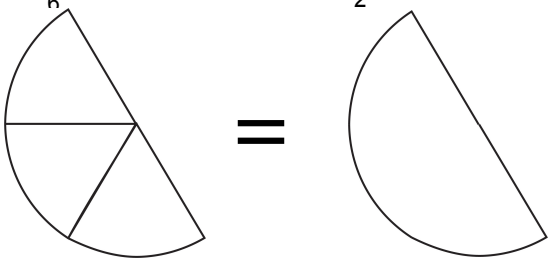
1. Invite a child to bring the fraction materials and place on a table so that there is space in front of the divided circles which are arranged in order from one to ten tenths. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fraction parts for the minuend at the front of the table. Tell the child that the fractions must be changed so that the denominators are the same. For example, in the problem $\frac{2}{3} - \frac{1}{6}$ =, two $\frac{1}{3}$ parts are placed at the front of the table.



3. Ask the child to determine the lowest common multiple in the minuend and subtrahend and to change the fractional parts so that there is a common denominator. Have the child rewrite the problem. In this example, each one third part is changed to two sixths and the problem is rewritten as $\frac{4}{6} - \frac{1}{6}$ =



4. Tell the child to subtract the subtrahend from the minuend by removing the quantity indicated and recording the answer, reducing if necessary. In this example, 1/6 is removed from the $\frac{4}{6}$, so the answer is $\frac{3}{6}$ which is reduced to $\frac{1}{2}$.

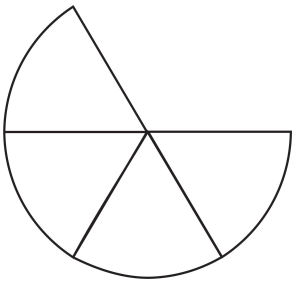


$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

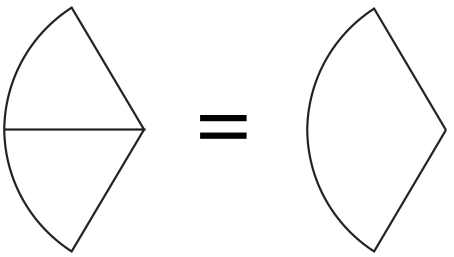
5. The child may continue to do additional problems or put the materials away.

Procedure for Multiplication of Fractions by Fractions

1. Invite a child to bring the problem booklet or card which is placed on the child's dominant side.
2. Have the child copy the problem and place the fraction parts for the multiplicand in the empty fraction frame. For example, for the problem $\frac{1}{2} \times \frac{4}{6}$, four one sixth fraction parts are placed in the empty frame. State that the multiplication sign can be read as " Qf." In this example, $\frac{1}{2} \times \frac{4}{6}$ is read as $\frac{1}{2}$ of $\frac{4}{6}$.dwg.



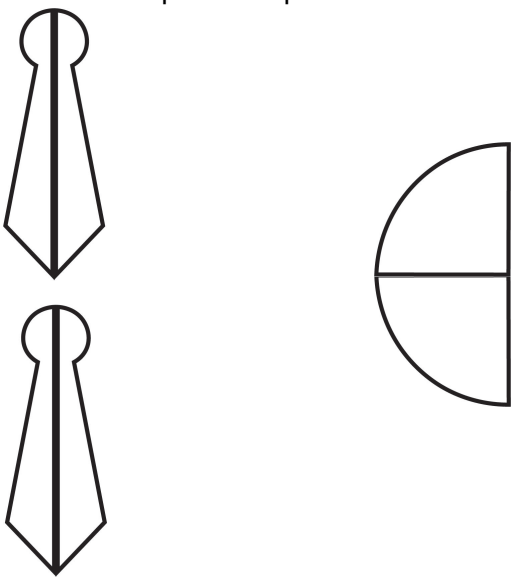
3. Tell the child to look at the multiplier to determine how many fraction parts are specified, then to divide the parts into groups accordingly, removing the groups not specified. In this example, one half of the four sixths are specified, so two groups of $\frac{2}{6}$ each are formed. One group is removed leaving $\frac{2}{6}$ in the frame which is reduced to $\frac{1}{3}$.



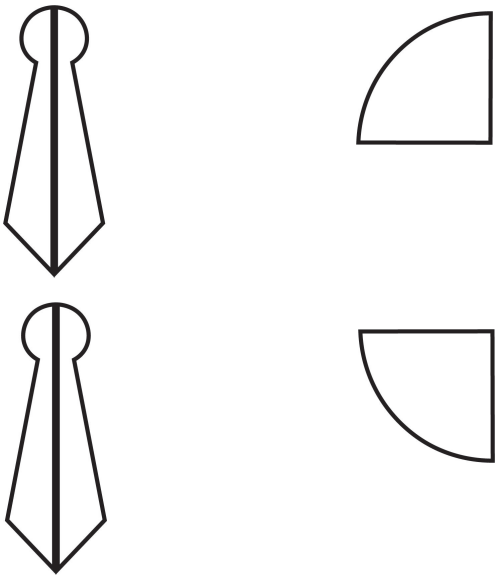
4. Ask the child to record the results. In this example, $\frac{1}{2} \times \frac{4}{6} = \frac{2}{6} = \frac{1}{3}$.
5. The child may continue to do additional problems or put the materials away.

Procedure for Division of Fractions by Fractions

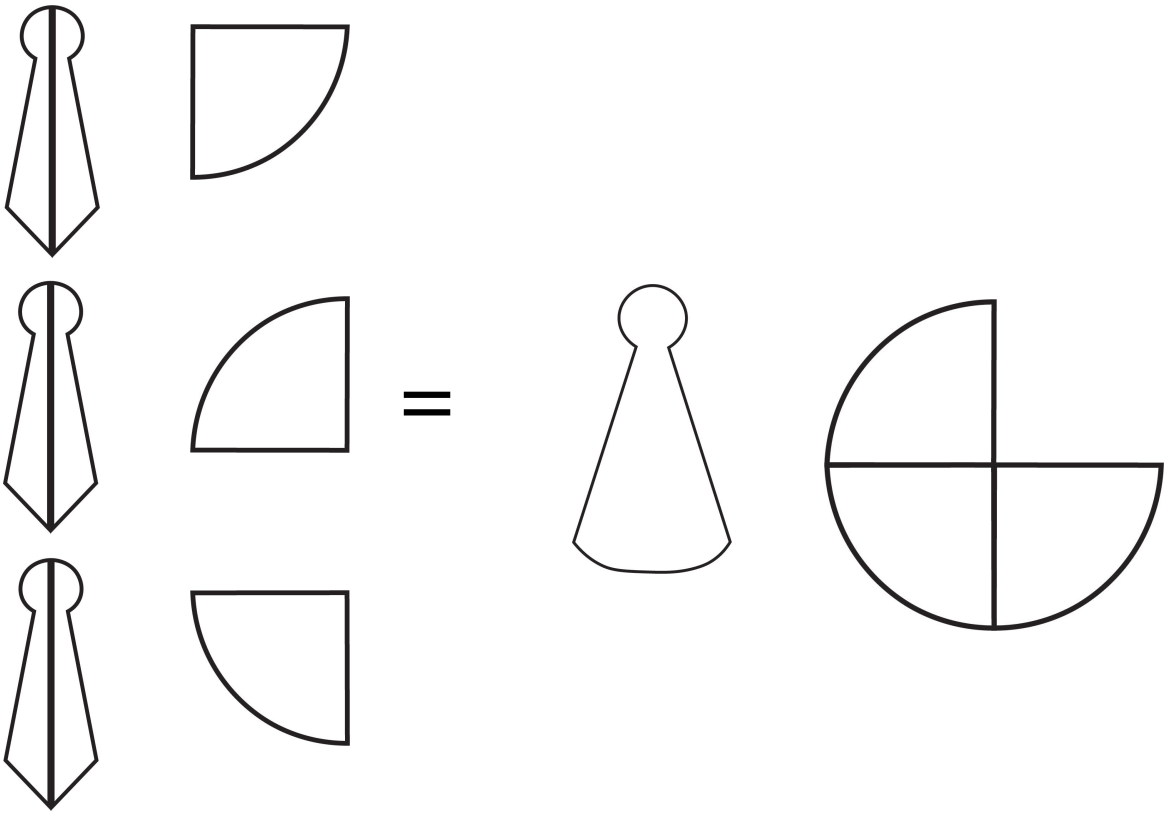
1. Invite a child to bring the fraction materials and place on a table so that there is space in front of the divided circles which are arranged in order from one to one tenth. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fraction parts for the dividend at the front of the table. Ask the child to place the appropriate divided skittles according to the divisor to the left of the fraction parts. For example, in the problem $\frac{2}{4} + \frac{2}{3} =$, two sections of the skittle (which is divided into three thirds) are placed to the left and two one fourth fraction parts are placed at the front of the table.



3. Have the child give each divided skittle section a fraction part and record the answer. In this example, $\frac{2}{4}$ is divided between $\frac{2}{3}$. Each of the two $\frac{1}{3}$ skittle sections gets $\frac{1}{4}$.



4. Remind the child that in division, the answer is what one gets. Ask the child to place the remaining section of the skittle with its other sections. In this example, it is the third section of the skittle divided into thirds. Each section of the skittle must receive the same number of fraction pieces. In this example, each $\frac{1}{3}$ section of the skittle will receive $\frac{1}{4}$, so the whole skittle will receive $\frac{3}{4}$.



5. Ask the child to record the results. In this example, $\frac{2}{4} + \frac{2}{3} = \frac{3}{4}$
6. The child may continue to do additional problems or put the materials away.

Control of Error

Answers in problem booklets or cards

Variation

Invite the child to read appropriate rules once operation has been practiced and there is evidence of understanding.

Note: See white pages for rules.

Improper Fractions

Purpose

To develop understanding of calculations involving mixed numbers

Preliminary Exercises

Practice in operations with like denominator fractions
Work with factors and multiples
Practice in operations involving fractions with unlike denominators

Materials

Five sets of cut-out labeled fractional parts for one half through ten tenths and ten complete circles
(Note: These are not in frames as are the beginning materials.)
Booklets or cards with mixed numbers; improper fractions; operations with mixed numbers
(Note: Be sure the problems can be worked with the material.)
Recording paper and pencil

Procedure for Changing Mixed Numbers to Improper Fractions

1. Invite a child to bring *the* fraction materials and place on a table so that there is space in front of the fraction parts which are arranged in order from one to one tenth. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fractional parts representing it at the front of the table. As an example, for $2\frac{3}{4}$, two complete circles each representing one and three $\frac{1}{4}$ parts are placed.
3. Tell the child to convert the whole circles to equivalent fractional parts according to the fraction in the mixed number. In this example, the two whole circles are changed to $\frac{4}{4}$ each because the fraction is $\frac{3}{4}$.
4. Ask the child to add the numerators to convert the mixed number to an improper fraction. In this example, $\frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$
5. The child may continue to do additional problems or put the materials away.

Procedure for Changing Improper Fractions to Mixed Numbers

1. Invite a child to bring the fraction materials and place on a table so that there is space in front of the fraction parts which are arranged in order from one to one tenth. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fractional parts representing it at the front of the table according to the denominator. As an example, for $\frac{10}{3}$, ten one third parts are placed.
3. Tell the child to form as many whole circles as possible by combining the fractional parts. In this example, three whole circles can be formed with $\frac{1}{3}$ remaining.
4. Ask the child to write the result. In this example, $\frac{10}{3} = 3\frac{1}{3}$
5. The child may continue to do additional problems or put the materials away.

Procedure for Addition of Mixed Numbers

1. Invite a child to bring the fraction materials and place on a table so that there is space in front of the divided circles which are arranged in order from one to one tenth. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fractional parts representing each quantity at the front of the table. As an example, for $2\frac{3}{4} + 3\frac{1}{2}$, two whole circles and three $\frac{1}{4}$ parts are placed, then three whole circles and a $\frac{1}{2}$ part are placed to the right of the first quantity.
3. Ask the child to determine the lowest common multiple of the fractions and exchange the fractions appropriately. In this example, it is four, so the $\frac{1}{2}$ is exchanged for two $\frac{1}{4}$ parts. The child records the conversion.
4. Tell the child to convert the whole circles to equivalent fractional parts according to the fraction in the mixed number. In this example, the two circles are replaced by four $\frac{1}{4}$ parts each and the three circles are replaced by four 1/4 parts each. The child records the conversion, in this example $\frac{11}{4} + \frac{14}{4}$
5. Have the child add the numerators of the improper fractions, then convert to a mixed number. The child records the operation. In this example, $\frac{11}{4} + \frac{14}{4} = \frac{25}{4} = 6\frac{1}{4}$ 6. The child may continue to do additional problems or put the materials away.

Procedure for Subtraction of Mixed Numbers

1. Invite a child to bring the fraction materials and place on a table so that there is space in front of the divided circles which are arranged in order from one to one tenth. The problem booklet or card is placed on the child's dominant side.
2. Have the child copy the problem and place the fractional parts representing each quantity at the front of the table. As an example, for $2\frac{3}{4} - 1\frac{1}{2}$, two whole circles and three 1/4 parts are placed, then one whole circle and a $\frac{1}{2}$ part are placed to the right of the first quantity.
3. Ask the child to determine the lowest common multiple of the fractions and exchange the fractions appropriately. In this example, it is four, so the $\frac{1}{2}$ part is exchanged for two $\frac{1}{4}$ parts. The child records the conversion.
4. Tell the child to convert the whole circles to equivalent fractional parts according to the fraction in the mixed number. In this example, the two circles are replaced by four $\frac{1}{4}$ parts each and the one circle is replaced by four $\frac{1}{4}$ parts, so there are eleven $\frac{1}{4}$ parts and six $\frac{1}{4}$ parts. The child records the conversion.
5. Have the child subtract the numerators of the improper fractions, then convert to a mixed number. The child records the operation. In this example, $\frac{11}{4} - \frac{6}{4} = \frac{5}{4} = 1\frac{1}{4}$ 6.

The child may continue to do additional problems or put the materials away.

Procedure for Multiplication of Mixed Numbers

1. Invite a child to bring the problem booklet or card which is placed on the child's dominant side.
2. Have the child copy the problem.
3. Tell the child to convert the mixed numbers to improper fractions.
   For example, $2\frac{1}{4} \times 3\frac{1}{2} = \frac{9}{4} \times \frac{7}{2}$. The child records the conversion.
4. Ask the child to multiply the numerators, then the denominators.
   In this example, $9 \times 7 = 63$ (numerator) and $4 \times 2 = 8$ (denominator). The child records the result.
5. Have the child divide the numerator by the denominator and record the result.
   In this example, 63 divided by 8 = $7\frac{7}{8}$.
6. The child may continue to do additional problems or put the materials away.

Procedure for Division of Mixed Numbers

1. Invite a child to bring the problem booklet or card which is placed on the child's dominant side.
2. Have the child copy the problem.
3. Tell the child to convert the mixed numbers to improper fractions. For example,

   $5\frac{1}{6}$ divided by $2\frac{1}{4}$ = $\frac{31}{6}$ divided. by $\frac{9}{4}$ =. The child records the conversion.
4. Ask the child to multiply the first fraction by the reciprocal of the second. The reciprocal is obtained by inverting the numerator and denominator.

   In this example, $\frac{31}{6} \times \frac{4}{9} = \frac{31 \times 4 = 124}{6 \times 9 \quad 54}$. The child records the result.

5. Have the child divide the numerator into to the denominator, reduce if necessary and record the result. In this example, the result is $2\frac{16}{54}$ or $2\frac{8}{17}$.
6. The child may continue to do additional problems or put the materials away.

Control of Error

Answers in problem booklets or cards

Variations

Invite the child to read appropriate rules once a procedure has been practiced and understanding of the concept is in evidence.
Invite the child to a lesson on cross reduction for multiplication of fractions by fractions.

Operations with Decimal Fractions

Purposes

To understand the relationship between common fractions and decimal fractions
To develop comprehension of operations involving decimal fractions

Preliminary Exercises

Practice with common fraction operations
Introductory work with decimal beads and cards
Exercises involving conversion of common fractions to decimal fractions with the centesimal circle
Practice with compound multiplication with golden beads

Materials

Container with ten green ten centimeter squares, twenty light blue strips one by ten centimeters, twenty light red or pink one centimeter squares
Note: The above materials are made from cover stock in the appropriate colors and laminated.
Containers of ten millimeter red, blue and green beads
Containers of eight millimeter light blue, light red or pink and light green beads
Containers of six millimeter light blue, light red or pink and light green beads
Small black laminated paper circles to be used as decimal points stored in the container with blue tenth decimal beads
Felt mat
Container of color-coded numeral cards for 100-900, 10-90, 1-9, 0.1- 0.9, 0.01-0.09, 0.001-0.009, 0.0001-0.0009
Recording paper with symbolically colored lines: red for hundreds, blue for tens, green for units, light blue for tenths, light red or pink for hundredths, light green for thousandths, light blue for ten thousandths
Color-coded problem booklets or cards
Large red, blue and green skittles, nine of each
Small red, blue and green skittles, nine of each

Introduction to Decimals

1. Invite a child to take the containers of green decimal squares, light blue strips and light red or pink squares, decimal points and numeral cards to a table.
2. Remind the child that green is units color as a large green ten centimeter square is placed on the table, that light blue is tenths color as a light blue strip is placed right of the green square and that light red or pink is hundredths color as a pink square is placed right of the light blue strip.
3. Place a decimal point to the right of the green unit square. Place a light blue strip to the right of the decimal and tell the child that it represents one tenth. Place a pink square to the right of the light blue strip and tell the child that it represents one hundredth.
4. Ask the child to lay out the numeral cards for unit under the green square. Place the numeral card for tenth under the light blue strip and the numeral card for hundredth under the pink square, then call attention to the position of the decimal on each card. Have the child return the materials to their containers.
5. Tell the child lay out all the decimal numeral cards with tenths in the first column to the left and hundredths to the right of tenths. Choose any card and lay it before the child, then request that the appropriate square or strip be placed above it. Have the child replace the material. Continue until it is apparent that the decimal squares, strips and cards are understood.

6. Beginning with the hundredths, have the child count pink hundredth squares through ten onto the table in a column and tell the child that these equal one tenth. Place a light blue tenths strip to cover the ten pink tenths, then remove. Remove the ten pink squares and place one blue tenth strip on the table. Have the child count tenth strips through ten, placing them side by side from right to left, and tell the child that these equal one. Place a green unit square to cover the ten tenths, then remove. Remove the ten tenth strips and place one green square on the table. Tell the child that this equals one. There is no need to count through units.

7. Repeat the "counting through" exercise with the decimal cards, beginning with one hundredth.

8. Place a decimal quantity in cards before the child and ask that the appropriate squares and strips be placed above them. For example, for 2.98 represented by decimal cards, the child places two green unit squares, nine light blue tenth strips and eight pink hundredth squares Continue until it is apparent that the child understands.

9. Place a quantity of squares and strips on the table and ask the child to find appropriate decimal cards to indicate the quantity. Continue until it is apparent that the child understands.

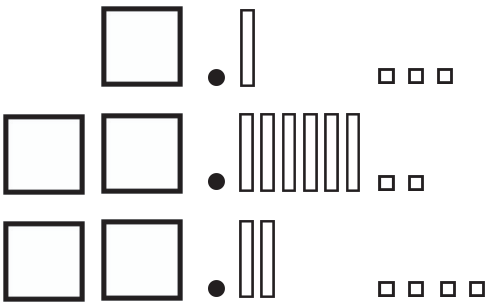
10. Have the child replace materials in their containers and return to the shelf.

Procedure for Addition

1. Invite a child to take the containers of decimal squares and strips, numeral cards and decimal points to a table
2. Using the numeral cards, set a problem in addition. (Note: Use problems which do not involve changing for introduction.)
   For example,

$$\begin{array}{r} 1.13 \\ 2.62 \\ \underline{4.24} \end{array}$$

3. Ask the child to place squares, strips and decimal points according to the problem. Tell the child that decimal points must always be in alignment. In this example, cards are arranged as follows:



4. Tell the child to add the squares, beginning with hundredths, then the tenth strips, then the unit squares and to form the answer with the cards. In this example, the answer is 7.99.

5. Invite the child may continue with additional problems from problem booklets or cards or to put the materials away.

Procedure for Subtraction

1. Invite a child to take the containers of decimal squares and strips, decimal points and numeral cards to a table.
2. Using the numeral cards, set a problem in subtraction. (Note: Use problems which do not involve changing for introduction.)
   For example,

$$\begin{array}{r} 4.74 \\ \underline{2.62} \end{array}$$

3. Ask the child to place squares, strips and decimal point according to the minuend. In this example, the arrangement is as follows:

4. Tell the child to subtract by removing squares and strips according to the subtrahend, beginning with the smallest amount, in this example, hundredths. Have the child count the squares and strips remaining at the top, moving them below the squares and strips which have been subtracted, then form the answer with the cards. In this example, the answer is 2.12

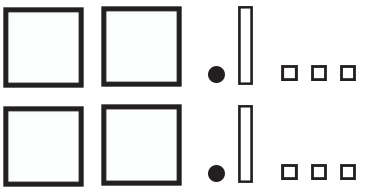
5. The child may continue to do additional problems from problem booklets or cards or to put the materials away.

Procedure for Multiplication by a Whole Number

1. Invite a child to take the containers of decimal squares and strips, decimal points and numeral cards to a table.
2. Using the numeral cards, set a problem in multiplication. For example,

$$\begin{array}{r} 1.13 \\ \times \quad 2 \end{array}$$

   (Note: Use problems which do not involve changing for introduction.)
   (Note: Place the multiplier in its proper place value position.)
3. Ask the child to place squares, strips and decimal points to represent the multiplicand of the problem according to the number of times designated by the multiplier. In this example,



4. Tell the child to count the hundredth squares as they are moved down , then the tens strips and the unit squares and to form the answer with the cards. In this example, the answer is 2.26.

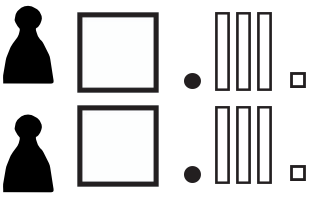
5. Invite the child to continue with other problems or to put the materials away.

Procedure for Division by a Whole Number

1. Invite a child to take the containers of decimal squares and strips, decimal points and numeral cards to a table.
2. Using the numeral cards, set a problem in division.

   For example, 2.62 + 2 =

   (Note: Use problems which do not involve changing for introduction.)
3. Have the child place large green unit skittles vertically at the left to indicate the divisor. In this example, there are two large green skittles.
4. Tell the child to count squares and strips as indicated by the dividend. In this example, there are two green unit squares, six light blue strips and two pink squares.
5. Ask the child to divide unit squares among the skittles, then the light blue tenths strips, and last the pink hundredth squares. In this example, each skittle receives one unit square,three tenth strips and one hundredth square and or 1.31.



6. Have the child place cards to indicate what one skittle gets.

7. Invite the child to continue to do additional problems from problem booklets or cards or to put the materials away.

Introduction to Decimal Beads

1. Invite a child to take the containers of beads, decimal beads, numeral cards and felt mat to a table.

2. Remind the child that green is units color as a large green bead is placed on the mat, that blue is tens color as a large blue bead is placed left of the green bead and that red is hundreds color as a large red bead is placed left of the blue bead.

3. Place a decimal point to the right of the green unit bead. Place an eight millimeter light blue bead to the right of the decimal and tell the child that it represents one tenth. Place an eight millimeter light red or pink bead to the right of the blue tenth bead and tell the child that it represents one hundredth. Place an eight millimeter light green bead to the right of the pink hundredth and tell the child that it represents one thousandth.

4. Ask the child to lay out the numeral cards for unit, ten and hundred in the appropriate places. Place the numeral cards for tenth, hundredth and thousandth under their appropriate beads and call attention to the position of the decimal on each card. Tell the child to return the beads and cards to their containers.

5. Have the child lay out all the decimal numeral cards with tenths in the first column to the left, then hundredths to the right of tenths and thousandths to the right of hundredths. Choose any card and lay it before the child, then request that the appropriate bead be placed on the mat above it. Have the child replace the bead and card. Continue until it is apparent that the decimal beads and cards are understood.

6. Beginning with the thousandths, have the child count green thousandths beads through ten onto the mat and tell the child that these equal one hundredth. Remove the ten green beads and place one light red or pink hundredth bead on the mat. Have the child count hundredth beads through ten and tell the child that these equal one tenth. Remove the ten pink hundredth beads and place one light blue tenth bead on the mat. Have the child count tenth beads through ten and tell the child that these equal one. Remove the ten tenth beads and place one green unit bead on the mat. There is no need to count through units, tens and hundreds.

7. Repeat the "counting through" exercise with the decimal cards, beginning with one thousandth.

8. Place a decimal quantity in cards before the child and ask that the appropriate beads be placed on the mat above them. For example, for 5.986 represented by cards, the child places five units, nine tenth beads, eight hundredth beads and six thousandth beads. Continue until it is apparent that the child understands.

9. Place a quantity of beads on the mat and ask the child to find appropriate cards to indicate the quantity. Continue until it is apparent that the child understands.

10. Have the child replace materials in their containers and return to the shelf.

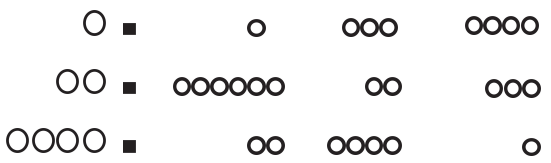
Procedure for Addition

1. Invite a child to take the containers of beads, decimal beads, numeral cards and a felt mat to a table.

2. Using the numeral cards, set a problem in addition with the decimal cards. (Note: Use problems which do not involve changing for introduction.)

For example,
1.134
2.623
4.241

3. Ask the child to place beads and decimal points on the mat according to the problem. Tell the child that decimal points must always be in alignment. In this example, beads are arranged as follows:



4. Tell the child to add the beads, beginning with thousandths, and to form the answer with the cards. In this example, the answer is 7.998.

5. Invite the child to continue to do additional problems from problem booklets or cards or to put the materials away.

Procedure for Subtraction

1. Invite a child to take the containers of decimal beads and numeral cards to a table.
2. Using the numeral cards, set a problem in subtraction with the decimal cards.

   For example,

   4.745
   - 2.623

   (Note: Use problems which do not involve changing for introduction.)

3. Ask the child to place beads and decimal point on the mat according to the minuend. In this example, beads are arranged as follows:

4. Tell the child to subtract beads according to the subtrahend, beginning with thousandths. Have the child count the beads remaining at the top, moving them below the the beads which have been subtracted and form the answer with the cards. In this example, the answer is 2.122.

5. Invite the child to continue to do additional problems from problem booklets or cards or to put the materials away.

Procedure for Multiplication by a Whole Number

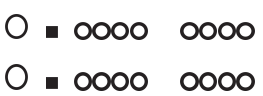
1. Invite a child to take the containers of decimal beads and numeral cards to a table.
2. Using the numeral cards, set a problem in multiplication.

   For example,

   1.134
   x 2

   (Note: Use problems which do not involve changing for introduction.) (Note: The multiplier must be in its proper place value position.)
3. Ask the child to place beads and decimal points on the mat to represent the multiplicand of the problem according to the number of times designated by the

   multiplier. In this example,

4. Tell the child to count the beads as they are moved down , beginning with thousandths, and to form the answer with the cards. In

   this example, the answer is 2.268.

5. Invite the child to continue with other problems or to put the materials away.

Procedure for Multiplication by Another Decimal Fraction

1. Invite a child to take the containers of decimal beads and numeral cards to a table. Using
2. the numeral cards, set a problem in multiplication with the decimal cards which involves a decimal fraction multiplier, for example, 1.13 X 2.3.
   (Note: To use bead material, problem answers must not exceed 0'.999 in decimal value.)
3. Ask the child to place beads, numeral cards and decimal point on the mat to represent the multiplicand. Tell the child that the first step is to multiply by tenths. Remind the child that one tenth of one hundredth is a thousandth, so pink hundredth beads are exchanged for green thousandth beads.
4. Remind the child that one tenth of one tenth is one hundredth, so blue tenth beads are exchanged for pink hundredth beads to multiply tenths by tenths.
5. Remind the child that one tenth of one unit is one tenth, so each large green unit bead is exchanged for blue tenth beads to multiply units by tenths.

6. Tell the child that the multiplicand has been multiplied by one tenth. Ask the child to determine how many tenths are in the multiplier. This is the number of times the amount is placed on the mat. In this example, it is three times.

7. Ask the child to count the beads and form the answer with the numeral cards for the first partial product.
   In this example, it is 0.339.

8. Tell the child to place beads and decimal points on the mat to represent the multiplicand again and to lay out the amount of the multiplicand the number of times indicated by the multiplier. In this example, it is two times.

9. Have the child count the amount and form the answer with the numeral cards for the second partial product. In this example, it is 2.26.

10. Ask the child to combine and count the beads representing the two partial products, then to place appropriate numeral cards indicating the answer. In this example it is 2.599.

11 Invite the child to continue to do additional problems from problem booklets or cards or to put the materials away.

Procedure for Division by a Whole Number

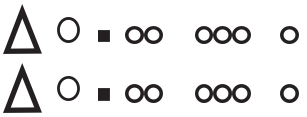
1. Invite a child to take the containers of red, blue and green unit beads, decimal beads, numeral cards and skittles to a table.

2. Using the numeral cards, set a division problem with a decimal fraction whose answer is a decimal fraction, for example, 2.462 + 2.

3. Have the child place large green unit skittles vertically at the left to to indicate the divisor. In this example, there are two large green skittles.

4. Tell the child to count into the box lid the number of beads indicated by the dividend. In this example, there are two green unit beads, four light blue eight millimeter tenth beads, six pink eight millimeter hundredth beads and two light green eight millimeter thousandth beads

5. Ask the child to give each skittle a bead. In this example, each skittle receives one unit, two tenths, three hundredths and one thousandth or 1.231.



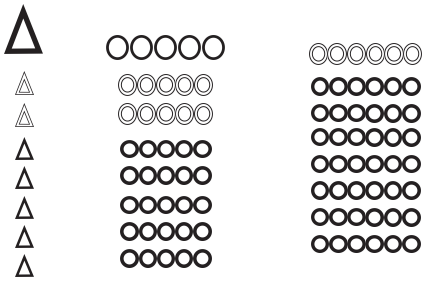
6. Have the child place cards to indicate what one skittle gets.

7. The child may continue to do additional problems from problem booklets or cards or to put the materials away.

Procedure for Division by a Decimal Fraction

1. Invite a child to take the containers of beads, decimal beads, numeral cards and skittles to a table.
2. Using the numeral cards, set a division problem with a fraction as the divisor, for example, 7 ÷ 1.25
3. Have the child place large green unit skittles, small blue tenth skittles and small red hundredth skittles vertically at the left to represent the divisor.
   In this example, there are one large green skittle, two small blue skittles and five small red skittles.

4. Tell the child to count into the box lid the number of beads indicated by the dividend.
   In this example, there are seven green unit beads.
5. Have the child divide the beads with each tenth skittle receiving a tenth bead and each hundredth skittle receiving a hundredth bead whenever the unit skittle receives a unit bead, changing one unit for ten tenth beads and one tenth for ten hundredth beads as necessary.
6. After dividing the units, there are tenths remaining to be divided. When the child gives each unit skittle a tenth bead, each tenth skittle is given a hundredth bead and each hundredth skittle is given a thousandth bead, obtained by changing one hundredth bead for ten thousandth beads
7. Remind the child that the answer in division is what one gets.
   In this example, the unit skittle receives five unit beads and six tenth beads.



8. Ask the child to form the answer with the numeral cards.
   In this example, it is 7 ÷ 1.25 = 5.6
9. Invite the child to continue to do additional problems or put the materials away.

Control of error

The teacher when problems are set with numeral cards
Answers in problem booklets or cards

Variations

Invite the child to write problems and record answers instead of using numeral cards.
(Note: Remind the child that decimal points must always be in alignment .)
Invite the child to do problems involving changing.

Converting Common Fractions to Decimal Fractions Using the Centesimal Circle

Purposes

To understand the relationship between common fractions and decimal fractions
To develop comprehension of operations involving decimal fractions

Preliminary Exercises

Practice with common fraction operations
Introductory work with decimal beads and cards Practice with compound multiplication with golden beads

Materials

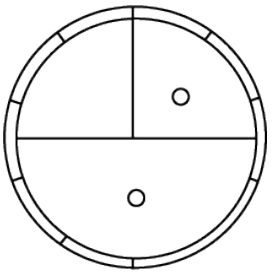
Ten fraction circles divided into equal parts from two to ten plus one undivided circle
Frame with centesimal circle the same dimension as the whole fraction circles, calibrated to indicate one hundred parts with each tenth labeled appropriately beginning with zero from which there is a line to the center
Recording paper with half-inch squares and pencil
Problem booklets with common fraction addition and subtraction problems

Introductory Procedure

1. Invite a child to bring to a table the divided fraction circles, recording paper and pencil and the calibrated centesimal circle.
2. Show the child the centesimal circle, pointing out that it is divided into one hundred parts with zero at the top, each part representing one hundredth.
3. Ask the child to move the fraction frame containing the ten fractional parts above the centesimal circle, then to place a one tenth fraction part in the right side of the centesimal circle with the edge of the fraction part aligned with the line at the zero position. Tell the child to read the numeral at the other edge of the tenth fractional part which is 0.1 . Have the child write $\frac{1}{10} = 0.1$
4. Ask the child to place a second tenth fraction part at the right side of and touching the tenth already in the frame . Tell the child to read the numeral at the edge of the second tenth fractional part which is 0.2 and write $\frac{2}{10} = 0.2$
5. Tell the child to continue placing tenth fractional parts and to record, being sure to keep decimal points aligned.
6. Invite the child to use any fractional parts to convert to decimal fractions.

Procedure for Addition of Fractions Using the Centesimal Circle

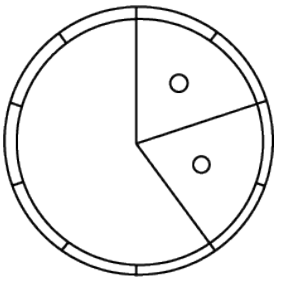
1. Invite a child to bring to a table the divided fraction circles, recording paper and pencil, frame with centesimal circle and addition problem booklet.
2. Have the child record a problem, for example, $\frac{1}{4} + \frac{1}{2} =$, and place the common fractions representing the addends into the centesimal circle, then record the decimal equivalent. In this example, it is 0.25 + 0.50=



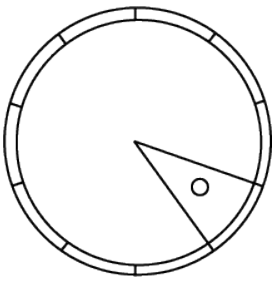
3. Ask the child to read the numeral at the edge of the second fraction and record the decimal fraction. In this example, it is 0.25 + 0.50= 0.75
4. Have the child continue with other problems or put the materials away.

Procedure for Subtraction of Fractions Using the Centesimal Circle

1. Invite a child to bring to a table the divided fraction circles, recording paper and pencil, frame with centesimal circle and subtraction problem booklet.
2. Have the child record a problem, for example, $\frac{2}{5} - \frac{1}{10} =$, and place the common fraction representing the minuend into the centesimal circle, then record the decimal equivalent. In this example, it is 0.4



3. Tell the child to remove the fractional part representing the minuend, then to place the fractional part representing the subtrahend so that the right edge is at the location where the minuend had been. In this example, the 1/10 is placed so that its right edge is at the 0.4 mark.



4. Have the child record the numeral at the left edge of the subtrahend fractional part. In this example, it is 0.3, so 0.4 - 0.1 = 0.3
5. Invite the child to continue with other problems or put the materials away.

Control of error

Answers in problem booklets or cards

Percentage

Purposes

To develop an understanding of the concept of computing by hundredths
To gain the ability to make calculations involving percent
To develop an understanding of percent in relation to common and decimal fractions
To learn how to convert percents to fractions

Preliminary Exercises

Work with common and decimal fractions
Practice in computation
Exercises for solving algebraic equations, for finding percent one number is of another and for finding a number when percent is known

Materials

Centesimal circle
Problem cards for conversion of fractions and decimals to percent, percent to fractions and decimals
Rule cards for conversions to and from percents
Problem cards involving all operations using percent
Paper and pencil

Procedure for Changing Percents to Fractions and Fractions to Percents

1. Invite a child to bring paper, pencil, conversion problem cards and conversion rule cards to a table.
2. Tell the child that the term *per cent* comes from *per centum,* Latin meaning *by the hundreds,* and percents represent common and decimal fractions.
3. Have the child read any rule card, choose practice problems appropriate for that rule and follow the procedure.

Procedure for Finding Percent of a Number

1. Invite a child to bring paper, pencil and problem cards to a table and to choose a problem to copy.
2. Remind the child that the percent in the problem represents a fraction by which another number is multiplied.
3. Tell the child to convert the percent into either a decimal or common fraction which becomes the multiplier, then to multiply the multiplicand by the fraction.

   For example,

   in the problem 5% of 45, 5% is converted to $\frac{5}{100}$ then 45 is multiplied by $\frac{5}{100}$.
   $\frac{5}{100}$ X 45 = $\frac{225}{100}$ = $2\frac{25}{100}$ = $2\frac{1}{4}$,
   or 5% may be converted to 0.05, then 45 is multiplied by 0.05. 0.05 X 45 = 2.25

Procedure for Finding Percent One Number Is of Another

1. Invite a child to bring paper, pencil and problem cards to a table and to choose a problem to copy.

2. Have the child write the problem as an algebraic equation, then divide the part by the whole.

   For example, in the problem,What percent of 72 is 18?, the equation is written as
   18 = x X 72 or 18 + 72 = 0.25 or 25%

3. Invite the child to continue with the problems.

Procedure for Finding a Number when Percent Is Known

1. Invite a child to bring paper, pencil and problem cards to a table and to choose a problem to copy.
2. Have the child write the problem as an algebraic equation with the known number equal to the percent times the unknown, x.
3. Tell the child to convert the percent to a decimal, then to divide it into the known number. For example, 5 is 25 % of what number? The equation is 5 =0.25 X x or 5 + 0.25 =20 , so 5 is 25% of 20..
4. Invite the child to continue with the problems.

Procedure for Calculating Discounts

1. Invite a child to bring paper, pencil and problem cards to a table and to choose a problem to copy. Tell the child that a "discount" is a percent of the original price which is deducted from the original price, giving a discount or "sale" price.
2. Have the child write the problem, change the per cent to a decimal fraction and multiply the original price by the decimal fraction. The answer is the amount to be subtracted from the original price to calculate the discount price.

   Example: An $800 computer is discounted 65%. What is the discount price?

   65%=0.65

   $$\begin{array}{r} \$800 \\ \times\ .65 \\ \hline \$520 \end{array} \qquad \begin{array}{r} \$800 \\ -\$520 \\ \hline \$280 \end{array}$$

   The discount price is $280.

Procedure for Calculating Interest

1. Invite a child to bring paper, pencil and problem cards to a table and to choose a problem to copy. Explain that interest is money paid for the use of money that has been borrowed from a bank, a lending institution or a person.
2. Have the child write the problem, change the per cent to a decimal fraction and multiply the principal by the decimal fraction to calculate the interest on the stated principal. For example, if the problem states that $12,000 was borrowed for one year at 6% interest, 6% is changed to 0.06. $12,000 X 0.06 =$720 interest.
3. Tell the child that if a loan is made for less than one year, the amount of interest is multiplied by the fractional part of the year. For example, if $12,00 is borrowed for six months at 6% interest, the interest of $720 is multiplied by 0.5 since the period of the loan is half a year. and the interest is $720 x 0.5 =$360
4. Tell the child that if a loan is made for more than one year, the amount of interest is multiplied by the number of years. For example, if $12,00 is borrowed for six years at 6% interest, the interest of $720 is multiplied by six since the period of the loan is six years. $720 X 6 =$4320.
5. Invite the child to continue with other problems or to put the materials away.

Decimal Fraction Board Operations

Purposes

To understand the relationship between common fractions and decimal fractions
To further develop comprehension of operations and skills involving decimal fractions

Preliminary Exercises

Practice with centesimal circle materials
Exercises with fraction bead materials

Materials

Pegboard with holes approximately one centimeter apart
Laminated strip four centimeters wide with colored sections one and one half centimeters wide (See white pages for pattern.)
Thin black elastic band about ninety-six centimeters long with ends sewn so that it can be placed around the entire board
Container with three compartments or three small containers for ten millimeter faceted plastic beads: one hundred green, fifty blue and fifty red
Container with six compartments or six small containers for eight and six millimeter faceted beads: fifty light blue, fifty pink and fifty light green in the eight millimeter size; fifty light blue, fifty pink and fifty light green in the six millimeter size
Container with three compartments or three small containers for nine green, nine blue and nine red skittles
Container for three sets of whole numeral cards for units, tens, hundreds and thousands in symbolic colors
Container for three sets of decimal numeral cards one tenth through nine millionths in the lighter symbolic colors
Problem booklets or cards without and with changing
Recording paper and pencil

Procedure for Addition

1. Invite a child to bring the materials except skittles to a table and indicate that the strip of symbolically colored discs is placed across the top right of the pegboard with the elastic band placed lengthwise between the unit and tenths discs.

2. Have the child copy a decimal addition problem, then place the addends on the pegboard using the appropriate symbolic colors and sizes of faceted beads. For example, in the problem 23.4653 + 14.3333 =, there are two blue ten beads, three green unit beads, four light blue eight millimeter beads, six pink eight millimeter beads, five light green eight millimeter beads and three light blue six millimeter beads in the first addend; one blue ten bead, four green unit beads, three light blue eight millimeter beads, three pink eight millimeter beads, three light green eight millimeter beads and three light blue six millimeter beads in the second addend.

3. Ask the child to count and move the beads toward the bottom of the pegboard, beginning at the far right, then to record the answer. In this example, there will be three blue ten beads, seven green unit beads, seven light blue eight millimeter beads, nine pink eight millimeter beads, eight light green eight millimeter beads and six light blue six millimeter beads or **37.7986.**

4. Invite the child to continue with other problems or to put the materials away.

Procedure for Subtraction

1. Invite a child to bring the materials except skittles to a table and indicate that the strip of symbolically colored discs is placed across the top right of the pegboard with the elastic band placed lengthwise between the unit and tenths discs.
2. Have the child copy a decimal subtraction problem, then place the minuend on the pegboard using the appropriate symbolic colors and sizes of faceted beads. For example, in the problem 73.8654 - 42. 7321, there are seven blue ten beads, three green unit beads, eight eight millimeter light blue beads, six pink eight millimeter beads, five light green eight millimeter beads and four light blue six millimeter beads.
3. Ask the child to subtract the quantity of beads indicated in the subtrahend by counting and moving the beads several spaces below the lowest bead in the minuend. In this example, four blue ten beads, two green unit beads, seven eight millimeter light blue beads, three pink eight millimeter beads, two light green eight millimeter beads and one light blue six millimeter bead are moved down.
4. Tell the child to determine the remainder by counting the beads left in what was the minuend and moving them to the bottom on the pegboard. In this example, there are three blue ten beads, one green unit bead, one eight millimeter light blue bead, three pink eight millimeter beads, three light green eight millimeter beads and three light blue six millimeter beads or 31.1333.
5. Invite the child to continue with other problems or to put the materials away.

Procedure for Multiplication by a Whole Number

1. Invite a child to bring the materials except skittles to a table and indicate that the strip of symbolically colored discs is placed across the top right of the pegboard with the elastic band placed lengthwise between the unit and tenth discs.
2. Have the child copy a decimal multiplication problem, then place the multiplicand on the pegboard the number of times indicated by the multiplier. For example, in the problem 23.4223 X 2 =, there are two blue ten beads, three green unit beads, four light blue eight millimeter beads, two pink eight millimeter beads, two light green eight millimeter beads and three light blue six millimeter beads placed two times on the pegboard.
3. Ask the child to count and move the beads toward the bottom of the pegboard, beginning at the far right, then to record the answer. In this example, there will be four blue ten beads, six green unit beads, eight light blue eight millimeter beads, four pink eight millimeter beads, four light green eight millimeter beads and six light blue six millimeter beads or 46.8446.
4. Invite the child to continue with other problems or to put the materials away.

Procedure for Division by a Whole Number

1. Invite a child to bring the materials and skittles to a table and indicate that the strip of symbolically discs is placed across the top right of the pegboard with the elastic band placed lengthwise between the unit and tenths discs.
2. Have the child copy a decimal division problem, then place the dividend on the pegboard using the appropriate symbolic colors and sizes of faceted beads. For example, in the problem 22.4684 + 2 =, there are two blue ten beads, two green unit beads, four light blue eight millimeter tenth beads, six pink eight millimeter hundredth beads, eight light green eight millimeter thousandth beads and four light blue six millimeter ten thousandth
3. Tell the child to place skittles, according to the divisor, in a column on the left of the pegboard with enough space between to accommodate the beads. In this example, there are two green skittles placed in a column at the left of the board below the beads representing the dividend.
4. Ask the child to divide the beads among the skittles, beginning with the highest place value and to record the result. In this example, each skittle gets one blue ten bead, one green unit bead, two light blue tenth beads, three pink hundredth beads, four light green thousandth beads and two light blue six millimeter tens of thousand beads or 11.2342.
5. Invite the child to continue with other problems or to put the materials away.

Decimal Checkerboard

Purposes

To reinforce the concept of place value
To prepare for abstract multiplication of decimal fractions

Preliminary Exercises

Practice with multiplication on the checkerboard
Use of decimal board
Work with decimal operations with beads

Materials

Decimal checkerboard with forty-nine squares symbolically colored to represent hierarchies from millions to millionths
Container of twenty of each colored bead bars one through nine
Container of white tiles and gray tiles with numerals one to nine in symbolic colors Container of black discs to indicate decimal points
Recording paper with symbolically colored lines
Graph paper
Red, blue and green pencils
Decimal multiplication problems with whole number multiplicands

Procedure for Introduction to the Decimal Checkerboard

1. Invite a child to bring the decimal checkerboard and container of bead bars to a table, placing them beside the checkerboard on the child's dominant side.
2. Ask the child to read the numerals across the bottom of the checker board beginning at the lower right corner with millionths, then to read the numerals at the right side from bottom to top, starting with units. Call the child's attention to the symbolic color code of the board.
3. Have the child name the values of the colored squares in the second row from right to left, then in the third and fourth rows, always from right to left.
4. Remove a bead bar from the container and place it on the light green millionths square at the lower right corner. Tell the child that millionths are indicated according to the number of beads on the bead bar. For example, if a three bead bar is placed on the light green millionths square, it represents three millionths. Move the bead bar to the left so that it is in the pink hundred thousandths square. Explain that the value is now hundred thousandths. For example, if a three bar is placed in the pink hundred thousandths square, it represents three hundred thousandths.
5. Continue to move the bead bar to the left, explaining the change in value in each square, then repeat by moving the bead bar vertically to each square, starting at the light green millionths square.
6. Place a bead bar in any square on the checkerboard and ask the child to name the value. Repeat until the child understands the concept.
7. Give the child a list of amounts to form on the checkerboard, for example,

5.324
26.7895
302.63217

8. Invite the child to continue as long as desired.

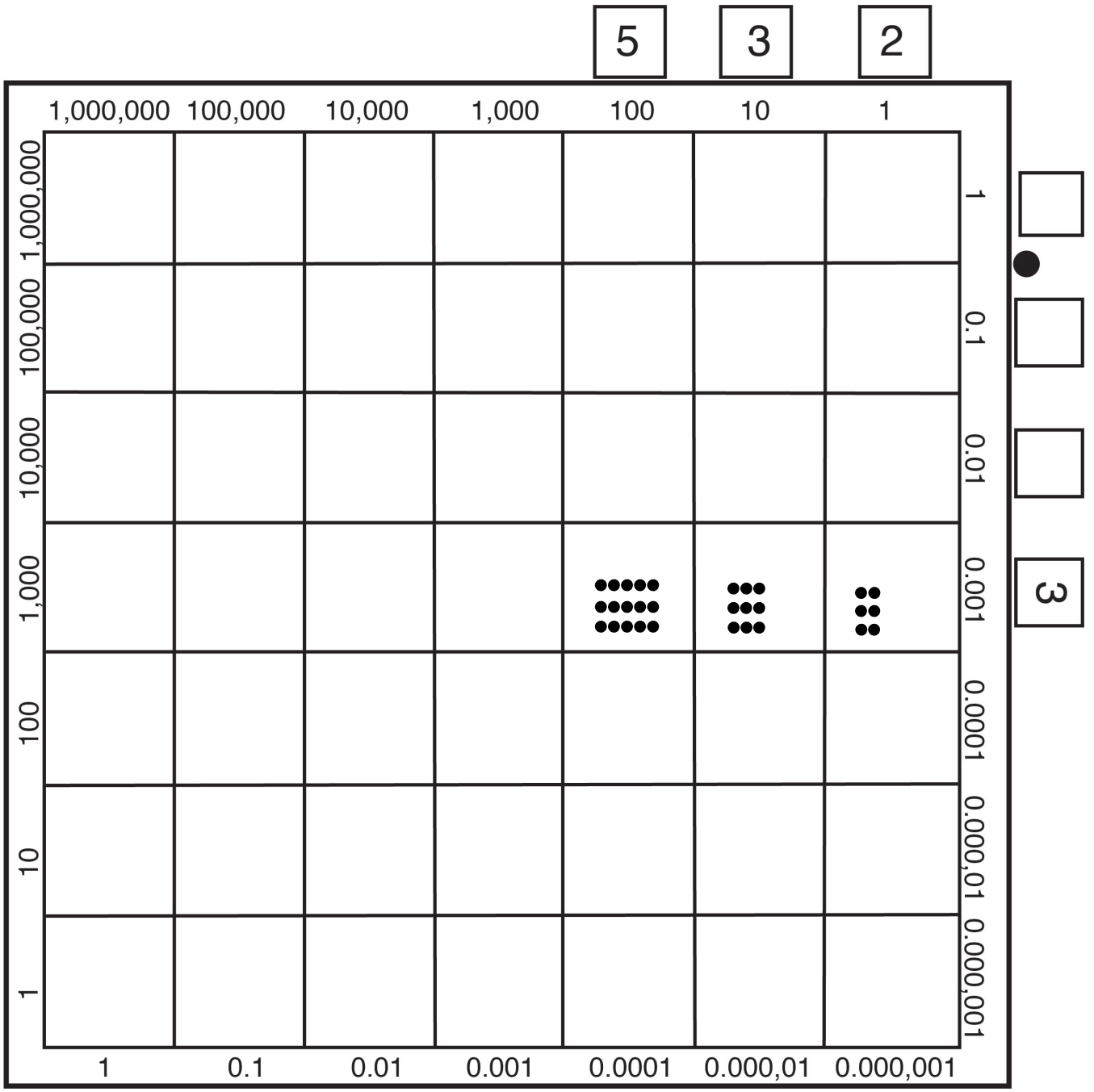
Procedure for Multiplication on the Decimal Checkerboard

(Note: Only whole number multiplicands can be used.)

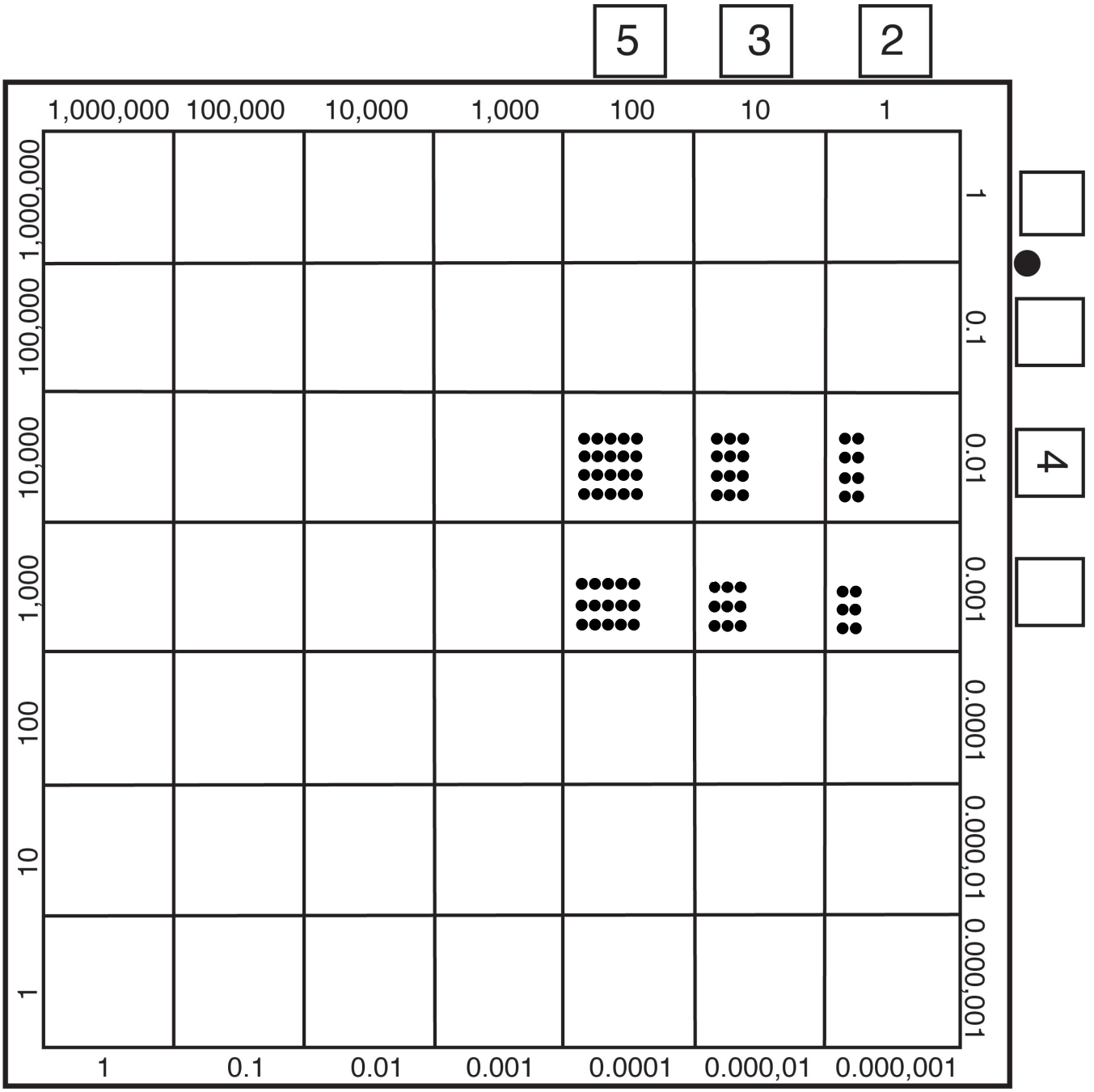
1. Invite a child to bring the decimal checkerboard, container of bead bars, container of numeral tiles, written problems with whole number multiplicands and recording paper to a table, placing them beside the checkerboard on the child's dominant side.

2. Choose a problem and have the child copy it on the recording paper. Ask the child to form the multiplicand with the white numeral tiles at the top of the checker board in the appropriate place value position, then to form the multiplier with the gray numeral tiles along the right edge of the checker board in the appropriate place value position. For example, in the problem 532 X 2.143, to form the multiplicand, the white tile with red numeral five is placed at the top of the board above the red hundred square, the white tile **with** blue numeral three above the blue tens square, the white tile with green numeral two above the green units square. For the multiplier, the gray tile with green numeral two is placed to the right of units square at the top right of the board, the gray tile with blue numeral one is placed to the right of tenths square, the gray tile with red numeral four to the right of hundredths square, the gray tile with green numeral three to the right of thousandths square. A black disc is placed between the tiles to indicate the decimal point.

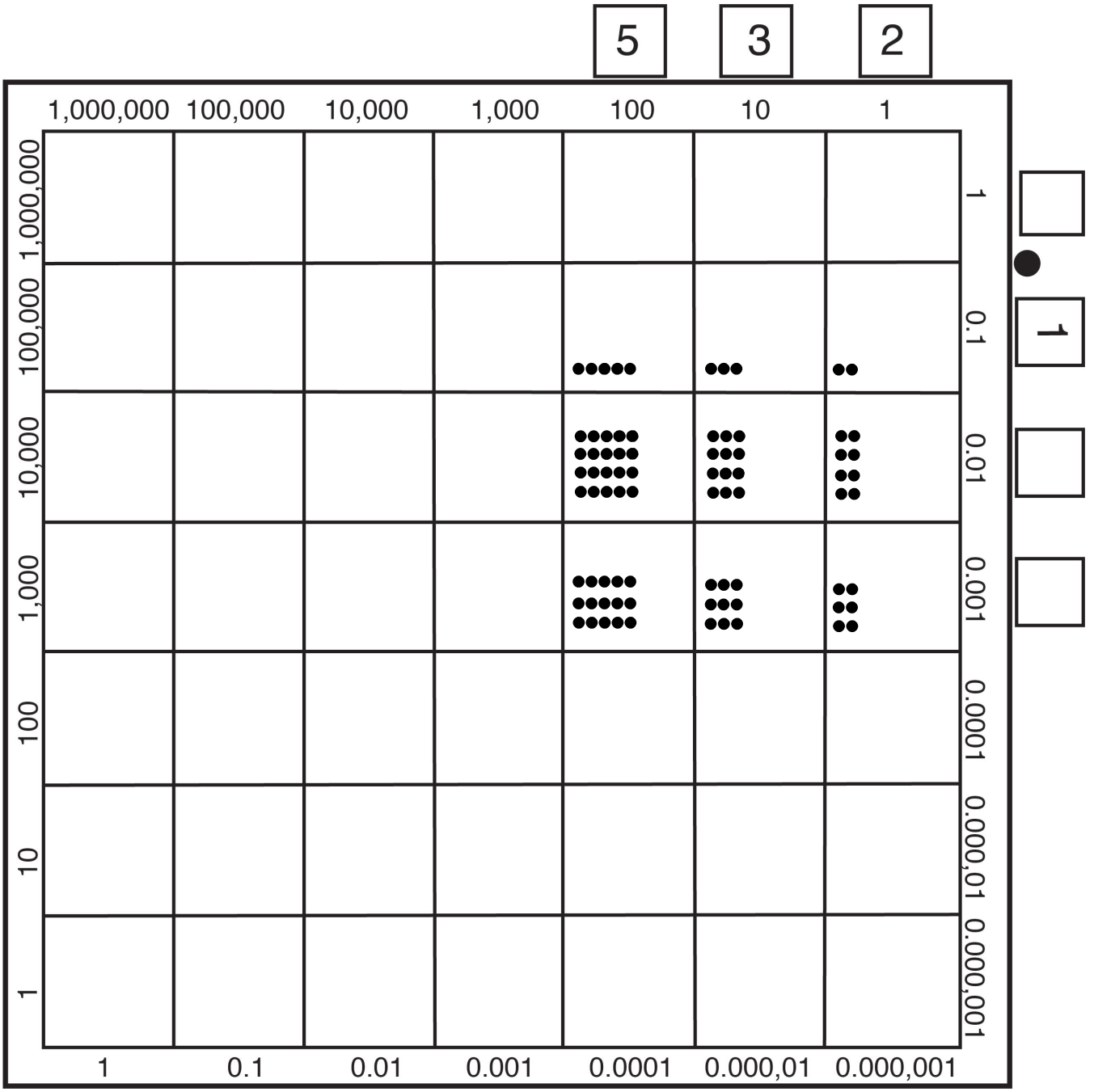
3. Tell the child that units in the multiplicand will be multiplied beginning with the smallest place value by placing bead bars on appropriate squares. Have the child turn face down all numerals in the multiplier except that with the smallest place value which is the first multiplier. For this example, all gray tiles are turned face down except for three thousandths. Have the child place three two bead bars on thousandths square (2 X 0.003), three three bead bars on hundredths square (30 X 0.003), three five bead bars on tenths square (500 X 0.003). The partial product is 1.596.


5
3
2
1,000,000
100,000
10,000
1,000
100
10
1
1,000,000
100,000
10,000
1,000
100
10
1
1
0.1
0.01
0.001
0.0001
0.000,01
0.000,001
0.000,001
0.000,01
0.0001
0.001
0.01
0.1
1
3

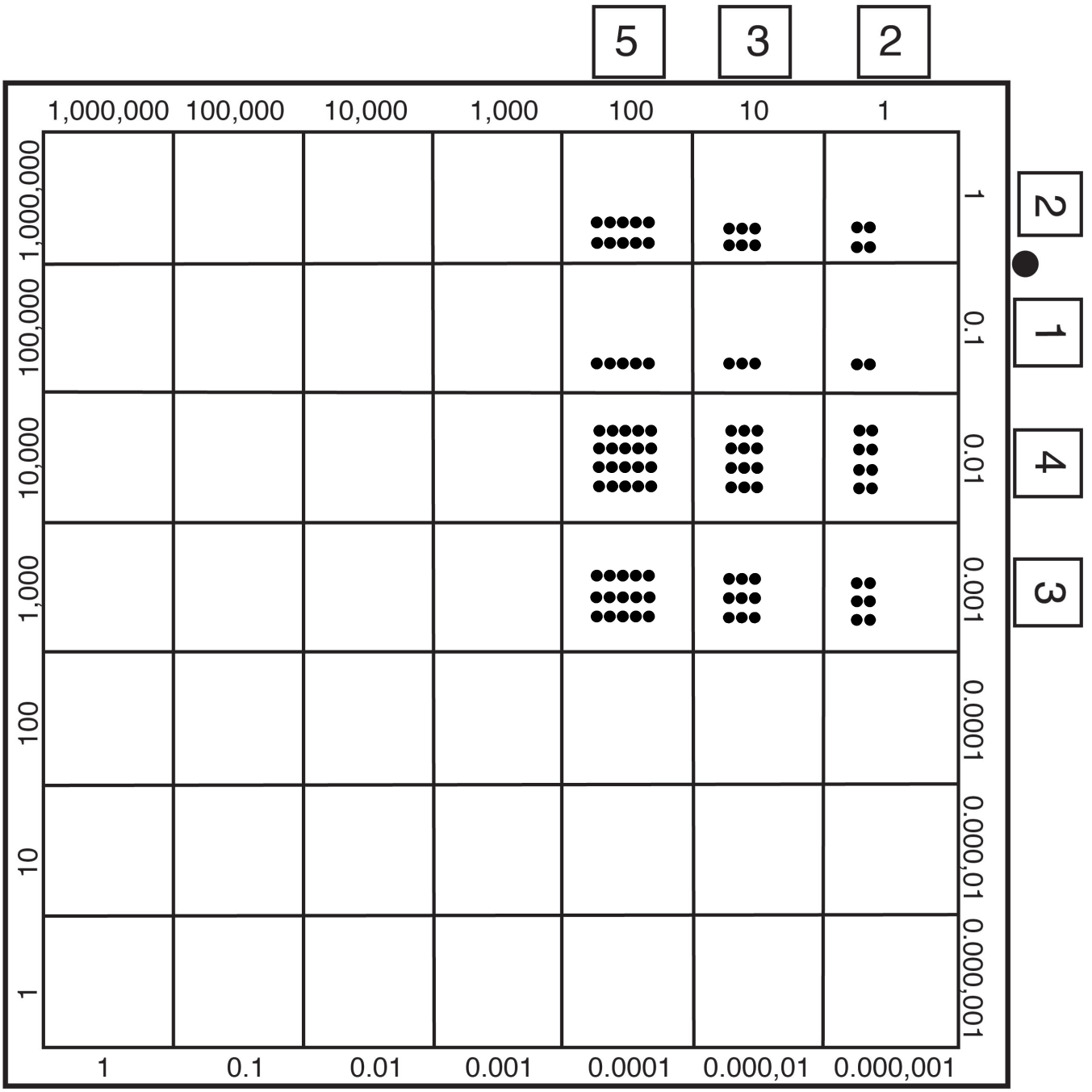
4. Have the child turn the gray tile just used for the multiplier faced down and the next tile face up in preparation for the next multiplication, then put appropriate bead bars in the second row of squares above those in which bead bars were previously placed. In this example, there are four two bead bars (2 X 0.04), four three bead bars (3 X 0.04), four five bead bars (5 X 0.04). The partial product is 21.28.



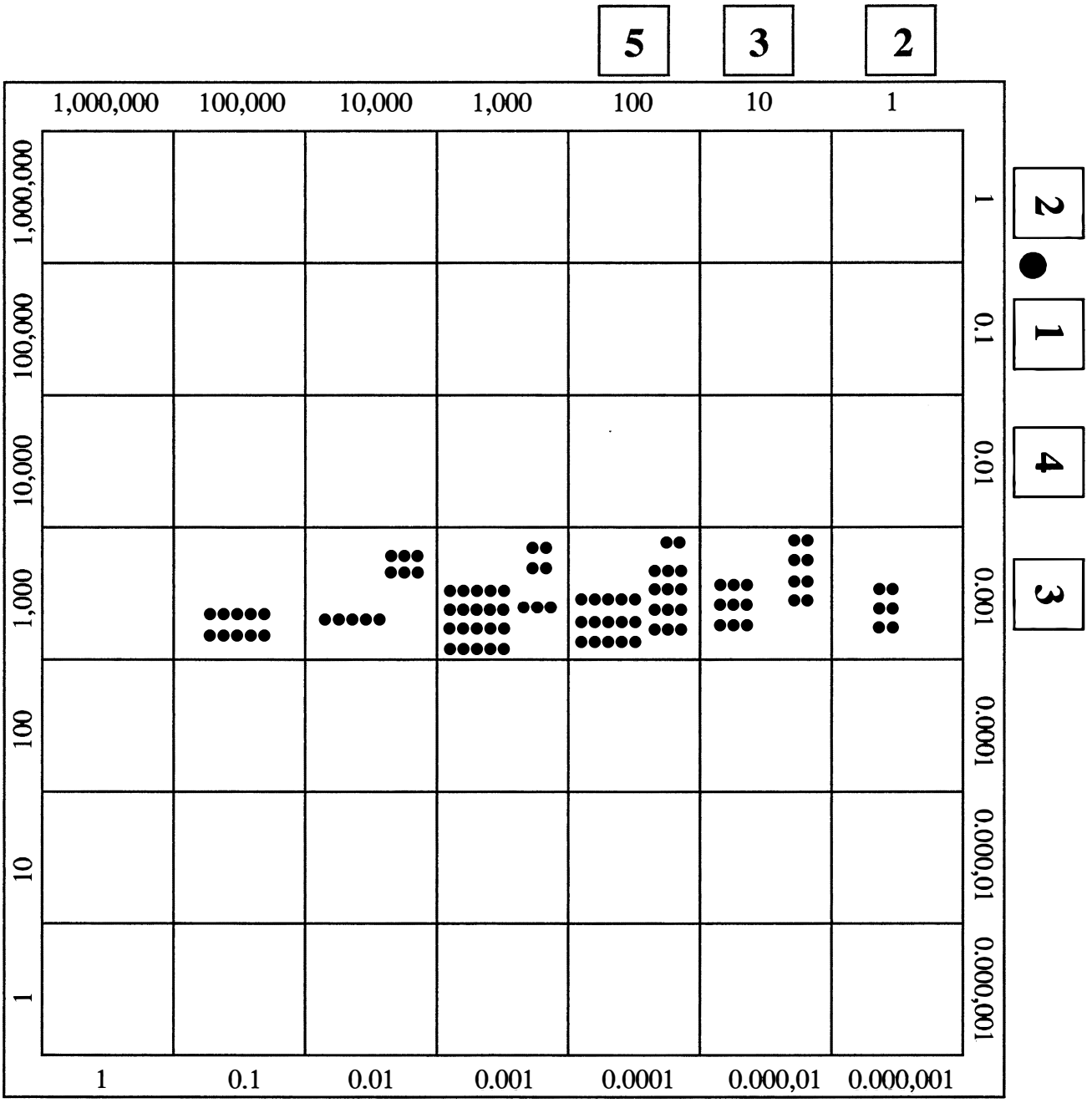
5. Have the child turn the gray tile just used for the multiplier faced down and the next tile face up in preparation for the next multiplication, then put appropriate bead bars in the third row of squares above those in which bead bars were previously placed. In this example, there is one two bead bar (2 X 0.1),one three bead bar (30 X 0.1), one five bead bar (500 X 0.1). The partial product is 53.200.



6. Tell the child to turn the gray tile just used for the multiplier face down and the next tile face up in preparation for subsequent multiplication, then to place appropriate bead bars in the fourth row of squares above those in which bead bars have been placed previously. In this example, there are two two bead bars (2 X 2), two three bead bars (30 X 2), two five bead bars (S00X 2). The partial product is 1064.

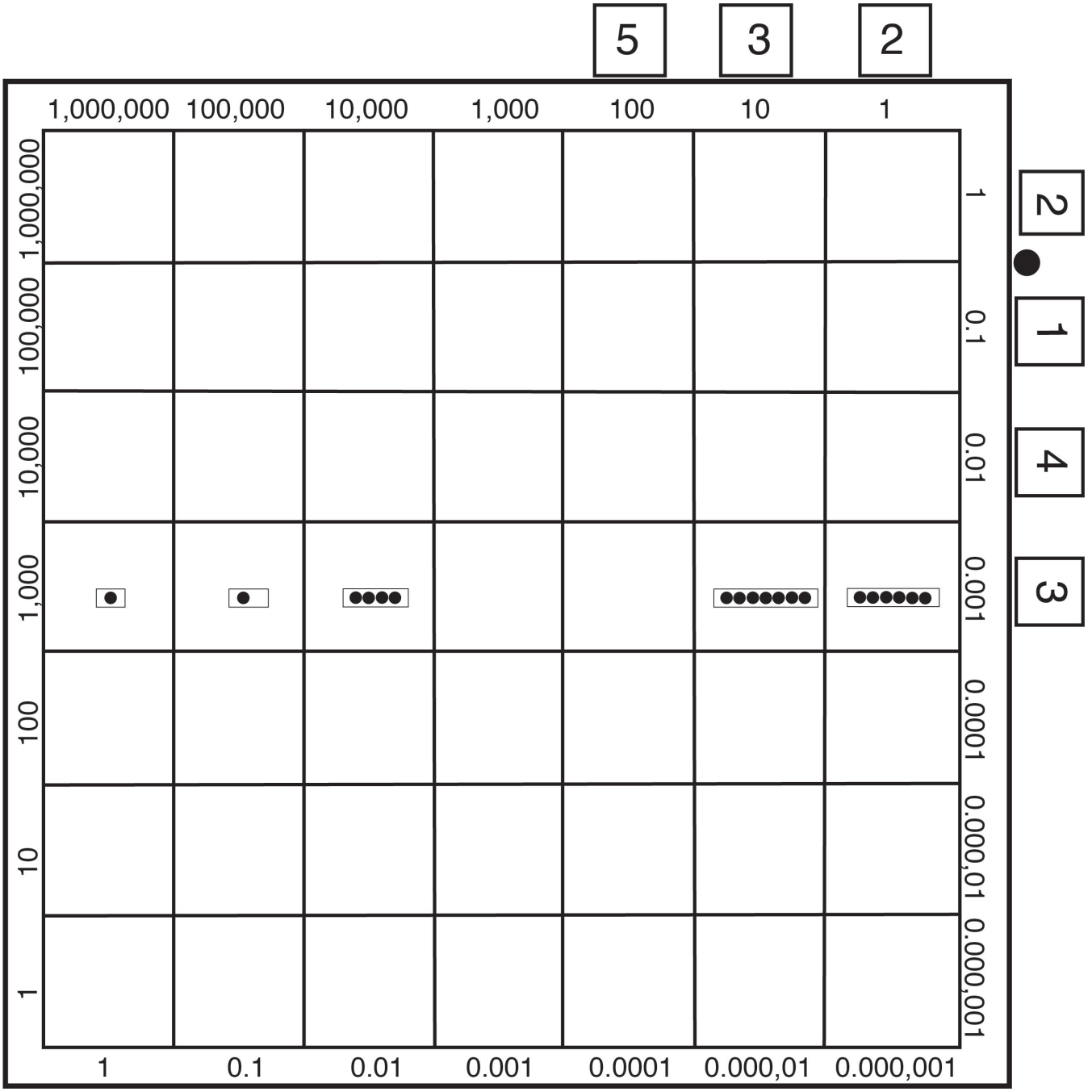


7. To obtain the product, ask the child to move all bead bars diagonally downward from right to left to the lowest same color square in which there are bead bars. In this example, the three two bead bars in thousandths place are not moved. There are three two bead bars in the thousandths place light green square; four two bars and three three bars in the pink square to its left; one two bar, four three bars and three five bars in the light blue square to the left of the pink square; two two bars, one three bar and four five bars in the green square to the left of the light blue square; two three bars and one five bar in the blue square to the left of the green square and two five bars in the red square at the left of the blue square.



8. Ask the child to count and change the bead bars in the lowest row of beads. In this example, the three two bars are changed for one six bar in the light green thousandths place. The three three bars and four two bars total seventeen, so a seven bar is placed in the pink hundredths square and one bar is placed in the light blue tenths square. The two bar, four three bars, three five bars and the one bar in tenths square total thirty, so a three bar is placed in the green unit square to the left and no bars remain in the light blue tenths square. The two two bars, one three bar, four five bars and one three bar in the green units square total thirty, so a three bar is placed in the blue tens square and no beads remain in the green units square. The two three bars, one five bar and one three bar in the blue tens square total fourteen, so a one bead bar is placed in the red hundreds square and a four bar is placed in the blue tens square. The two five bars and the one bar total eleven, so a one bead bar is placed in the green thousands square and a one bead bar is placed in red hundreds square.

9. Have the child read and record the entire equation. In this example, it is 532 X 2.143 = 1,140.076.



10. Invite the child to continue with other problems or to put away the materials.

Variation

Invite the child to count and record partial products, then combine to determine a final product.

Control of Error

Color coding of board
Place value numerals around the perimeter of the board
Answers in problem booklets

Building Squares of Numbers with Golden Beads

Purposes

To develop understanding of forming squares of numbers and finding square roots
To prepare for work with squares of numbers and finding square roots

Preliminary exercises

Work with bead chains
Practice with individual golden bead exercises
Exercises with division by quotition
Practice with geometrical multiplication with golden beads
Experience in forming figurate numbers
Work with binomial square

Materials

Tray of unlimited golden bead hundreds, tens and units
Recording paper and pencil
Felt mat
Problem booklets or cards

Introductory Procedure

1. Invite a child to place a felt mat on a table with the tray of golden beads to the left of. the mat.
2. Have the child form squares with golden bead units, starting at the lower right corner of the mat. After 1^2 is formed with one bead, additional beads are added to form 2^2 or 4, continuing to 11^2.
3. When 11^2 is reached, tell the child to exchange one hundred unit beads for one hundred square and exchange the ten unit beads on the bottom and right side of the square for ten bead bars with one unit remaining in the lower right corner.
4. Have the child count the quantity of beads.

Procedure for Building Squares

1. Invite a child to place a felt mat on a table with the tray of golden beads to the left of the mat and to choose a problem, for example, 32^2.
2. Tell the child lay out golden bead tens and units vertically and horizontally as indicated by the problem, starting at the lower right corner of the mat with units, then placing ten bead bars vertically and horizontally. The one unit in the lower right corner counts in both the vertical and horizontal configuration. In this example, two unit beads are placed horizontally from the lower right corner and another unit bead is placed above the corner bead. Three ten bars are placed to the left of the two horizontal unit beads at the bottom and three ten bars are placed above the two vertical units at the right side.

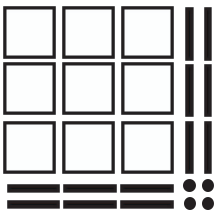
3. Ask the child to place unit beads to complete the square of units and record the result. In this example, only one bead is required.

$2^2 = 4$

4. Tell the child to complete the rectangles of tens according to the units square and record the result. In this example, there will be three more ten bars placed horizontally at the bottom above the existing ten bars and three placed vertically to the left of the vertical ten bars.

6 X 10 = 60 X 2 = 120

5. Have the child complete the large square by placing hundred squares according to the ten bars and record the result. In this example, there will be nine hundred squares required to complete the large square.



3 X 100=300x3=900

6. have the child add the amounts counted, in this example 4 + 120 + 900 = 1024.

7. The child may continue to do additional problems or put the materials away.

Control of Error

Answers in problem booklets or cards

Building Squares of Numbers with Stamps

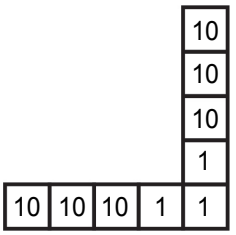
Purposes
To develop understanding of forming squares of numbers and finding square roots
To prepare for work with squares of numbers and finding square roots

Preliminary exercises
Work with bead chains
Practice with individual golden bead exercises
Exercises with division by quotition
Practice with geometrical multiplication with golden beads
Experience in forming figurate
ate numbers
Work with binomial square
Practice in building squares of numbers with golden beads

Materials
Container of color coded stamps for units, tens hundreds, thousands
Felt mat
Problem booklets or cards

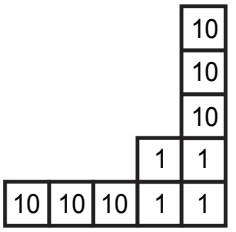
Procedure

1. Invite a child to place a felt mat on a table with the container of stamps behind the mat and to choose a problem, for example, 32^2.
2. Tell the child lay out green unit stamps vertically and horizontally as indicated by the problem, starting at the lower right corner of the mat , then placing blue ten stamps vertically and horizontally according to the unit stamps.. The one unit stamp in the lower right corner counts in both the vertical and horizontal configuration. In this example, two unit stamps are placed horizontally from the lower right corner and another unit stamp is placed above the corner stamp. Three blue ten stamps are placed to the left of the two horizontal unit stamps at the bottom and three blue ten stamps are placed above the two vertical units at the right side.



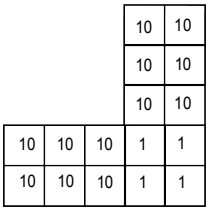
3. Ask the child to place unit stamps to complete the square of units and record the result. In this example, only one stamp is required.

$2^2 = 4$



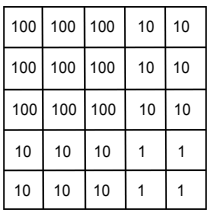
4. Tell the child to complete the rectangles of tens according to the units square and record the result. In this example, there will be three more ten stamps placed horizontally at the bottom above the existing ten stamps and three placed vertically to the left of the vertical ten stamps.

30 X 2 = 60, 60 X 2 = 120



5. Have the child complete the large square by placing hundred stamps according to the ten stamps and record the result. In this example, there will be nine hundred stamps required to complete the large square.

3 X 100 = 300, 300 X 3 = 900



6. Have the child add the amounts counted, in this example 4 + 120 + 900 = 1024.
7. The child may continue to do additional problems or put the materials away.

Control of error

Answers in problem booklets or cards

Building Squares of Numbers with Pegs and Peg Board

Purposes

To develop understanding of forming squares of numbers and finding square roots
To prepare for work with squares of numbers and finding square roots Preliminary exercises
Work with peg board operations
Practice in building squares of numbers with golden beads and with stamps

Materials

Containers of color coded pegs for units, tens, hundreds, thousands Large peg board
Problem booklets or cards
Recording paper and pencil

Introductory Procedure

1. Invite a child to place the peg board on a table with the opened containers of peg sat the left in the usual order then to get recording paper, pencil and problem booklet or card. Review the value of pegs if necessary.
2. Tell the child to build the square of any number between one and nine with green unit pegs, for example 5^2.

ggggg
ggggg
ggggg
ggggg .
ggggg

3. Have the child count downward, starting at the right, and change ten units for a blue ten bead as necessary. In this example, there will be two blue tens pegs and five green units pegs, or $5^2 = 25$.

bbggggg

4. After building squares with numbers nine and under, have the child build squares of numbers over nine, for example, 11^2.

gb
gb
gb
gb
gb
gb
gb
gb
gb
gb
gb

5. Have the child count downward, starting at the right,and change ten units for a blue ten bead and ten tens for a red hundred as necessary.

   In this example,
   there will be one red hundreds peg, two blue tens pegs and one green units peg,
   or 11^2 =121.

rb
bg

6. Tell the child that there is a faster way to find squares by placing the quantity of pegs at a right angle with a unit peg shared at the bottom right.
   For example, 12^2 is represented as

b
g
bgg

7. Have the child complete the square by adding units, tens and hundreds pegs.
   In this example, 12^2=144.

rbb
bgg
bgg

8. The child may continue with other problems or put the work away.

Procedure for Building Squares of Numbers with Tens and Units

1. Invite a child to place the peg board on a table with the opened containers of pegs at the left in the usual order then to get recording paper, pencil and problem booklet or card.

2. Tell the child lay out green unit pegs vertically and horizontally as indicated by the problem, starting at the lower right corner of the board, then placing blue ten pegs vertically and horizontally according to the unit pegs. The unit peg in the lower right corner counts in both the vertical and horizontal configuration.

   For example, 32^2 requires two unit pegs to be placed horizontally from the lower right corner and another unit peg is placed above the corner peg. Three blue ten pegs are placed to the left of the two horizontal unit pegs at the bottom and three blue ten pegs are placed above the two vertical units at the right side.

b
b
b
g
bbbgg

3. Ask the child to place unit pegs to complete the square of units and record the result. In this example, only one peg is required, giving a square of four green pegs.

$2^2 = 4$

b
b
b
gg
bbbgg

4. Tell the child to complete the rectangles of tens according to the units square and record the result. In this example, there will be three more tens pegs placed horizontally at the bottom above the existing ten pegs and three placed vertically to the left of the vertical tens pegs.

bb
bb
bb
bbbgg
bbbgg

	30 X 2 = 60
30 x 2 = 60	2 x 2 = 4

5. Have the child complete the large square by placing hundreds pegs according to the tens pegs and record the result. In this example, there will be nine hundred pegs required to complete the large square.

r r r r r r r r r	bb bb bb
bbb bbb	gg gg

$30^2 = 900$	30 x 2 = 60
30 x 2 = 60	$2^2 = 4$

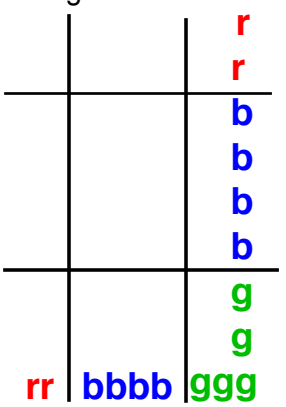
6. The child may continue to do additional problems or put the materials away.

Control of error

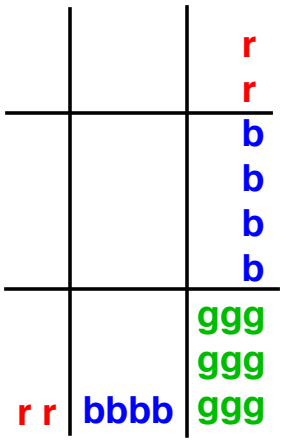
Answers in problem booklets or cards

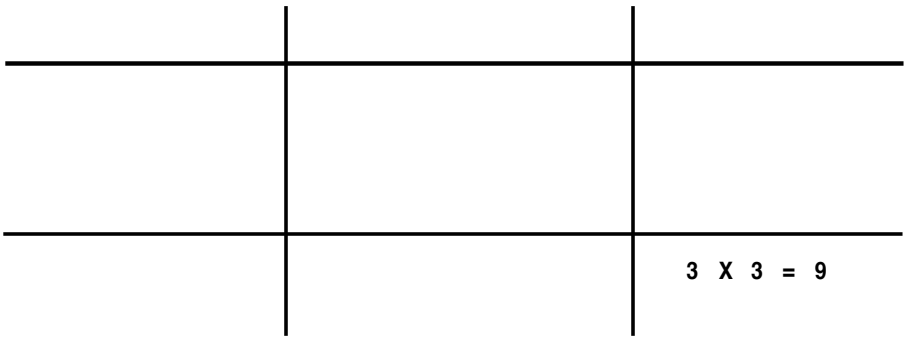
Procedure for Building Squares of Numbers with Hundreds, Tens and Units

1. Invite a child to place the peg board on a table with the opened containers of pegs at the left in the usual order then to get recording paper, pencil and problem booklet or card.
2. Tell the child lay out green unit pegs vertically and horizontally as indicated by the problem, starting at the lower right corner of the board , then placing blue ten pegs vertically and horizontally according to the unit pegs. The unit peg in the lower right corner counts in both the vertical and horizontal configuration. For example, 243^2 requires three units pegs to be placed horizontally from the lower right corner and another two units pegs are placed above the corner peg. Four blue tens pegs are placed to the left of the three horizontal unit pegs at the bottom and four blue tens pegs are placed above the three vertical units at the right side. Two red hundreds pegs are placed left of the tens on the bottom row of pegs and two are placed above the tens at the right.

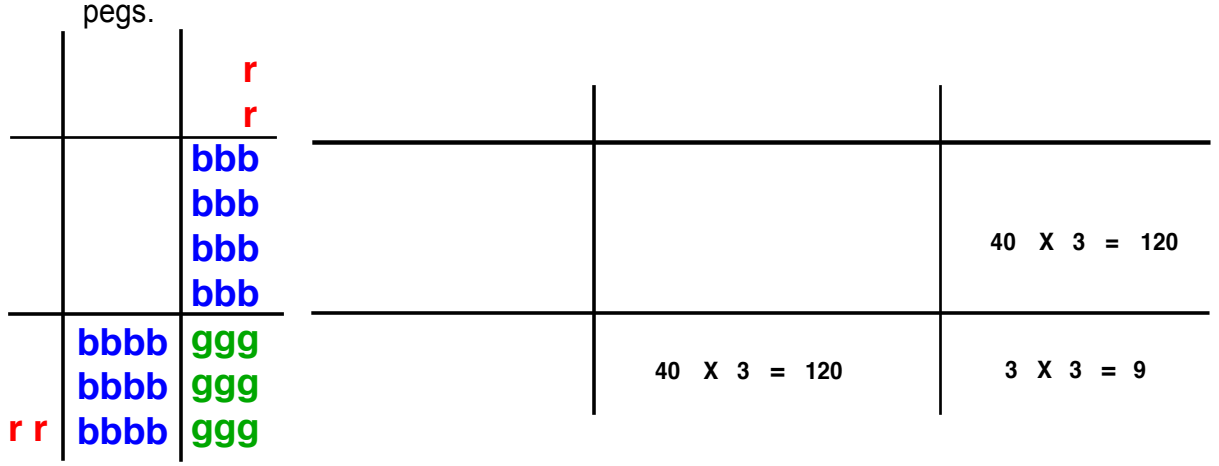


3. Ask the child to place unit pegs to complete the square of units and record the result. In this example, four more pegs are required, giving a square of nine green pegs.

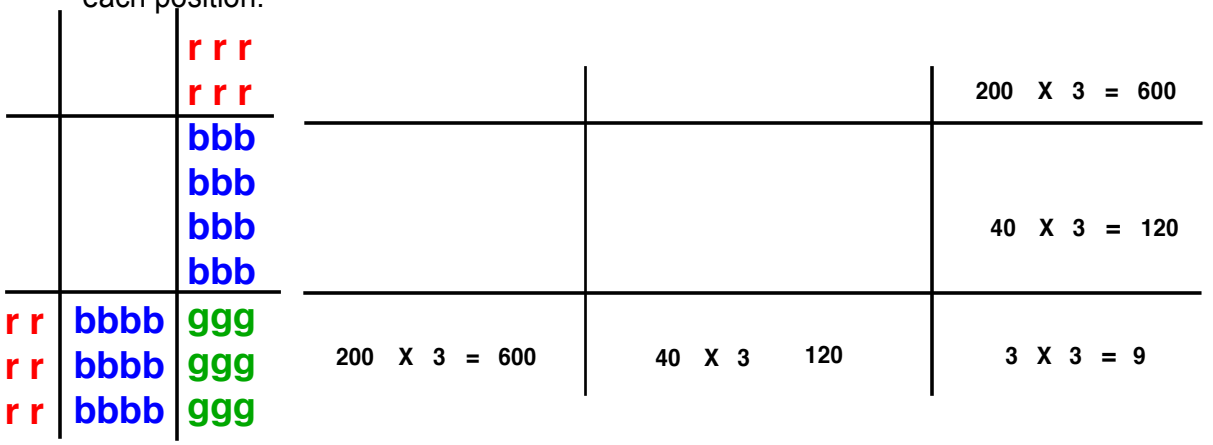




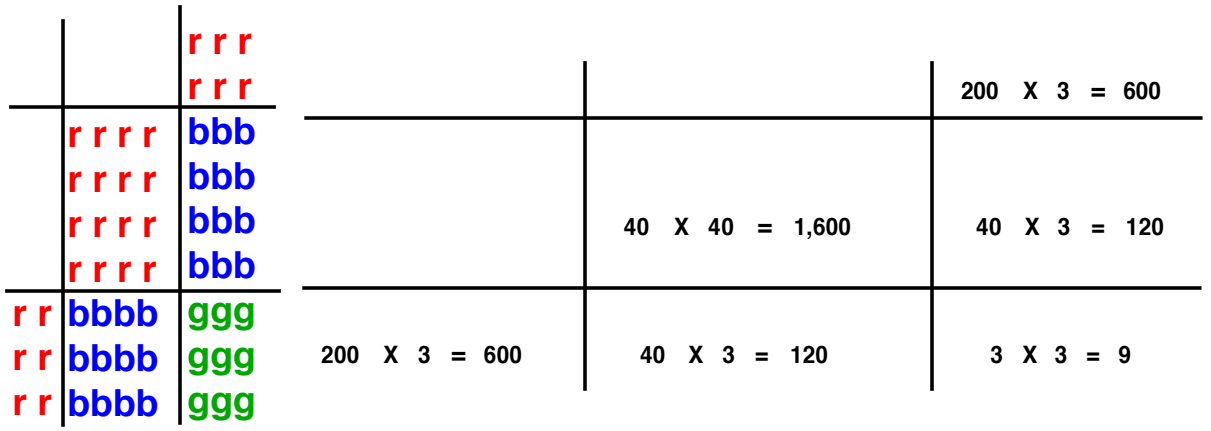
4. Tell the child to complete the rectangles of tens according to the units square and record the result. In this example, there will be eight more tens pegs placed horizontally at the bottom above the existing ten pegs and eight placed vertically to the left of the vertical tens pegs.



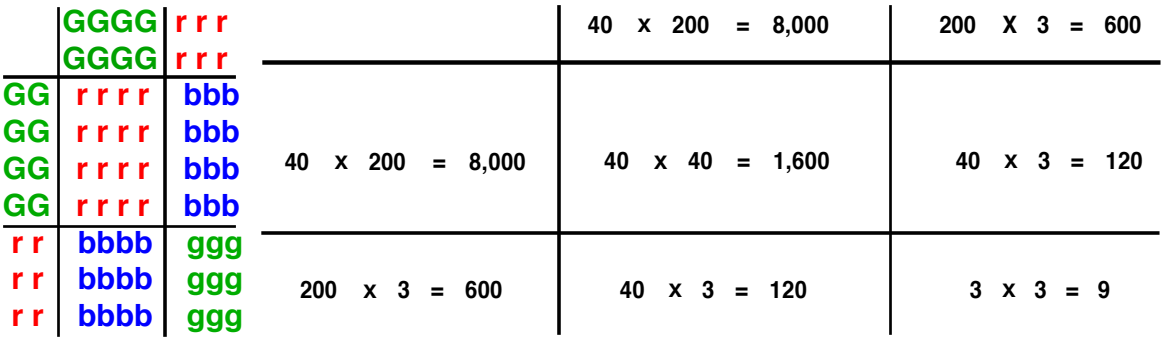
5. Ask the child to complete the rectangles of hundreds pegs above the hundreds at the left and above the hundreds at the right. In this example, there will be four added in each position.



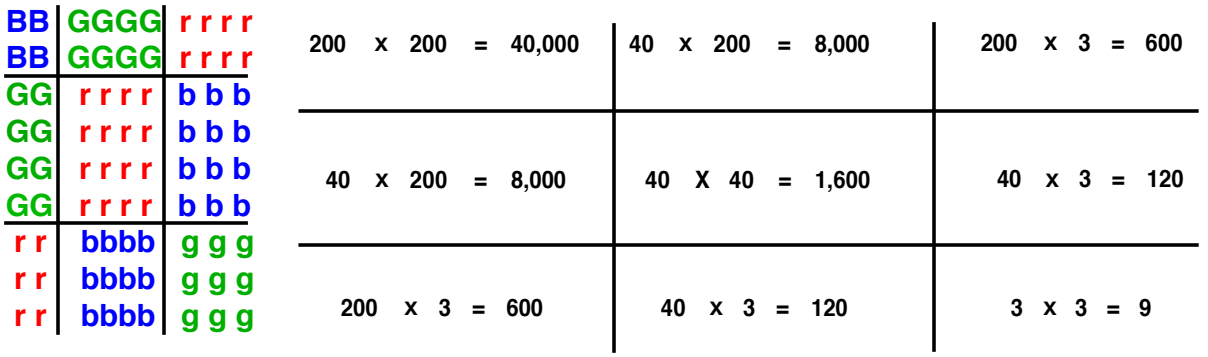
6. Have the child complete the large square by placing hundreds pegs according to the tens pegs and record the result. In this example, there will be sixteen hundred pegs required to complete the center square.



7. Tell the child to form rectangles with green units of thousands pegs to the left and above the rectangles of hundreds pegs. In this example, there will be eight thousands pegs in each position.



8. Ask the child to place blue tens of thousands pegs in the square above and left of the green units of thousands pegs. In this example there will be four tens of thousands pegs.



9. Have the child count the pegs, changing as necessary, and tabulate the results. In this example, it will be 59,049.

BBBBBGGGGGGGGGbbbbgggggggg

10. The child may continue to do additional problems or put the materials away.

Control of error

Answers in problem booklets or cards

Finding Square Roots with the Golden Bead Material

Purposes

To provide experience in extracting square roots
To increase knowledge and understanding of mathematics

Preliminary exercises

Work in forming figurate numbers
Experience in building squares of numbers
Practice with division by quotition

Materials

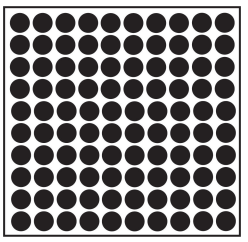
Unlimited golden bead material
Note: Use the wooden representational squares and cubes.
Felt mat
Problem booklets or cards from $\sqrt{1}$ to $\sqrt{81}$ having exact square roots
Problem booklets or cards for square roots of numbers over one hundred
Recording paper and pencil

Introductory Procedure

1. Invite a child to bring recording paper, pencil, a mat, exact square root problems and a tray of golden beads to a table, placing the tray to the left of the felt mat which is at the front of the table.
2. Have the child choose any exact square root problem, place the beads representing the square root on the mat in the proper place value locations, then build a square of the number after changing tens to units For example, for $\sqrt{16}$, sixteen beads are arranged in a square with four beads to a side. Tell the child to count the beads on the right side of the square. This represents the square root of the number. In the example, there are four beads on the side, so $\sqrt{16} = 4$.

3. Ask the child to examine a hundred square to determine the number of beads represented on the right side, which is ten. Tell the child that the square root of one hundred is ten.

$\sqrt{100} = 10$



4. The child may continue to do other problems or put the work away.

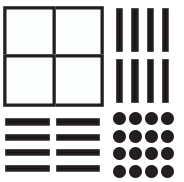
Procedure for Square roots of Three Digit Numbers

1. Invite a child to bring recording paper, pencil, a mat, problems for square roots over one hundred and a tray of golden beads to a table, placing the tray to the left of the felt mat which is at the front of the table.
2. Tell the child to choose a problem and place the beads representing the square root on the mat in the proper place value locations. For the example, for $\sqrt{5,76}$, there are five hundred squares, seven ten bars and six units.

3. Explain to the child that when extracting square root , a comma is placed after every two numerals, starting from units. This indicates that squares can be made with units and hundreds but not tens and thousands. For example, for the square root of 1736, it is written as $\sqrt{17,36}$ to indicate that seventeen hundreds are to be used rather than one thousand seven hundred.
4. Have the child form a square with the hundreds at the upper left of the mat. In this example, four of the hundred squares will be used so there are 176 beads remaining.

remaining

5. If there are hundreds remaining, tell the child to change them into tens and place tens to the right and below the square formed from the hundreds, always placing unit beads in the lower right corner after each placement of ten bars. In this example, there are seventeen ten bars and six units. These are placed so that there are four ten bars to the right and below the square formed from the hundreds squares, a total of sixteen ten bars, with unit beads added to form a square at the lower right corner. Remind the child that the ten bars not used are changed into units. In this example, one ten bar not used is changed to ten units so there are sixteen units to place.



6. Have the child count the beads along the right side of the square to determine the square root. In this example, there are two ten bars and four units, so $\sqrt{5,76} = 24$.
7. The child may continue to do other problems or put the work away.

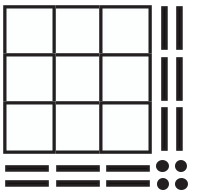
Procedure for Square Roots of Four Digit Numbers

1. Invite a child to bring recording paper, pencil, a mat, problems for square roots over one thousand and a tray of golden beads to a table, placing the tray to the left of the felt mat which is at the front of the table.
2. Remind the child that when extracting square root , a comma is placed after every two numerals, starting from units. This indicates that squares can be made with units and hundreds but not tens and thousands.
3. Have the child place the quantity of golden beads on the mat according to the problem. For example, in the problem $\sqrt{10,24}$ there are a one thousands cube, no hundreds, two tens bars and four units on the mat.

4. Remind the child that a square cannot be made with the thousands cubes, so have the child change thousands for hundreds then build the largest square possible with the hundreds squares. In this example, the one thousand cube is changed for ten hundreds squares which are built into a square, and remaining are a one hundred square, two tens and four units.

remaining

5. Tell the child to change remaining hundreds for tens and to place the tens to the right and below the square built with hundreds squares and to place units in the lower right comer to complete the formation of the large square. In this example, the child changes one hundred for ten tens bars, then places these to the right and below the hundreds square with units in the lower right comer. There are twelve ten bars and four units in the formation.



6. Have the child count the beads along the right side of the square to determine the square root. In this example, there are three tens and two units, so $\sqrt{10,24}$ = 32.
7. The child may continue with other problems or put the work away.

Control of Error

Answers in problem booklets or cards

Finding Square Roots with Stamps

Purposes
To provide experience in extracting square roots
To increase knowledge and understanding of mathematics

Preliminary exercises
Work in forming figurate numbers
Experience in building squares of numbers Practice
with finding square roots with golden beads

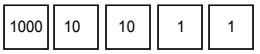
Materials
Container with green stamps labeled 1000, red stamps labeled 100, blue stamps labeled 10 and green stamps labeled 1
Problem booklets or cards for finding square roots
Recording paper and pencil

Procedure for Square Roots of Four Digit Numbers

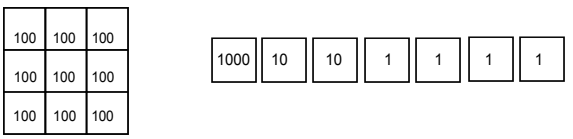
1. Invite a child to bring problems for square roots over one thousand, recording paper, pencil and the container of stamps to a table with the container behind the area at the front of the table in which stamps will be placed .

2. Remind the child that when extracting square root , a comma is placed after every two numerals, starting from units. This indicates that squares can be made with units and hundreds but not tens and thousands.

3. Have the child place the quantity of stamps according to the problem. For example, in the problem $\sqrt{10,24}$ there are a one thousands green stamp, no hundreds stamps, two blue tens stamps and four green units stamps.



4. Remind the child that a square cannot be made with the thousands , so tell the child to change thousands for hundreds and to build the largest square possible with the hundreds stamps. In this example, the one thousand stamp is changed for ten hundreds stamps which are built into a square, and remaining are a one hundred stamp, two tens stamps and four units stamps.



remaining

5. Tell the child to change remaining hundreds for tens and to place the tens to the right and below the square built with hundreds stamps and to place units stamps in the lower right comer to complete the formation of the large square. In this example, the child changes one hundred for ten tens stamps, then places these to the right and below the hundreds stamps with units in the lower right comer. There are twelve tens stamps and four units stamps in the formation.

100	100	100	10	10
100	100	100	10	10
100	100	100	10	10
10	10	10	1	1
10	10	10	1	1

6. Have the child count the stamps along the right side of the square to determine the square root. In this example, there are three tens stamps and two units stamps, so

$$\sqrt{10,24} = 32.$$

7. The child may continue with other problems or put the work away.

Control of Error

Answers in problem booklets or cards

Finding Square Roots with Pegs and Pegboard

Purposes

To provide experience in extracting square roots
To increase knowledge and understanding of mathematics

Preliminary exercises

Work in forming figurate numbers
Experience in building squares of numbers
Practice with finding square roots with golden beads and with stamps

Materials

Container of symbolically colored pegs with large peg board
Problem booklets or cards for finding square roots
Recording paper and pencil
Color-coded chart for quadranomial square

Procedure for Square Roots of Four Digit Numbers

1. Invite a child to bring problems for square roots over one thousand, recording paper, pencil, the container of pegs and a peg board to a table with the opened containers of pegs placed to the left of the peg board which is at the front of the table.

2. Remind the child that when extracting square root, a comma is placed after every two numerals, starting from units. This indicates that squares can be made with units and hundreds but not tens and thousands.
3. Have the child place the quantity of pegs into box lids according to the problem. For example, in the problem $\sqrt{10,24}$ there are a one thousands green peg, no hundreds pegs, two blue tens pegs and four green units pegs.

G bbgggg

4. Remind the child that a square cannot be made with the thousands , so tell the child to change thousands for hundreds and to build the largest square possible with the hundreds pegs. In this example, the one thousand peg is changed for ten hundreds pegs which are built into a square, and remaining are a one hundred peg, two tens pegs and four units pegs

rrr
rrr
rrr

rbbgggg
remaining

5. Tell the child to change remaining hundreds for tens and to place the tens to the right and below the square built with hundreds pegs and to place units pegs in the lower right corner to complete the formation of the large square. In this example, the child changes one hundred for ten tens pegs, then places these to the right and below the hundreds pegs with units pegs in the lower right corner. There are twelve tens pegs and four units pegs in the formation.

rrrbb
rrrbb
rrrbb
bbbgg
bbbgg

6. Have the child count the pegs along the right side of the square to determine the square root. In this example, there are three tens pegs and two units pegs, so

$\sqrt{10,24} = 32.$

7. The child may continue with other problems or put the work away.

Control of Error

Answers in problem booklets or cards

Procedure for Square Roots of Large Numbers

1. Invite a child to bring problems for square roots of large numbers, recording paper, pencil, quadranomial square chart, the container of pegs and a peg board to a table with the opened containers of pegs placed to the left of the peg board which is at the front of the table.
2. Remind the child that when extracting square root , a comma is placed after every two numerals, starting from units. This indicates that squares can be made with units, hundreds and tens of thousands but not tens, thousands or hundreds of thousands.
3. Have the child place the quantity of pegs into box lids according to the problem. For example, in the problem $\sqrt{17,72,41}$ there are a one red hundreds of thousands peg, seven blue tens of thousands pegs, seven green units of thousands pegs, two red hundreds pegs, four blue tens pegs, one green units peg.

RBBBBBBBGGGGGGGrrbbbbg

4. Remind the child that hundreds of thousands must be changed for tens of thousands since squares cannot be built with hundreds of thousands. Have the child build a square with the blue tens of thousands pegs, referring to the quadranomial square chart. In this example, there are seventeen blue tens of thousands pegs after the change has been made. These are built into a square with four pegs to a side representing 16 tens of thousands with one tens of thousands peg not used for a total value of $\sqrt{17,72,41}$ pegs not used.

BBBB
BBBB **BGGGGGGGrrbbbbg**
BBBB **remaining**
BBBB

5. Tell the child to change any remaining tens of thousands pegs for green units of thousands pegs, placing these to the right and below the square built with tens of thousands pegs and placing hundreds pegs in the right corner to complete the formation. In this example, the child changes the one tens of thousands peg for ten units of thousands pegs, then places these to the right and below the square built with blue tens of thousands pegs. There are sixteen green units of thousands pegs and four red hundreds pegs used with a total value of 841 pegs not used.

BBBBGG
BBBBGG
BBBBGG
BBBBGG **rrrrrrrrbbbbg**
GGGGrr **remaining**
GGGGrr

6. Have the child place the remaining hundreds pegs to the right and below the units of thousands pegs, the remaining tens pegs to the right and below the hundreds pegs and the units pegs in the lower right corner to complete the building of the square. In this example, there are eight hundreds pegs placed to the right and below the units of thousands pegs, four tens pegs placed to the right and below the square of hundreds pegs and the unit peg placed in the corner.

BBBBGGr
BBBBGGr
BBBBGGr
BBBBGGr
GGGGrrb
GGGGrrb
rrrrbbg

7. Have the child count the pegs along the right side of the square to determine the square root. In this example, there are four hundreds, two tens and one unit.

Control of Error

Answers in problem booklets or cards

Exponents

Purposes

To develop an understanding of the concept of exponents

To provide practice with exponential notation

Preliminary Exercises

Practice with multiplication
Experience using bead chains for squaring and cubing
Work with binomial and trinomial cubes

Materials

Container of definition cards
Container of rule cards
Problem cards: Set 1 Factor repetition for exponential notation; Set 2 Exponential notation for repeated factors; Set 3 Standard numerals or product in exponential notation; Set 4 Scientific notation; Set 5 Recording of scientific notation;
Set 6 Changing scientific notation to standard form
Recording paper and pencil

Procedure

1. Invite a child to bring containers of rules, definitions, problem cards for exponents, paper and pencil to a table, placing the cards near the center and paper and pencil on the dominant side.
2. Ask the child to find and read the card with the definition of exponents, then to practice writing the exponential notation for repeated factors, using problem cards from Set 1.
   For example, $7 \times 7 \times 7 \times 7 = 7^4$
3. Tell the child to find the Set 2 problem cards and write repeated factors for exponential notations. For example, $7^4 = 7 \times 7 \times 7 \times 7$.
4. Have the child find the Set 3 problem cards and calculate the product (or standard numeral) for exponential notations. For example, $7^4 = 2401$
5. Invite the child to continue with the problems or to put the materials away.

Variations

Invite the child to solve problems in multiplying two expressions with the same base by referring to the Law of Exponents card.
Invite the child to solve problems in dividing two expressions with the same base by referring to the Law of Exponents card.

Procedure for Scientific Notation

1. Invite a child to bring containers of rules, definitions, problem cards for scientific notation, paper and pencil to a table, placing the cards near the center and paper and pencil on the dominant side.
2. Have the child find and read the definition card for scientific notation, then sort the cards from Set 4.
3. Once it is observed that the sorting exercise is understood, ask the child to convert the problems from Set 5 into scientific notation.
4. Tell the child to solve the problems from Set 6 to change each scientific notation into a product or standard form.
5. Invite the child to continue with the problems or to put the materials away.

Logarithms

Purposes
- To further the understanding of exponents
- To learn the use of logarithms to reduce multiplication to addition, division to subtraction, raising a number to a power by simple multiplication, extracting a root by simple division
- To become acquainted with logarithmic scales

Preliminary Exercises
- Practice with equations
- Work with number bases
- Experience in using exponents
- Practice with scientific notation

Materials
- History of Logarithms booklet
- Logarithm information booklet
- Chart showing exponential and logarithmic forms of the powers of ten
- Decibel scale for sound intensity
- Richter scale for earthquakes
- Frequencies of tone, classical musical scale (C = 512)
- Frequencies of tone, international musical scale (A = 440)
- Logarithm tables
- Procedure for finding logarithms of numbers
- Procedure for finding antilogarithms of numbers
- Log rules for multiplication, division, raising to a power, extracting a root
- Problems for multiplication using logarithms
- Problems for division using logarithms
- Problems for raising to a power using logarithms
- Problems for extracting a root using logarithms
- Paper and pencil

Introductory Procedure

1. Invite the child to read the history booklet and the information booklet, then to view the chart of powers of ten and examine the log tables.
2. Review the terms and their meanings to ensure that the child understands terminology.
3. Show the child how to determine the characteristic of a number by finding the whole number part of the log. Demonstrate the procedure for locating the mantissa.
4. Invite the child to continue using the materials or to return them to the shelf.

Procedure for Finding the Logarithm of a Number

1. Invite the child to bring the directions for finding logs, log tables, problems, paper and pencil to a table.
2. Tell the child to choose a problem and write the number in scientific notation.

   For example, **232** is written **232 = 2.32** x $\mathbf{10}^2$.

3. Have the child determine and write the characteristic. In this example, it is 2.
4. Ask the child to locate the mantissa by finding the first two significant figures of the number in the Table of Logarithms in column N, then finding the third significant figure in the column under the third number. In this example, **23** is the number located in the N column and in the **2** column on the row numbered **23,** the mantissa is located which is **3655.** A decimal point is placed before the mantissa. In this example, it is **0.3655.**
5. Have the child add the mantissa to the characteristic to produce the logarithm of the number. In this example, it is **2.3655.** Tell the child to record the answer. In this example, log10 **232 = 2.3655.**
6. Invite the child to continue with other problems or to put away the materials.

Procedure for Finding the Antilogarithm of a Number

1. Invite the child to bring the directions for finding antilogs, log tables, problems, paper and pencil to a table.
2. Tell the child to choose a problem , write the mantissa and locate it in the log tables. Remind the child that the characteristic is not part of the mantissa. For example, in the problem **3.3874.,** the mantissa is **.3874.**
3. Tell the child to locate the significant figures that correspond to the mantissa first in the N column, then in the row of whole numbers across the top. In this example, **24** is in the N column and 4 is in the horizontal row of numerals at the top, making the significant figure 244.
4. Have the child write the significant figure as a decimal number between **1** and **10.** In this example, it is written **2.44.**
5. Multiply the significant figure by 10 raised to the power indicated by the characteristic to obtain the antilog. In this example, **2.44** is multiplied by **10** to the third power since the characteristic is **3.** **2.44** x $\mathbf{10}^{\mathbf{3}}$ **= 2,440.** Therefore, the antilog of **3.3874 = 2,440.**
5. Invite the child to continue with other problems or to put away the materials.

Procedure for Multiplication

1. Invite the child to bring the log tables, multiplication problems, paper and pencil to a table.
2. Have the child choose a problem, for example, **389** x **9,** then find the log of the multiplier and of the multiplicand. The logs are added and the antilog of the sum is located in the tables.

   In this example, log10 **389 = 2.5899**
   log10 **9 = +0.9542**
   3.5441 antilog **= 3500**

   Note: When four place log tables are used, there is error due to rounding. Here the answer obtained by multiplication with logs is one less than the actual product , **3501.**
3. Invite the child to continue with other multiplication problems or put the work away.

Procedure for Division

1. Invite the child to bring the log tables, division problems, paper and pencil to a table.
2. Have the child choose a problem, for example, 3476 + 43, and find the log of the dividend and of the divisor. The log of the divisor is subtracted from the log of the dividend, and the antilog of the difference is located in the tables.

   In this example, log10 3470 = 3.5403
   log10 43 = - 1.6335
   1.9068 antilog 80.7

   Note: There is no value for 9068 in the four place log table. The closest value is 9069. The antilog of 1.9068 is 80.7. In conventional division, the answer is 80.69721, which when rounded to the nearest tenth agrees with the answer obtained using logs.

Procedure for Raising to a Power (Involution)

1. Invite the child to bring the log tables, problems for raising to a power, paper and pencil to a table.
2. Have the child choose a problem and find the log of the number.

   For example, in the problem 14^3 , the log10 14 is 1.1461.

3. Tell the child to multiply the log by the power according to the exponent.

   In this example, the power is 3, so 1.1461 x 3 = 3.4384

4. Ask the child to find the antilog of the product resulting from the multiplication of the log of the number by its power.

   In this example, the antilog of 3.4384 is 2744.

Procedure for Extracting a Root (Evolution)

1. Invite the child to bring the log tables, problems for extracting a root, paper and pencil to a table.
2. Have the child choose a problem and find the log of the number.

   For example, in the problem $\sqrt[2]{25}$ $\log_{10} 25 = 1.3979$

3. Tell the child to divide the log by the index number in the radical.

   In this example, the index is 2, so 1.3979 + 2 = .6990

4. Ask the child to find the antilog of the quotient resulting from the division of the log of the number by its index.

   In this example, the antilog of .6990 is 5, so $\sqrt[2]{25} = 5$.

Number Bases

Purposes

To develop understanding of mathematical operations in bases other than ten
To broaden knowledge of various number systems used to program computers
To establish a foundation for developing programming skills
To provide a basis for comprehension of computer functioning
To increase knowledge and understanding of mathematics

Preliminary Exercises

Work with bead chains.squares and cubes
Practice in finding squares of numbers

Materials

Felt mat
Booklets with problems for base conversions
Container with single beads in the colors of the bead bars
Note: Single beads are obtained by removing beads from the bars.
Bead bars, bead squares and bead cubes for one to ten
Tray for bead material
Chart divided into columns with space for heading strips at the top
Container of color-coded heading strips with values of number bases for two through ten
Note: Heading strips are color coordinated to match bead material, for example, gold for the ten heading strip.
Container of counters or small discs in any color other than green, blue or red if possible

Introductory Procedure

1. Invite a child to the lesson and place the tray of beads on a table to the left of the chart with the container of heading strips above it. Tell the child that counting in bases other than ten will be demonstrated, but base ten counting will be done first to show the use of the chart with the system already known.
2. Have the child place the gold ten heading strip on the top of the chart and begin counting gold unit beads through nine by placing vertically in the units column.
3. Remind the child that one more unit makes ten, so the units are removed and a tens bar is placed in the "group of 10^1" column. Have the child count tens bars through nine onto the chart, placing them side by side from right to left.
4. Remind the child that one more ten makes one hundred, so tens are removed and a one hundreds square is placed in the "group of 10^2" column. Have the child count and stack hundreds through nine onto the chart.
5. Remind the child that one more hundred makes one thousand, so hundreds are removed and one thousand cube is placed in the "group of 10^3" column. Have the child remove the cube and the heading strip.
6. Ask the child to choose another heading strip, for example, the light blue five strip, and place it across the top of the chart.
7. Tell the child to begin counting the single beads, in this example, light blue, by placing vertically in the units column. Beads are counted through one less than the number chosen. In this example of five, only four single light blue beads are counted.
8. Remind the child that one more single bead makes the quantity of its corresponding bead bar, so the single beads are removed and the corresponding bar is placed in the "group of base number" column. In this example, the four single light blue beads are removed and a five bar is placed in the "group of 5^1" column.
9. Have the child count bars onto the chart through one less the the number chosen, placing them side by side. In this example, four ten bars are counted.

10. Remind the child that one more bead bar makes the quantity of its corresponding bead square, so the bead bars are removed and its square is placed in the "group of base number ^{2}" column. In this example, four five bars are removed and a five square is placed in the "group of 5^2" column.
11. Have the child count and stack squares onto the chart through one less than the number chosen. In this example, four five squares are placed on the chart.
12. Remind the child that one more square makes the bead cube, so the squares are removed and the cube is placed in the "group of base number^{3}" column. In this example, the five cube is placed in the "group of 5^3" column.
13. The child may continue counting in other bases or put the work away.

Procedure for Base Conversions

1. Invite a child to the lesson and place the container of counters or discs on a table to the left of the chart with the container of heading strips above it. Have the child choose a problem, for example, 2134_5 (2134 base five), and place the base strip of appropriate color across the top of the chart. In this example, the blue strip is used for base five.

2. Tell the child to begin at the right and place counters in appropriate columns as indicated by the problem. In this example, four counters are placed in units column, three in "group of 5^1" column, one in "group of 5^2" column and two in "group of 5^3" column.

Bases					
group of 5^5 3125	group of 5^4 625	group of 5^3 125	group of 5^2 25	group of 5^1 5	units 1
		○ ○	○	○ ○ ○	○ ○ ○ ○

3. Explain that conversion to base 10 is accomplished by tabulating the decimal value of each column and adding. In this example, $250 + 25 + 15 + 4 = 294_{10}$
4. When using base 2, point out that this represents the binary system used in computers with 1 for on and 0 for off.
5. The child may continue to do problems or put the work away.

Procedure for Addition in Bases Other Than Ten

1. Invite a child to the lesson and place the container of counters or discs on a table to the left of the chart with the container of heading strips above it. Have the child choose an addition problem, for example, $2123_5 + 2313_5 =$ and place the base strip of appropriate color across the top of the chart. In this example, the light blue strip is used for base five.

2. Tell the child to place discs on the chart to represent the addends. In this example:

Bases

group of 5^5 3125	group of 5^4 625	group of 5^3 125	group of 5^2 25	group of 5^1 5	units 1
		○ ○	○	○ ○	○ ○ ○
		○ ○	○ ○ ○	○	○ ○ ○

3. Remind the child that when adding, the change is made when the base number is reached. In this example, when units are added, five are changed for the next group, leaving one in units place.

Bases

group of 5^5 3125	group of 5^4 625	group ofs^3 125	group of 5^2 25	group of 5^1 5	units 1
		○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ [○]	○

moved from units place

4. Have the child record the answer. In this example,
$2123_5 + 2313_5 = 4441_5$

5. The child may continue to do problems or put the work away.

Procedure for Subtraction in Bases Other Than Ten

1. Invite a child to the lesson and place the container of counters or discs on a table to the left of the chart with the container of heading strips above it. Have the child choose a subtraction problem, for example, $2443_5 - 1244_5 =$, and place the base strip of appropriate color across the top of the chart. In this example, the blue strip is used for base five.

2. Tell the child to place discs on the chart to represent the minuend. In this example:

Bases					
group of 5^5 3125	group of 5^4 625	group of 5^3 125	group of 5^2 25	group of 5^1 5	units 1
		○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○

3. Ask the child to subtract the subtrahend, changing as necessary. In this example, one disc from "group of 5^1 " is exchanged for five units, and one disc from "group of 5^2" is exchanged for five discs placed in "group of 5^1

". Base 5

group of 5^5 3125	group of 5^4 625	group of 5^3 125	group of 5^2 25	group of 5^1 5	units 1
		○ ○	○ ○ ○ ○	○ ○ ○ [○] ➔	○ ○ ○ [○] [○] [○] [○] [○]

(after changing one from "group of 5^1 ")

Base 5

group of 5^5 3125	group of 5^4 625	group of 5^3 125	group of 5^2 25	group of 5^1 5	units 1
		○ ○	○ ○ ○ [○] ➔	○ ○ ○ [○] [○] [○] [○] [○]	○ ○ ○ ○ ○ ○ ○ ○

(after changing one from "group of5^2")

Base5

group of5^5 3125	group of5^4 625	group of5^3 125	group of5^2 25	group of5^1 5	units 1
		○	○	○ ○ ○ ○	○ ○ ○ ○

(answer)

4. Have the child record the answer. In this example,

   2443_5 - 1244_5 = 1144_5

5. The child may continue to do problems or put the work away.

Negative Numbers

Number Line

Purposes
- To develop understanding of numbers less than zero
- To become familiar with rules of signs
- To develop skills for doing operations involving negative numbers

Preliminary Exercises
- Practice with arithmetic problems in all four operations

Materials
- Number line with positive and negative numbers
- Definition booklet
- Booklets or cards with problems for operations with negative numbers
- Booklet or cards with Rules of Signs
- Negative bead stairs

Introductory Procedure

1. Invite a child to the lesson and place the number line near the front of the table. Explain that it represents a continuous sequence of numbers with positive numbers which are greater than zero stretching to infinity to the right of zero and negative numbers which are less than zero stretching to infinity to the left of zero, while zero is neither positive or negative. Ask the child to locate several numbers, for example, "Find - 7," "Find 5." Tell the child that unsigned numbers, that is, those with no sign indicated, are assumed to be positive.
2. Have the child locate several pairs of numbers, one positive and the identical negative number, for example, "Find 5 and -5." Tell the child that the numbers in such pairs are opposites or additive inverses.
3. Ask the child to determine the distances from zero for pairs of opposite numbers such as -6 and 6 so that there is awareness that they are equidistant from zero. Tell the child that both numbers in pairs of opposites have the same absolute value.
4. Invite the child to read the definition booklet and examine the number line.

Negative Numbers Operations

Purposes

To develop understanding of operations involving negative numbers
To prepare for more abstract work with negative numbers

Preliminary Exercises

Experience with opposites or inverse numbers
Work with the number line

Materials

Container of short bead stairs
Container of gray negative bead bars
Green division skittles
Small felt mat
Problem booklets or cards
Rule cards or booklets for work with negative numbers
Pencil
Recording paper

Preliminary Procedure for Finding Inverses or Opposites

1. Invite the child to bring a felt mat, and both containers of bead bars, then to lay out one set of the negative bead bars and one set of short bead stairs near the top of the felt mat. Tell the child that the short bead stair bars represent positive numbers.
2. Have the child select a bead bar from either set, place it on the mat, then find its inverse or opposite. Remind the child that when a negative number and a positive number are added, the result is zero. Continue until it is observed that the child understands the concept, then proceed to the next exercise.

Preliminary Procedure for Simplifying

1. Have the child lay out several bars of the short bead stair and several negative bead bars of the same quantity, that is, the inverses of the short bead bars which represent positive numbers. Remind the child that when added together, opposites equal zero. Have the child match the opposites and remove them. Explain that this process of removing zeros is called simplifying.
2. Have the child continue with additional selections of beads or return the materials to the shelf.

Addition of Negative Numbers

1. Invite the child to bring a felt mat, container of negative bead bars, paper and pencil to a table then lay out a set of negative beads near the top of the felt mat.
2. Have the child choose any two negative bead bars and place them horizontally on the felt mat, for example, - 2 and - 6.
3. Tell the child to add the quantity of beads on negative bead bars by counting, then to record the problem and answer. In this example, - 2 + - 6 = - 8
4. Invite the child to continue with addition of negative bead bars or to put the materials away.

Note: The child has had experience in the addition of positive numbers through extensive work with the short bead stair.

Addition of a Positive and a Negative Number

1. Invite the child to bring a felt mat, problem booklet or cards, pencil, paper, container of short bead stairs and container of negative bead bars, then lay out two sets of short bead stairs and two sets of negative bead bars near the top of the mat.
2. Have the child choose and record a problem, then place the appropriate bead bars horizontally one below the other. For example, for the problem +9 + -7 =, the child places a nine bar from the short bead stairs above a seven bar from the negative beads.
3. Remind the child that a number and its inverse equal zero. Ask the child to determine if there is a zero in this problem. In this example, the short bead stair nine bar can be changed into short bead stair seven and two bars so that the child can remove the zero in the form of +?(short bead seven bar) and -7 (gray negative seven bar).
4. Have the child record the answer, in this example, +9 + -7 =+2.
5. Invite the child to continue with additional problems or return the materials to the shelf.

Subtraction of a Negative Number from a Negative Number with a Negative Difference

1. Invite the child to bring the container of negative bead bars, a felt mat, problems, paper and pencil to a table, then lay out one set of gray negative bead bars near the back of a felt mat.
2. Have the child choose and record a problem, then place the bead bar representing the minuend on the mat. For example, in the problem - 8 - -4 =, an eight gray bar is placed on the mat.
3. Tell the child to count and cover with the fingers the number of beads for the subtrahend, using the right hand, and beginning at the right of the bead bar. In this example, four beads will be covered.
4. Ask the child to count the beads not covered by the fingers of the right hand, starting at the left and using the fingers of the left hand. In this example there will be a difference of -4.
5. Have the child record the answer. In this example, - 8 - - 4 = - 4
6. Invite the child to continue with additional problems or return the materials to the shelf.

Subtraction of a Negative Number from a Negative Number with a Positive Difference

1. Invite the child to bring the container of negative bead bars, container of short bed stairs, felt mat, problems, pencil and paper to a table, then lay out one set of gray negative bead bars and one set of short bead stairs on the felt mat near the back.
2. Have the child choose and record a problem.
3. Tell the child to select the bead bar for the minuend and place it near the front of the mat. For example, in the problem -3 - -8 =, a - 3 bar is the minuend.
4. Have the child count the amount of the subtrahend and cover with the fingers of the right hand. The child sees that there are not enough beads on the bead bar. Ask the child to determine how many additional negative beads are needed. In this example, five addi-tional negative beads are needed.
5. Tell the child to select a negative bead bar to give the quantity required to complete subtraction and a short bead stair bead bar of the same quantity (adding zero). Remind the child that opposites equal zero which is also known as *adding zero.* In this example, both bars have five beads. Have the child place the negative bead bar at the left of the negative bead bar representing the minuend so that the gray negative bead bars are in a line and simultaneously place the short bead stair bar behind its inverse.
6. Have the child count and cover the number of gray negative beads for the subtrahend, in this example eight.
7. Tell the child to count the difference indicated by the short bead stair bar. In this example there is a difference of five.
8. Have the child record the answer. In this example, - 3 - - 8 = + 5
9. Have the child continue with additional problems or return the materials to the shelf.

Subtraction of a Positive Number from a Negative Number

1. Invite the child to bring the container of negative bead bars, container of short bed stairs, felt mat, problems, pencil and paper to a table and lay out one set of gray negative bead bars and one set of positive bead bars near the back of the felt mat.
2. Have the child choose and record a problem, for example, -2 - +7 =.
3. Tell the child to place the bead bar for the minuend at the front of the mat, in this example, the two gray negative bar.
4. There are no positive beads to count for the subtrahend. Have the child determine the quantity of the short bead bar needed and simultaneously place the appropriate gray negative bar and short bead bar beside the minuend (adding zero) so the gray bead bars are in a line and the short bead bar is behind its inverse. Remind the child that opposites equal zero which is also known as *adding zero.* In this example a seven short bead bar and a seven gray negative bar are needed.
5. Using the right hand, have the child count and cover the number of beads for the subtrahend, in this example, the seven bead bar.
6. Tell the child to count the gray negative beads which are not covered. In this example there are nine gray negative beads.
7. Have the child record the answer. In the example, -2 - +7 = - 9.
8. Have the child continue with additional problems or return the materials to the shelf.

Subtraction of a Negative Number from a Positive Number

1. Invite the child to bring the container of negative bead bars, container of short bead stairs, felt mat, problems, pencil and paper to a table and lay out one set of gray negative bead bars and one set of short bead stair bars on the felt mat near the back.
2. Have the child choose and record a problem, for example 3 - -5 =.
3. Tell the child to select the short bead stair bar for the minuend and place it near the front of the mat, in this example, three.
4. There are no negative beads to count for the subtrahend. Have the child determine the quantity of negative beads needed and simultaneously place the appropriate negative and short bead bars beside the minuend (adding zero) so the short bead bars are in a line and the negative bead bar is behind its inverse. In this example, a five gray negative bead bar and a five short bead stair bar are used.
5. Have the child count and cover the quantity of gray negative beads for the subtrahend, using the right hand, in this example, five gray negative beads.
6. Tell the child to count the short bead bars which are not covered. In this example, there are eight beads.
7. Ask the child record the answer. In this example, 3 - - 5 = 8
8. Have the child continue with additional problems or return the materials to the shelf.

Multiplication of Negative Numbers by Positive Numbers

1. Invite the child to bring the container of negative bead bars, a felt mat, problems, paper and pencil to a table, placing the container of negative bead bars toward the back of the felt mat.
2. Have the child choose and record a problem, for example, -2 x 6 =.
3. Tell the child to place the gray negative bars multiplicand on the mat the number of times indicated by the multiplier. In this example, the gray negative two bars are laid out six times.
4. Have the child count the number of beads on the gray negative bars which have been placed on the mat. In this example, there are twelve gray negative beads..
5. Have the child record the answer. - 2 x 6 = - 12
6. Have the child continue with additional problems or return the materials to the shelf.

Division of Negative Numbers by Positive Numbers

1. Invite the child to bring the container of negative bead bars, green division skittles, felt mat, problems, paper and pencil to the table, placing the beads and skittles at the back of the mat.
2. Ask the child to choose and record a problem, for example, -8 ÷ 4 =.
3. Tell the child to place skittles to represent the divisor along the left side of the mat. In this example there will be four skittles.
4. Have the child count out the gray negative unit beads to represent the dividend and place them at the bottom of the mat. In this example, there are eight gray negative unit beads.
5. Tell the child to divide the beads evenly among the skittles. The answer is what one skittle receives. In this example, each skittle receives two gray negative beads.
6. Have the child record the answer. In this example, - 8 ÷ 4 = - 2.
7. Invite the child to continue with additional problems or return the materials to the shelf.

Division of Negative Numbers by Negative Numbers

1. Invite the child to bring the negative bead bars, felt mat, problems, paper and pencil to a table, placing the beads at the back of the mat.
2. Have the child choose and record a problem, for example, -10 ÷ -2 =.
3. Tell the child to count gray negative unit beads to represent the dividend and place them at the bottom of the mat, in this example, ten gray negative unit beads. State that division by quotition will be used.
4. Ask the child to form groups of beads as represented by the divisor. In this example, five groups of two each gray negative beads are formed.
5. Have the child record the answer. In this example, -10 ÷ - 2 = 5

Variation

After the child has experience doing operations with negative numbers, introduce the rule cards for negative numbers.

Negative Snake Game

Purpose

To reinforce skills for addition and subtraction

Preliminary Exercises

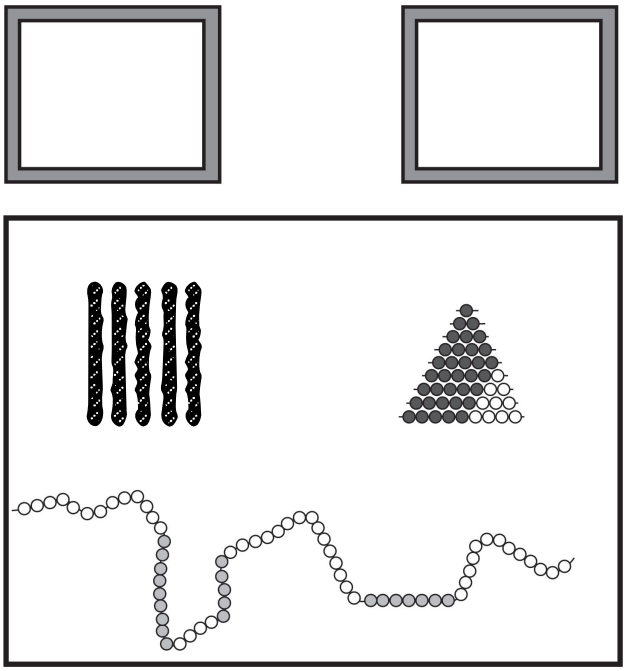
Practice with snake game
Work with addition and subtraction materials

Materials

Container with two sets of short bead stairs and place holder, if desired
Container with ten golden bead ten bars
Container with one set of black and white bead bars
Container with one set of gray bead bars
Felt mat

Procedure

1. Invite a child to bring to a table a felt mat and containers of short bead stairs, golden bead ten bars, black and white bead bars and gray bead bars.
2. Ask the child to place the felt mat at the center front of the table and to make an equilateral triangle with black and white bead bars at the upper right of the mat, starting with the one at the top and having the white beads at the right.
3. Tell the child to form a snake with the short bead stairs with gray bars placed at random within the snake arrangement., then to close the bead boxes.



4. Beginning at the left end of the snake, have the child count beads toward the right in sequence using the first finger of the dominant hand, changing for ten bars, placing black and white bars as needed to complete the ten and moving the counted short bead stair to the top center of the mat as in regular snake game.
5. When a gray bar is reached, tell the child to count the gray beads on the bar and to subtract that number of beads from the preceding bead bars by counting from right to left, starting at the left end of the gray bar, using black and white bars as needed to complete the amount. The gray bead bar is removed from the snake and placed at the top left of the mat. Ask the child to continue counting and subtracting.
   Note: If the placement of a gray bead bar requires additional beads to be subtracted from short bead stair bars but there are none, have the child place the appropriate gray bead bar which is then subtracted from the next short bead stair bars to the right.
6. Once the snake has been converted to ten bead bars and black and white bars, ask the child to find combinations of the short bead stairs on the mat which make ten and place horizontally above the ten and black and white bar combination.
7. Have the child place the gray bars horizontally at the right of the short bead bar combinations, count the amount and subtract from the total amount of combined short bead stairs. This sho1,.1ld be the same amount as the golden bead-black and white bead bar combination.
8. Invite the child to continue with the negative snake game or to replace the materials.

Control of Error

Amount of short bead stairs minus amount of gray bead bars is the same as the quantity of golden beads with black and white bead bars.

Volume

Purposes

To develop an understanding of the concept of volume as an amount of three-dimensional space and a measure of capacity

To learn procedures for calculating amount of space occupied by various solids

Preliminary Exercises

Practice in measuring liquid and dry volume involving cups, pints, quarts, metric units

Work with volume box containing two hundred fifty two centimeter cubes Experience with volume box containing one thousand centimeter cubes

Exercises in cubing and in multiplication

Materials

Box of two hundred fifty cubes measuring two cubic centimeters

Box of one thousand cubes measuring one cubic centimeter

Tray with metal volume containers: large rectangular prism, small rectangular prism, pyramid

Calibrated measuring containers, in both metric and standard systems

Container of dry material such as grits or parakeet gravel

Container of formula cards for calculating volume of geometric solids

Container of command problem cards for calculating volumes of geometric solids

Paper and pencil

Preliminary Procedure

1. Invite a child to bring the volume box of two centimeter cubes , paper, pencil and formula card for finding volume of a rectangular solid to a table, placing the formula card, paper and pencil on the dominant side with the box of cubes near the center front.
   Note: The length of the box parallels the front edge of the table.
2. Have the child open the box, remove and count the cubes from the top front row, placing them in a row at the front of the box. Say that this represents the length, then ask the child to record that number as l= the number. Have the child return the cubes to their proper places.
3. Tell the child to remove the cubes from the top right edge, placing them in a row as above. Say that this represents the width, then ask the child to count and record that number as w = the number. Have the child return the cubes to their proper places.
4. Ask the child to remove the cubes from the front right corner, placing them in a column. Say that this represents the height, then ask the child to count and record that number as h= the number.
5. Have the child look at the formula for finding volume of a rectangular prism, then substitute the numbers previously recorded and multiply to obtain the volume of the box.
6. Invite the child to repeat the procedure using the box with one thousand cubes or to put away the materials.

Control of Error

Number of cubes in each box

Variations

Invite the child to use the metal volume containers for filling with dry material, then to measure the volume of dry material used to fill each

Calculation of Volume by Water Displacement

Purposes

To learn how to calculate the volume of irregular solids without using a mathematical formula
To gain further understanding of volume

Preliminary Exercises

Practice with liquid measurement

Maferials

Calibrated containers such as a measuring cup and a graduated beaker or cylinder
Container of irregular solid objects which will not float such as rocks, each marked with a number
Calibrated container such as a measuring cup
(Note: If possible, obtain a container with both metric and U.S. Standard calibrations.)
Water
Tray
Card marked U.S. Customary System stating that one ounce of water occupies 1.805 cubic inches
Card marked Metric System stating that one gram of water occupies one cubic centimeter
Control card with the number of each object recorded with its appropriate volume.
Paper and pencil

Procedure

1. Invite a child to place two calibrated cups and the container of rocks on a tray, to place it at the middle of a table, then to fill one cup with water to the eight ounce line line, recording that amount of water , for example, 8 fluid ounces.
2. Ask the child to place a rock in the other measuring cup, then fill it with water from the other cup up to the one cup calibration.
3. Have the child record the amount of water remaining in the cup that does not contain the rock and subtract it from the original amount. For example, if the remaining amount is six ounces, 8 - 6 = 2, so two ounces of water have been displaced by the rock.
4. Ask the child to bring the U.S. Customary System card and record the cubic inches per ounce which is 1.805, then to multiply that by the number of ounces of water displaced by the rock, in this example, two to determine the volume of the rock. In this example, 1.805 cu. in. X 2 oz. = 3.610 cubic inches is the volume of the rock whose number is also recorded.
5. Invite the child to continue the procedure with other rocks or to put the materials away.

Control of Error

Control card
Variations
Invite the child to repeat the procedure using the metric system materials.

Perimeter

Purposes
- To develop understanding of the concept of perimeter
- To be able to calculate the perimeter of circles and polygons

Preliminary Exercises
- Practice with geometry materials
- Experiences with addition, subtraction, multiplication and division Work with measuring equipment such as ruler, compass

Materials
- Container of jar lids of several sizes with the location of the diameter marked and pieces of string that exactly fit the circumference of each lid rolled and placed in the appropriate lid Definition cards with formulas and examples
- Problem cards with answers on the reverse
- Paper and pencil

Procedure for Finding Perimeters of Polygons

1. Invite a child to bring definition cards and problem cards for polygons to a table, then to read the definitions.
2. Have the child choose a polygon problem and calculate the perimeter of the figure. For example, a triangle with sides of three, four and five has a perimeter of twelve. P=3+4+5=12
3. Invite the child to continue finding perimeters of polygons or to put away the materials.

Procedure for Relating Circumference to Diameter

1. Invite a child to bring container of jar lids with strings to a table.
2. Tell the child to remove the jar lids with strings from the container and to choose any lid with string.
3. Have the child place the string around the circumference of the lid to see that it matches, then remove it and place one end of the string across the marked diameter of the same lid. Ask the child to determine the number of times the string can be measured across the diameter.
4. Invite the child to continue the exercise with other jar lids or to return the materials to the shelf, being sure to return the strings to their appropriate lids.

Procedure for Finding the Circumference of a Circle

1. Invite a child to bring definition cards, problem cards, paper and pencil to a table, then to read the definitions, paying close attention to the formulas.
2. Have the child choose a circle problem and calculate the circumference (perimeter). For example, a circle with a diameter of ten has a circumference of thirty-one and four tenths. C=3.14X10=31.4
   For an example involving the radius, a circle with a radius of fifteen has a circumference of ninety-four and two tenths. C=3.14X15X2= 94.2

Geometric Construction

Purposes
To develop skill in use of drawing instruments To enhance understanding of geometry
To use for practical applications

Preliminary Exercises
Work with geometry classification materials

Materials
Ruler Compass
Paper and pencil Problem booklet

Procedure

1. Invite a child to bring the materials to a table.
2. Have the child read the booklet in sequence and follow directions for construction.

Polyominoes

Purposes:

To develop an understanding of the relationship of perimeter to area
To develop an understanding of optimizing
To develop an appreciation of the function of graphing
To develop appropriate vocabulary

Preliminary exercises:

Exercises in finding area
Exercises in finding perimeter
Exercises in graphing

Materials:

Container of square tiles (2 cm^2 or 1 in^2)
Graph paper
Recording paper
Pencil
Scissors
Container of command cards for pentominoes
Container of pentominoes
Latin and Greek root materials (from Language area)
Dictionary

Introductory Procedure

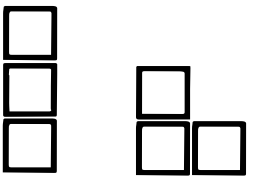
1. Invite the child to bring the container of square tiles, graph paper, recording paper, pencil and scissors to a table.

2. Have the child place the container of tiles at the front of the table with the graph paper, recording paper, pencil and scissors on the dominant side.

3. Explain that a polyomino is a shape that is made by joining squares along their edges. A domino is a good example of a polyomino.

4. Tell the child to make as many shapes as possible using three tiles, then draw these trominoes on the graph paper.

5. Ask the child to cut out the drawn trominoes and determine if any can be flipped to the reverse side or rotated to fit others. Explain that reversing and rotating do not create new polyominoes. Have the child record the number of tiles used (three) and the number of possible tromino shapes. (There will be only two).



6. Invite the child to continue the activity by constructing polyominoes of four, five and six squares,drawing these constructions on graph paper, recording the number of squares used for each and the number of possible shapes or put the materials away.

Procedure for Area and Perimeter

1. Invite the child to bring the container of tiles, graph paper, recording paper, and pencil to the table, placing the container of tiles near the front with graph paper, recording paper and pencil on the dominant side.

2. On the recording paper have the child draw a chart with two columns each one inch wide and as long as the paper. One column is labeled "Area" and the other is labeled "Perimeter."

3. Ask the child to build all the possible polyominoes beginning with one square through five squares and draw each on the graph paper.

4. Remind the child of the procedure for finding area and perimeter. Have the child record the area and perimeter of each polyomino on the chart. For each area have the child find the shortest and the longest perimeter

5. Ask the child to draw a graph with area as the horizontal axis and perimeter as the vertical axis. Tell the child to record the shortest and longest perimeter for each area, using two different colors.

Variations:

Invite the child to use the pentomino commands.
Invite the child to determine the optimal polyomino shape which has the shortest perimeter.
Invite the child to determine the polyomino shape which has the longest perimeter.
Invite the child to research the prefixes used in polyominoes and make a list of other words which use the same prefixes, referring to the Greek and Latin root materials in the Language area and to a dictionary.

Vocabulary:

tromino tetramino pentomino hexamino heptamino octomino nonimo decamino polyomino

Similarity

Purposes

To develop understanding of the concept of similarity
To increase recognition of similar figures

Preliminary Exercises

Work with geometric cabinet Exercises
with constructive triangles Experience
with geometric construction

Materials

Container of drawings of similar and dissimilar figures, heading cards for each of the two categories
Container of definition cards
Drawing materials
Directions for indirect measurement

Procedure

1. Invite a child to bring the drawings, labels and definition cards to a table, and read the definitions.
2. Have the child sort the geometric drawings under the labels **similar or not similar,** then mix the cards before returning to container.

Control of Error

Definition cards with drawings

Variations

Invite the child to construct similar figures.
Invite the child to conduct indirect measurements of tall structures by the principal of similarity by following the directions in the booklet. or on cards.

Congruence in Geometry

Purposes

To develop understanding of the concept of congruence
To be able to recognize congruent figures
To acquire the technique for constructing congruent figures
To exercise the imagination in order to visualize in three dimensions

Preliminary Exercises

Practice with geometric cabinet, fraction insets, insets for design and geometric solids
Experience in constructing geometric figures

Materials

Definition cards with drawings
Cards with figures which are congruent and not congruent, heading cards for each category
Cards with directions for construction of congruent figures
Pencil, straight edge, compass, paper

Procedure

1. Invite the child to bring the definition cards and drawings to a table for matching, then to classify the figures under congruent or not congruent headings.
2. Have the child construct congruent figures as directed on the cards.
3. Invite the child to continue the work or to put the materials away.

Control of Error

Answer on reverse of congruent/ not congruent cards

Congruence in Modular Arithmetic

Purposes

To develop understanding of the concept of congruence
To provide the knowledge necessary for modular arithmetic operations
To introduce the exercise of casting out nines
Paper, pencil

Preliminary Exercises

Practice with addition and subtraction
Familiarity with telling time on a conventional clock

Materials

Clock faces with 1 to 12, 1 to 5
Problem cards for modulo 12 and modulo 5
Cards with problems for casting out nines

Procedure with Clock Faces

1. Invite a child to bring paper, pencil, a clock face with 1 to 12 and modulo 12 problems.
2. Tell the child that calculations are to be made in modulo 12 which means that problems are limited to a finite set of twelve numbers.
3. Have the child copy a problem and calculate it by using the clock face.

   For example, with the problem 9 + 6 =, have the child locate 9 on the clock face, then count six , moving clockwise. The answer is $9 + 6 = 3 \pmod{12}$

4. Introduce the clock face with 1 to 5 and modulo 5 problems which are limited to a finite set of five numbers. Have the child begin at the first numeral and count clockwise according o the second.

   For example, with the problem 4 + 2 =, have the child locate 4 on the clock face, then count 2, moving clockwise. The answer is $4 + 2 = 1 \pmod{5}$.

5. Invite the child to continue with problems or to put the materials away.

Control of Error

Answers on reverse of problem cards

Procedure for Casting Out Nines

1. Invite a child to bring paper, pencil and cards with problems for casting out nines. Say that this is a method for checking accuracy of addition problems
2. Tell the child that the sum of an addition problem should be congruent modulo 9 to the sum of the remainders modulo 9 of the addends.
   For example,

1342	$\equiv 1 \pmod 9$	$3 + 4 + 2 = 9$,	1 remains
1571	$\equiv 5 \pmod 9$	$1 + 7 + 1 = 9$,	5 remains
+ 6811	$\equiv 7 \pmod 9$	$8 + 1 = 9$, $6 + 1$ or	+ 7 remains
9724	$\equiv 4 \pmod 9$	$9 \& 7 + 2 = 9$,	$13 \equiv 4 \pmod 9$

3. Have the child continue with other problems.

Control of error

Answers on back of problem cards

Tessellations

Purposes

To understand the concept of tessellations
To become aware of the use of tessellations
To relate tessellations to forms in art and the environment

Preliminary Exercises

Use of geometric insets for design
Practice with geometry materials and congruent figures

Materials

Booklet or cards containing examples of mosaic patterns from different artists, cultures and time periods, labeled as to country and historic period, with pertinent information
Container or file of control cards with outlines of tessellated designs
Container of several geometric shapes (tessera) in various colors and sizes which correspond to the outlines on control cards
Templates of small geometric shapes for drawing tessellations
Geoboards for construction of tessellations
Paper, colored pencils

Procedures for Tessellation Activities

Note: These activities take place over a period of time, according to interest.

1. Invite a child to bring the booklet or cards with examples of mosaic patterns to a table, to examine the illustrations and to read the information which accompanies each example.
2. Invite the child to collect pictures of tessellations for a personal booklet or series of cards, if interested.
3. Invite the child to use the control cards and tessera to build designs according to the given pattern.
4. Invite the child to construct tessellations on a geoboard.
5. Invite the child to make original tessellations or to copy designs from the illustrations on cards or in a booklet, using geometric templates and colored pencils.
6. Invite the child to construct tessellations from colored paper cut into geometric shapes and glued to a backing.
7. Invite the child to research any artist, such as Escher, or period, such as early Greece, in preparation for an illustrated written or oral report.
8. Invite the child to prepare a time line on the history of mosaics.

Measure of Angles

Purposes

To develop skills necessary for measuring angles
To understand the concept of degree as the unit of measurement for angles
To establish a foundation for calculus through measuring angles by radians

Preliminary Exercises

Work with fraction exercises
Practice with geometrical classification materials
Use of definition booklet for measuring angles

Materials

Divided fraction circles
Frame with circle inset calibrated with 360 degrees on outer edge of circle
Recording paper or small booklet and pencil

Procedure

1. Invite a child to bring paper or booklet, pencil, tray of divided fraction circles and the three hundred sixty degree measuring frame to a table.
2. Show the child the three hundred sixty divisions called degrees on the inset circle within the frame and place one of the one-half fraction parts within the inset on the right side with the intersection of the arc and the diameter at zero. Explain that the fractions are to be considered according to their angles in this exercise rather than as the previously used fractional parts. Demonstrate how to write the symbol for degree.
3. Ask the child to determine the number of degrees which are occupied by the one-half and to record it. Tell the child that the number of degrees represent the measure of the angle. (drawing) 1/2 = 1800 angle.
4. Have the child continue to place fraction parts within the inset circle and to record the name of the fraction part and its angle measure in degrees.

Control of Error

Answers in back of booklet

Triangle Facts

Purposes

To develop understanding of properties of triangles
To understand relationships within triangles
To aid the study of geometry
To observe and appreciate the use of triangles in many fields

Preliminary Exercises

Work with geometry classification materials
Practice in analysis of geometric figures

Materials

Heavy paper such as cover stock
Scissors
Straightedge, paper and pencil
Metric ruler
Several classifications of triangles with the measure of each angle stated in degrees
Paper triangles divided into square unit regions (See white pages.)
Material for theorem of Pythagoras
Triangle information booklet

Procedure for Triangle Inequality

1. Invite a child to bring the geometry materials which illustrate different classifications of triangles, a ruler, paper and pencil to a table.
2. Ask the child to choose any triangle and to measure any two of its sides, recording each of the two dimensions, then adding them.
3. Tell the child to measure the third side and record, then to compare this dimension with the sum of the first two.
4. Have the child continue to measure and add dimensions of any two sides of any triangle, then compare with the third side.
5. Ask the child to state the conclusion that can be drawn.

Procedure for Sum of Angles Using Triangle Models

1. Invite a child to bring cover stock, pencil, scissors and a straightedge to a table.
2. Tell the child to draw any size or kind of triangle on the cover stock, using the straightedge, then to cut out the triangles.
3. Have the child label the angles a, b, and c, then cut the triangle apart without cutting through any angle.
4. Ask the child to place the lettered angles together and observe that the sum of all the angles **is 180°.**
5. Invite the child to repeat the procedure with triangles of other sizes and kinds and draw a conclusion about the sum of angles in any triangle.

Procedure for Sum of Angles

1. Invite a child to bring the triangles of each classification with the measure of each angle stated in degrees, paper and pencil to a table.
2. Ask the child to **write** the name of any chosen triangle, to record the measure of each angle within the chosen triangle, and to find the sum of all three angles.
3. Have the child repeat the procedure with the other triangles.
4. Ask the child to state the conclusion that can be drawn.

Area of a Triangle

1. Invite a child to bring the triangles divided into square unit regions, scissors, paper and pencil to a table. State that the area of a triangle is determined by taking one-half of the product resulting form the multiplication of the base and height. A= 1/2 bh

2. Ask the child to chose any triangle, record its name together with the number of units representing the base and the number of units representing the height, then to cut it into parts along the lines indicated.

3. Tell the child to assemble the cut units so that each unit forms a complete square and all the units together form a rectangle. Have the child count and record he number of units, then compare with the product obtained by multiplying the number of units counted on the base and on the height.

4. Remind the child that the area of a triangle is one-half of the base times the height. The rectangle constructed from the cut-up triangle will be 1/2 b h.

5. Have the child repeat the procedure with the other triangles.

Theorem of Hero

1. Invite a child to bring any triangle, paper, pencil and ruler to a table.

2. Ask the child to measure and record the sides of any triangle.

3. Tell the child that area of any triangle can be calculated using the formula $A = \sqrt{s(s - a) (s - b) (s - c)}$ where s represents half the perimeter, also known as semi perimeter, and a, b, and c represent the sides.

4. Have the child add the dimensions of the sides of the triangle, then take one-half of that sum to obtain the semi perimeter,

   for example, 6 + 4 + 8= 18; 112 of 18= 9, so s=9

5. Ask the child to substitute the dimensions of the triangle in the formula, in this example,

   $A = \sqrt{9(9-6) (9-4) (9-8)}$ or $\sqrt{9 \times 3 \times 5 \times 1}$ which is $\sqrt{135}$ so A= 11.61895.

The Theorem of Pythagoras

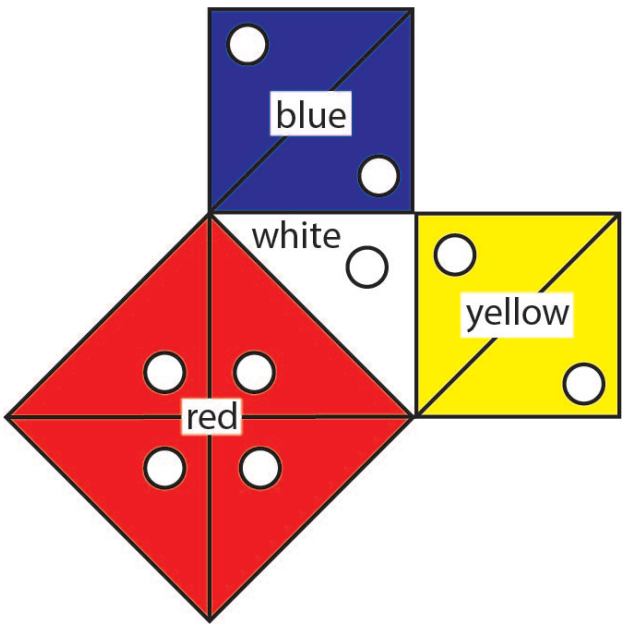
Purposes
- To develop understanding of the Pythagorean Theorem
- To become familiar with its proofs
- To be able to use its applications

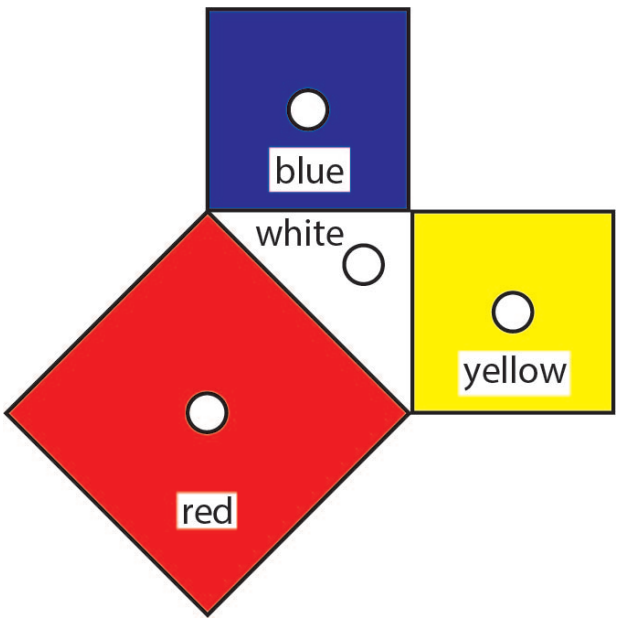
Preliminary Exercises
- Practice with geometry materials
- Experience with construction of geometric figures

Materials
- Three metal plates with insets for the Pythagorean Theorem
- Container of constructive blue triangles
- Paper, pencil, ruler and compass

Procedure for First Case: Two Equal Sides of Right Triangle

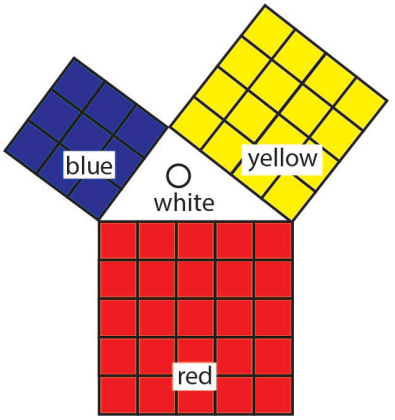
1. Invite a child to a table and bring the first set of metal insets in which two sides of the triangle are equal.
2. Call attention to the squares of two sides being divided in half by a diagonal line, thus forming two equal triangles in each square. Call attention to the two diagonals which divide the square of the hypotenuse into four equal triangles.
3. Have the child *remove* all of the triangles, place those from the square of the hypotenuse into the squares of the sides and those from the squares of the sides into the square of the hypotenuse.
4. Upon completion, tell the child to return the triangles to their original locations according to the insets.





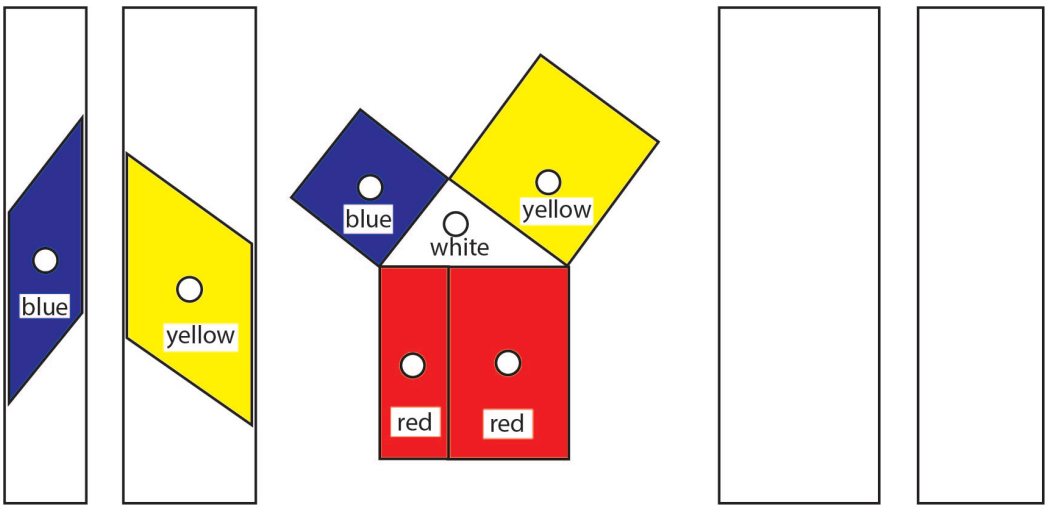
Procedure for Second Case: Sides with Proportion of Three to Four in a Right Triangle

1. Invite a child to a table and bring the second set of metal insets in which two sides of the triangle are in the proportion of three to four.
2. Call attention to the nine squares on the shorter side being the square of three, the sixteen squares on the longer side being the square of four and the twenty-five squares on the hypotenuse being the square of five. $3^2 + 4^2 = 5^2$
3. Have the child remove all the of squares, then place those from the hypotenuse into the insets of the sides and those from the sides into the inset of the hypotenuse.
4. Upon completion, tell the child to return the squares to their original locations.

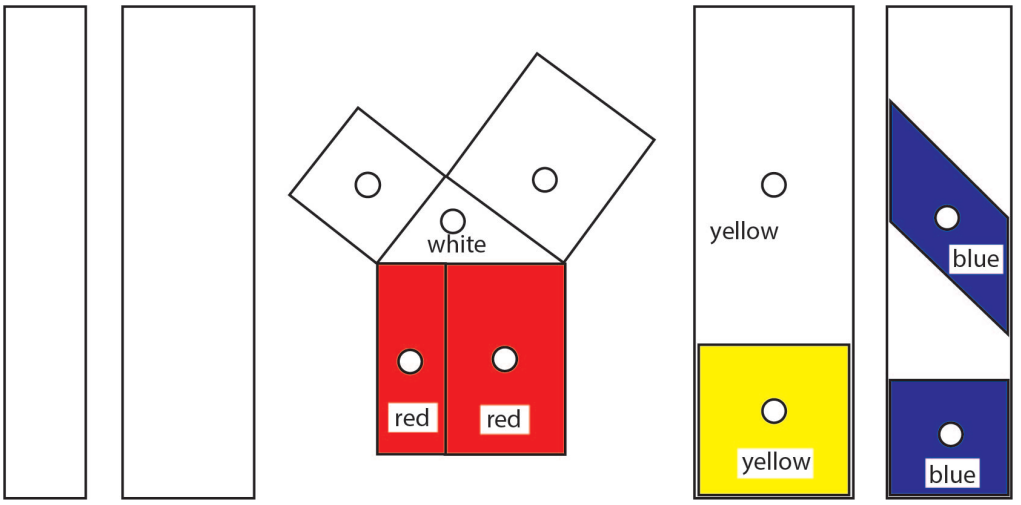


Procedure for Third Case: Euclidean Demonstration 1

1. Invite a child to a table and bring the third set of metal insets in which there are four rectangular insets with one small blue rhomboid and one large yellow rhomboid.
2. Call attention to the dimensions of rhomboids. The blue rhomboid has one pair of sides equal to the shorter side of the triangle and equal to any side of the blue square and one pair of sides equal to the hypotenuse. The yellow rhomboid has one pair of sides equal to the longer side of the triangle and equal to any side of th eyellow square and one pair of sides equal to the hypotenuse.
3. Call attention to the two rectangles in the square of the hypotenuse formed by the perpendicular constructed from the apex of the triangle through the hypotenuse.



4. Tell the child that this material is used to demonstrate the Pythagorean theorem that the two quadrilaterals with equal base and equal altitude are equivalent. Ask the child to move the yellow square and the yellow rhomboid to the wider empty inset at the right so that it can be seen that these are equal in altitude, then remove the pieces and match the bases to determine that these are equal. Therefore, the two quadrilaterals are equivalent.
5. Have the child repeat the procedure with the blue square and blue rhomboid in the narrow empty inset at the right to demonstrate that these two quadrilaterals are equivalent.



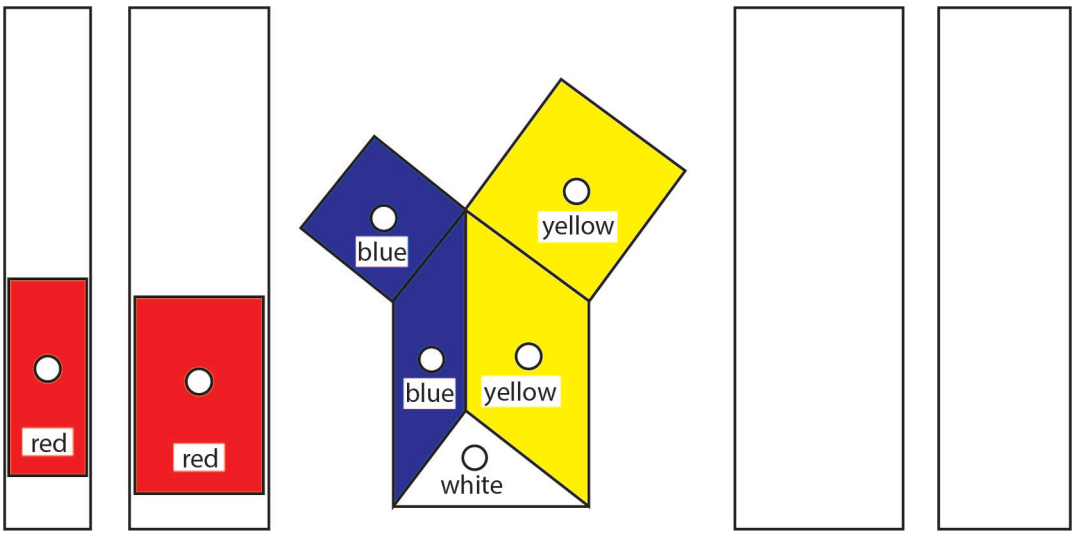
6. Ask the child to return the quadrilaterals to their original locations when finished with the exercise.

Procedure for Third Case: Euclidean Demonstration 2

1. Invite a child to a table and bring the third set of metal insets in which there are four rectangular insets with one small blue rhomboid and one large yellow rhomboid.
2. Tell the child to place the two rectangles of the hypotenuse into the appropriate empty spaces at the left, then to slide the triangle to the base of the empty space below it.
3. Have the child place the two rhomboids in the space above the triangle.
4. Tell the child that this demonstrates in the Pythagorean that the sum of the two rhomboids is equal to the sum of the two rectangles.

Procedure for Proof with Rhombi

1. Invite a child to a table and bring the third set of metal insets in which there are four rectangular insets with one small blue rhomboid and one large yellow rhomboid.
2. Have the child remove the blue square and place it in the white space at the far right of the frame.
3. Tell the child to slide the white triangle to the upper left into the space previously occupied by the blue square, then to place the blue rhomboid into the area above the red rectangles.



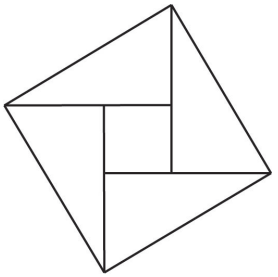
4. Ask the child to return the insets to their original locations, then remove the yellow square and place it in the white space to the right.
5. Have the child slide the white triangle to the upper right into the space previously occupied by the yellow square, then place the yellow rhomboid in the area above the red rectangles.
6. Ask the child to note that the squares and rhomboids are respectively equivalent and that rectangles and squares which are equivalent to the same rhomboids are equivalent to each other which is further proof of the Pythagorean theorem.

Procedure for Bhaskara's Proof with Constructive Blue Triangles

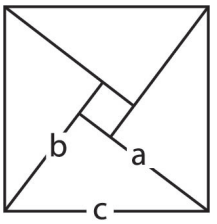
1. Invite a child to bring the container of constructive blue triangles to a table.
2. Tell the child to remove four of the blue triangles from the container and ask that they be identified. (right-angled scalene triangles)
3. Have the child construct a square with the hypotenuse of each triangle forming a side, then identify the figure formed in the center. (square)
4. Have the child replace the triangles in the container and return it to the shelf. If interest is shown, immediately proceed with the construction of Bhaskara's proof.

Procedure for Construction of Bhaskara's Proof

1. Invite a child to bring paper, pencil, straight edge and compass to the table.
2. Tell the child to construct a right triangle with sides of three, four and five units, then to construct perpendiculars at the acute angles the length of the hypotenuse which is five units. Have the child construct a line between the end points of the perpendiculars which will be five units. The result is a square with sides of five units each.
   Note: Units of measurement may be inches or centimeters.
3. Have the child construct a perpendicular to the four unit side of the triangle from the opposite vertex of the square, thus creating a second triangle equivalent to the first.



4. Tell the child construct a perpendicular to the four unit side of the second triangle from the opposite vertex of the square, thus creating a third triangle equivalent to the other two.
5. Ask the child to extend the three unit side of the first triangle to the four unit side of the third triangle, thus forming a fourth equivalent triangle.



6. Choose one right triangle and label the four inch side **a,** the three inch side b and the hypotenuse c. The equation is $c^2 = (a - b)^2 + 4(1/2ab)$

   $= a^2 - 2ab + b^2 + 2ab$

   $= a^2 + b^2$

   In this example, a = 4, b = 3, c = 5

   $c^2 = a^2 + b^2$ $c^2 = 4^2 + 3^2 = 16 + 9 = 25$ $c^2 = 5^2 = 25$

7. Call attention to the interior square formed by the four triangles, the sides of which are equivalent to a - b.
8. Tell the child that this is another proof of the Pythagorean theorem: The square of the hypotenuse is equal to the sum of the squares of the sides.

Procedure for Converse of the Pythagorean Theorem

1. Invite a child to bring paper and pencil to a table and bring the the card with the statement of the Converse of the Pythagorean Theorem and drawings of several triangles, some not right triangles, with dimensions. Have the child read the card.
2. Tell the child to square the side and hypotenuse dimensions, then to add the squares of the sides to determine if the sum is equal to the square of the hypotenuse. Have the child read the card and determine if the triangle is a right triangle . For example, if the sides are six and eight, $6^2 + 8^2 = 36 + 64 = 100$ and the hypotenuse is ten, $1a^2 = 100$. Since the sum of the squares of the sides equals the square of the hypotenuse, it is right triangle.
3. Invite the child to continue with the other triangles.

Converse of the Pythagorean Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right triangle.

Equivalent Plates

Purposes

To develop understanding of equivalent figures and of similar figures
To become aware of the differences between identical and equivalent figures
To further the understanding of fractional parts and relative sizes
To prepare for the use of formulas in abstract calculations

Preliminary Exercises

Practice with the geometric cabinet
Experience with insets for design
Exercises with the fraction circles
Practice in measuring angles
Work with exercises on similarity and congruence

Materials

Square frames which contain insets of ten square centimeters divided as follows:
- one whole square two
- equal rectangles two
- equal triangles four
- equal squares four
- equal triangles eight
- equal rectangles eight
- equal triangles sixteen
- equal squares sixteen
- equal triangles

Tray for carrying frames to table

Introductory Procedure

1. Invite the child to bring the materials to the table and arrange the frames in two rows near the front of the table.
2. Have the child explore possibilities of arranging the insets within the frames to gain familiarity with the equivalent plates.
3. Tell the child to return the insets to the appropriate frame before replacing the materials on the shelf.

Procedure for Congruent and Equivalent Figures

1. Invite the child to bring the divided square material to the table and arrange the frames in two rows near the center of the table.
2. Have the child choose two insets from any one divided frame and place them with the undersides together. State that the chosen insets are congruent. Have the child follow the same procedure using different frames as long as desired.
3. Tell the child to place the square with two equal rectangles and the square with eight triangles at the front of the table, then to remove one rectangle and four triangles from either side and place them above their frames. There are four triangles remaining in one frame and one rectangle remaining in the other.
4. Ask the child to place the triangles into the frame with the rectangle and the rectangle into the frame with the triangles.
5. Explain that since the four triangles exactly fill the space previously occupied by one rectangle, the four triangles are equivalent to one of the rectangles, and since the rectangle exactly fills the space previously occupied by the four triangles, one of the rectangles is equivalent to the four triangles.
6. Have the child place the frame with two triangles next to the frame with eight triangles at the front of the table. Tell the child to remove one of the two triangles, placing it above its frame, then to remove four smaller triangles from the adjacent frame and place them into the space left by the larger triangle.
7. Explain that since the four small triangles fit into the space of one large triangle, four small triangles are equivalent in area to one large triangle. Remind the child that the four small triangles are equivalent to one rectangle, therefore the rectangle and the large triangle are equivalent in area.
8. Invite the child to discover other relationships using the equivalent plates, returning the insets in the proper frames before replacing the materials on the shelf.

Variations

Invite the child to make booklets by drawing around divided figures from each frame, one frame per page, or by drawing the figures on colored paper to be cut and pasted in a booklet, one frame per page.

Invite the child to make geometric designs on sheets of paper using the insets from the equivalent plates.

Divided Triangles

Purposes

To develop understanding of the relationships of the angles of triangles
To provide a means for an analytical study of triangles
To prepare for abstract calculations involving triangles

Materials

Four frames with triangles divided as follows:
one whole triangle
two congruent right-angled scalene triangles three
congruent obtuse-angled isosceles triangles four
congruent equilateral triangles
Tray for carrying material to table
Protractor
Paper
Pencil

Procedure

1. Invite the child to bring the divided triangle material to a table and arrange the frames horizontally near the front of the table with the other materials on the child's dominant side.
2. Show the child how to use a protractor to measure the angles of a triangle.
3. Have the child choose any triangle, write its name, measure and record its angles, then add the angles. Ask the child to repeat the procedure for each of the triangles.
4. Have the child return the insets to their frames before replacing the materials to the shelf.

Triangle and Rectangle

Purpose

To develop understanding of the formula for finding the area of a triangle
To prepare for abstract calculations involving triangles

Materials

Ruler
Frame with an equilateral triangle divided into two small right-angled scalene triangles and a trapezoid whose base is the same length as the base of the rectangle into which the parts are placed

Procedure

1. Invite the child to bring the frame and ruler to the table.
2. Tell the child to remove the insets from the triangle and fit them into the space of the rectangle.
3. Invite the child to measure the rectangle and the triangle, then to compare the measurements.
4. Have the child return the insets to their places before replacing the materials on the shelf.

Trapezoid and Rectangle

Purpose

To develop understanding of the formula for finding the area of a trapezoid
To prepare for abstract calculations

Materials

Frame with an isosceles trapezoid divided into two rectangles and two triangles and an empty rectangular space into which the pieces from the trapezoid will fit
Ruler

Procedure

1. Invite the child to bring the materials and place the frame near the bottom of a table.
2. Tell the child to remove the pieces from the trapezoid and fit them into the space of the rectangle.
3. Ask the child use the ruler to measure the base and the top of the trapezoid and the base of the rectangle, then compare the measurements.
4. Ask the child use the ruler to measure the altitude of the trapezoid and the height of the rectangle, then compare the measurements.
5. Have the child return the insets to their places before replacing the materials on the shelf.

Rhombus and Rectangle

Purpose

To develop understanding of the formula for finding the area of a rhombus
To prepare for abstract calculations involving triangles

Materials

Frame with a whole rhombus and a rectangle divided into an equilateral triangle and two right-angled scalene triangles
Frame with a rhombus divided into an equilateral triangle and two right-angled scalene triangles and a whole rectangle
Ruler

Procedure

1. Invite the child the bring the rhombus materials and place near the front of a table.
2. Have the child remove the whole rectangle from one frame and the whole rhombus from the other frame and compare the altitudes by placing the undersides together.
3. Tell the child to place the whole rectangle and the whole rhombus above their frames.
4. Ask the child to fit the parts of the divided rectangle into the rhombus space and the parts of the divided rhombus into the rectangle space.
5. Invite the child to use the ruler to measure the sides of the rectangle and sides of the rhombus, then compare the measurements.
6. Have the child return the pieces to their places before replacing the materials on the shelf.

Rectangle and Two Rhombi

Purpose

To develop understanding that triangles with equal bases and equal altitudes are equal in area.
To prepare for abstract calculations involving triangles

Materials

Frame with a rectangle divided diagonally, two rhombi divided by opposite diagonals, all with equal bases and an empty rectangle having an altitude equal to that of the other three figures and a length equivalent to all the combined parts of the rectangle and the two rhombi
Ruler

Procedure

1. Invite the child to bring the materials and place near the front of a table.
2. Have the child compare the bases of the triangles from the two rhombi and the square by placing the undersides together.
3. Tell the child to compare the altitudes of one triangle from each figure by placing them in the rectangular space.
4. Ask the child to use the ruler to measure the base and the altitude of one triangle from each inset, then compare the measurements.
5. Remind the child that the triangles are equivalent since they are each half of equivalent figures.
6. Have the child return the parts to their appropriate spaces before replacing the materials on the shelf.

Decagon

Purpose

To develop understanding of the formula for the area of a decagon
To prepare for finding the area of a circle

Materials

Frame with a whole decagon
Frame with a decagon divided into ten isosceles triangles
Frame with nine isosceles triangles, two right angled scalene triangles and a rectangle with a width equal to the bases of five isosceles triangles and the height equal to the altitude of the triangles
Frame with ten trapezoids, nine isosceles triangles, two right-angled scalene triangles and a rectangle as long as the the bases of the ten trapezoids and as high as the altitude of the trapezoids
Drawer of polygons from the geometric cabinet Drawer
of circles from the geometric cabinet
Frame for the largest circle from the geometric cabinet
String
Scissors

Procedure 1

1. Have the child bring the frame with the whole decagon, the frame with decagon divided into ten isosceles triangles and the frame with nine isosceles triangles, two right angled scalene triangles and a rectangle and place near the front of a table.
2. Invite the child to compare the areas of the two decagons by removing the whole decagon and transferring the ten isosceles triangles into the empty frame.
3. Ask the child to remove the rectangle from the frame and rearrange the triangles to form ten isosceles triangles with their bases touching along the width of the rectangular space.
4. Have the child remove the triangles from the rectangular space, then place the triangles from the divided decagon into the rectangular space with bases touching
5. Indicate that the area of the triangles is one half of the area of the rectangular space and the area of the decagon is one half the area of the rectangular space.
6. Invite the child to use the ruler to measure the length and height of the rectangular inset. Remind the child that the rectangular inset is equivalent to the decagon and to the triangles occupying the other half of the large rectangular space.
7. Have the child return the parts to their appropriate spaces before replacing the materials on the shelf.

Procedure 2

1. Have the child bring the frame with the divided decagon and the frame with ten trapezoids, nine isosceles triangles, two right-angled scalene triangles and a rectangle whose base is equivalent to that of the ten trapezoids and whose altitude is equivalent that of the trapezoids and place near the front of a table.
2. Ask the child to remove all the parts from the rectangular frame, then to place the ten isosceles triangles from the divided decagon into the rectangular space with bases touching.
3. After replacing the decagon parts into their frame, have the child form ten isosceles triangles in the large rectangular space by placing the triangles above the trapezoids whose bases are touching along the base of the rectangular space.
4. Indicate that the area of the triangles and trapezoids is one half of the area of the large rectangular space and the area of the decagon is one half the area of the rectangular space.
5. Invite the child to use the ruler to measure the length and height of the rectangular inset. Remind the child that the rectangular inset is equivalent to the decagon and to the triangles occupying the other half of the large rectangular space.
6. Have the child return the parts to their appropriate spaces before replacing the materials on the shelf.

Variations

Invite the child to compare the metal decagon to the decagon from the geometric cabinet. Invite the child to fit each polygon from the geometric cabinet into the frame for the largest circle to compare the area of each polygon to the area of the circle.

Invite the child to compare the diameters and the circumferences of the circles in the geometric cabinet by cutting string to fit around the circumference of each of the circle, then placing each string across its circle to determine the number of diameters equivalent to the circumference. It is always three and a small fraction for each circle, no matter what the size. Tell the child that this is pi whose value is 3.1416.

Invite the child to calculate the circumference of any circle in the geometric cabinet by measuring the diameter then multiplying by pi, 3.1416.

Golden Section

Purposes

To develop appreciation for the esthetics of the Golden Section
To learn the construction of the Golden Section
To be able to recognize the proportions of the Golden Section in art and architecture

Preliminary Exercises

Work with geometric shapes
Experience in using a straight edge and a compass
Practice in geometric construction
Exercises for ratio and proportion
Reading of the history of the Golden Section

Materials

Recording paper and pencil
Straightedge and compass
Strips of paper one inch wide and eight and one half inches long
Booklet or card with information for construction of the Pentagram
Booklet or card with information for construction of the Golden Section
Booklet or card with information for measuring human proportions
Booklet or card with information for constructing the Golden Rectangle

Procedure for the Pentagram of Pythagoras

1. Invite a child bring to a table a paper strip, pencil, straightedge, recording paper and the booklet with information on construction of the pentagram.
2. Tell the child to read the booklet and follow the directions for construction of a pentagram.
3. Have the child label two opposite points lying on the same line A and B, then label D and C the points at which that line is intersected.
4. Ask the child to measure AB, then AC and record.
5. Tell the child to measure AC and AD, then record.
6. Have the child determine the proportion of the measurements recorded.

Procedure for the Golden Section

1. Invite a child bring to a table paper, pencil, straightedge, compass and booklet for construction of the Golden Section.
2. Tell the child to read the booklet and follow the directions for construction of the Golden Section.
3. Have the child measure and record the distances AB, AC and CB, then determine the proportion.

Procedure for the Golden Rectangle

1. Invite a child bring to a table paper, pencil, straightedge, compass and booklet for construction of the Golden Rectangle.
2. Tell the child to read the booklet and follow the directions for construction of the Golden Rectangle.
3. Have the child measure and record the distances AC and CB, then determine the proportion.

$$\frac{AC}{CB} + \frac{1+\sqrt{5}}{2} = \phi$$

Statistical Measures of Location

Purposes

To develop understanding of the concepts of measurements used in statistics To acquire the ability to calculate statistical measures
To be able to interpret statistics
To increase comprehension of mathematics

Preliminary Exercises

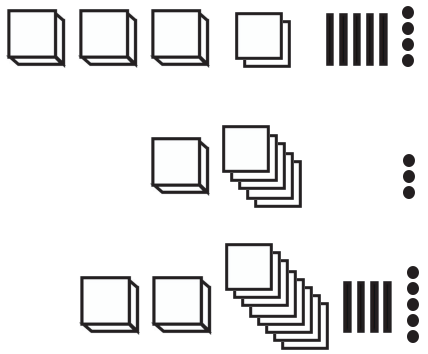
Work with golden bead material
Practice with addition, multiplication, subtraction and division Experience with place value materials

Materials

Tray of golden bead material
Felt mat
Small numeral cards
Container with definition, rule card and sets of problem cards for finding averages Container with definition, rule card and sets of problem cards for finding mode Container with definition, rule card and sets of problem cards for finding median Container with definition, rule card and sets of problem cards for finding range
Note: Mount definitions, rules and problems on a different color for each set of the above cards.
Percentile charts and definition
Paper and pencil
Card with directions for Gauss's method of adding consecutive numbers

Procedure for Determining Averages

1. Invite a child to bring a tray of golden bead material, problems without changing, paper and pencil to a table, placing paper and pencil on the dominant side.
2. Ask the child to choose a problem and to place the quantities of golden beads as indicated.
   For example, 3254, 1503, 2845.



3. Tell the child to observe the number of thousands in each quantity, then move the thousand cubes from the greater quantities to the lesser until each quantity has the same amount. In this example, there are two thousand cubes for each quantity.
4. Have the child repeat the procedure with hundreds, then tens and finally units so that each quantity is the same. In this example, there are two thousand cubes, five hundred squares, three tens and four units for each quantity.
5. Tell the child to count the beads on the bottom row which is the average of the three original quantities. In this example, the average is 2534.
6. Invite the child to continue with problems or to return the materials to the shelf.

Control of Error

Answers on reverse of problem

Variations

Invite the child to calculate averages of groups of numbers using golden beads with problems involving changing.

Invite the child to calculate averages abstractly by dividing the sum of the addends by the number of addends.

Invite the child to use Gauss's method to add consecutive numbers.

Procedure for Determining Median

1. Invite a child to bring numeral cards, container with median definition, rule cards, problems, paper and pencil to a table, placing the container at the center and the paper on the dominant side.
2. Ask the child to read the definition of median and the rule card for odd number data, then place the numeral cards for a problem in a row from left to right.
3. Tell the child to rearrange the cards in numerical order, then to find the middle number which is the median.
4. Have the child copy the series of numerals and underline the median.
5. Ask the child to read the rule card for even number data, then place the numeral cards for the problem in a row from left to right.
6. Tell the child to locate the middle two numerals, add them then divide by two to determine the average.
7. Have the child copy the series, underline the two middle numbers, then write their average, which is the median, below .
8. Invite the child to continue with problems or to put the material away.

Procedure for Determining Mode

1. Invite a child to bring container with mode definition, rule cards, problems, paper and pencil to a table, placing the container at the center and the paper on the dominant side.
2. Have the child read the definition and the rule card for mode, then place the numeral cards for the problem in a row from left to right.
3. Ask the child to rearrange the cards in numerical order with repeated numerals arranged in a column. Tell the child to locate the numeral most often repeated which is the mode, then to copy the series of numerals, underlining the mode.
4. Invite the child to continue with problems or to put the material away.

Procedure for Finding the Range

1. Invite a child to bring container with range definition, rule cards, problems, paper and pencil to a table, placing the container at the center and the paper on the dominant side.
2. Have the child read the definition and the rule card for range, place the numeral cards for the problem in a row from left to right and rearrange them in sequence.
3. Tell the child to copy the sequenced numerals and underline the those which are first and last, then subtract to obtain the range.
4. Invite the child to continue with problems or to put the material away.

Methods of Organizing Data

Purposes

To develop an understanding of the concept of presenting data by graphs and tables To acquire the skills necessary to produce graphs and tables
To understand how to interpret graphs and tables

Preliminary Exercises

Practice with statistical measurements
Work with determining means, median, mode, percentile Use of ruler, compass, graph paper

Materials

Container with examples of various tables and graphs, labeled and unlabeled, separate labels and definitions
Problems for making tables and for graphing
Cards with raw data on topics of interest such as sports statistics
Paper, ruler, compass, graph paper and pencil

Procedure

1. Invite a child to bring container of definition cards and examples of graphs with labels to place on unlabeled ones and place at the center of a table.
2. Ask the child to match each graph with its appropriate label and definition.
3. Have the child choose any problem card, collect the necessary materials and prepare the graph or table as directed.
4. Invite the child to continue graphing or to return the materials to the shelf.

Ratio, Proportion and Sequence

Purpose

To develop an understanding of ratio, proportion and sequence

Preliminary Exercises

Work with common fraction equivalents Practice with decimal fraction equivalents Exercises in solving algebraic equations Experience in solving problems involving percent

Materials

Definition booklet
Recording paper and pencil
List of fraction equivalents
Problem booklets or cards for solving unknowns
Booklet with possibilities for forming arithmetic progressions
Booklet with possibilities for forming geometric progressions
Card with formula for finding the nth term of an arithmetic progression
Card with formula for finding the nth term of a geometric progression
Problem booklets or cards for finding n th terms in progressions
Shallow box with lid and one hundred pennies
Dice
Deck of cards
Bag with five balls of one color and four of another

Introductory Procedure

1. Invite a child to bring the list of fraction equivalents to a table.
2. Tell the child to look at the list of equivalent fractions and to note that one is to two as four is to eight; three is to six as five is to ten, etc.
3. Explain that a fraction expresses a ratio. Say that the ratio of numerator to denominator is the same in all equivalent fractions. Show the child how to express the ratios, for example, $\frac{3}{6} = \frac{4}{8}$ or 3:6::4:8. which is represented by X.
4. Invite the child to read the definition booklet as needed.

Procedure for Solving Unknowns with Ratios

1. Invite a child to bring a problem booklet or card, recording paper and pencil to a table.
2. Tell the child that an unknown can be expressed as a ratio with the numerator as the unknown For example, 60% of 275 is expressed as $\frac{60}{100} = \frac{X}{275}$ with Xthe unknown. 3.

Have the child solve for Xby first multiplying the means and the extremes.

In this example, 100 X= 60 x 275 or 100 X= 16500.

4. Ask the child to divide the numeral on the left of the equation into the product on the right.

In this example, 100 divided into 16500 is 165, so 60% of 275 equals 165.

5. The child may continue to do additional problems or put the materials away.

Procedure for Ordering Sets of Elements in Geometric Progression

1. Invite a child to bring booklet with exercises in geometric progression, recording paper and pencil to a table.
2. Explain that geometric progression is a sequence in which there is a common ratio between successive terms.
   For example, in the progression {2, 4, 8, 16...}, each term following the first is twice the preceding term.
3. Tell the child to develop and record geometric progressions by using the booklet.

Procedure for Ordering Sets of Elements in Arithmetic Progression

1. Invite a child to bring booklet with exercises in arithmetic progression, recording paper and pencil to a table.
2. Explain that arithmetic progression is a sequence in which there is a common difference between successive terms. For example, in the progression {2, 4, 6, 8,...}, each term following the first is two more than the preceding term.
3. Tell the child to develop and record arithmetic progressions by using the booklet.

Procedure for Finding the nth Term of an Arithmetic Progression

1. Invite a child to bring recording paper, pencil and formula card for arithmetic progression nth term = $a+ (n - 1)d$ to a table.
2. Explain that n represents the ordinal number of the unknown term to be determined, a represents the first term in the sequence and d represents the common difference.

   For example, to find the forty-first term of { 2, 5, 8, 11,....], a = 2, d = 3 and n= 41, therefore 2 + (41 - 1)3 equals 2 + (40 x 3) or 2 + 120 = 122 so the forty-first term is 122.

3. Invite the child to continue finding unknown terms in progressions.

Procedure for Finding the *n* th Term of a Geometric Progression

1. Invite a child to bring recording paper, pencil and formula card for geometric progression nth term = $a r^{n-1}$ to a table.
2. Explain that n represents the ordinal number of the unknown term to be determined, a represents the first term in the sequence and r represents the common

   ratio. For example, to find the eleventh term of { 2, 4, 8, 16,...],

   a = 2, r = 2 and n = 11, 2 x 2<11 -1) or 2 x 1024 = 2048
   so the eleventh term is 2048.

3. Invite the child to continue finding unknown terms in progressions.

Procedure for Probability Experiments

1. Invite a child to bring recording paper, pencil, timer and box of pennies to a table.
2. Have the child turn all pennies heads up, replace the lid and shake the box ten times. Ask the child to open the box and count the number of pennies which are heads up and record the number.
3. Tell the child to turn all pennies heads up again, replace the lid, shake the box ten times
   before opening the box and repeating the counting and recording procedure.
4. Have the child repeat the procedure at least ten times, then write a statement about the results.
5. Ask the child to write the results as a fraction, a decimal and percent.

Permutations

Purposes:
To gain an insight to statistics
To develop understanding of calculating permutations To develop understanding of factorials

Preliminary Procedures:
Practice with multiplication Practice with division

Materials:
Sets of cards with matching pictures such as colors, numbers or letters
Note: There should be six different items in each set with six matching pictures of each item in the set.
Set of permutation definition cards Permutation information booklet Set of permutation rule cards Permutation problem cards
Recording paper and pencil

Procedure for finding permutations using all items one time

1. Invite the child to bring the permutation definitions rules, and materials to the table, placing the sets of cards in front and the recording paper and pencil on the dominant side. Tell the child to read the definitions and rules.
2. Explain that permutations are the possible arrangements of items in a set.
3. Ask the child to select three different pictures from one set of cards and arrange them in any order. Have the child record the arrangement. Ask the child to continue making and recording different arrangements of the selected cards until no further arrangements can be found. There are six different arrangements or permutations.
4. Have the child select four cards from a set, find arrangements and record them. There are twenty-four arrangements or permutations. Explain that this method will work when there are only a few items to arrange, but it would be difficult find all the permutations for large groups of items.
5. Have the child read Rule Card 1.
6. The child may compute permutations following the rule or put the materials away.

Procedure for finding permutations in which not all items are used

1. Invite the child to bring the materials to the table, placing the sets of cards in front and the recording paper and pencil on the dominant side.
2. Have the child select five different pictures from one set of cards.
3. Explain that sometimes not all of the items are used for the arranged set.
4. Ask the child to arrange any three of the selected pictures and record the arrangement or permutation. The child continues arranging and recording the permutations which are made with any three of the five cards. There are sixty permutations.
5. Have the child read Rule Card 2.
6. The child may compute permutations following the rule or put the materials away.

Procedure for finding permutations with repetition

1. Invite the child to bring the materials to the table, placing the sets of cards in front and the recording paper and pencil on the dominant side.
2. Have the child select three of three different pictures from a set of cards.
3. Explain that sometimes items are used more than once for the arranged set.
4. Ask the child to arrange any three of the selected pictures and record the arrangement or permutation, then to continue to arrange and record the permutations made with any three of the cards. There are twenty-seven permutations.
5. Have the child select three each of four different pictures from a set of cards.
6. Ask the child to arrange any three of the selected pictures and record the arrangement or permutation, then child continue to arrange and record the permutations made with any three of the cards. There are eighty-one permutations.
7. Have the child read Rule Card 3.
8. The child may compute permutations following the rule or put the materials away.

Procedure for finding combinations without regard to order

1. Invite the child to bring the materials to the table, placing the sets of cards in front and the recording paper and pencil on the dominant side.
2. Ask the child to select five different pictures from one set of cards. Have the child arrange any three and record the arrangement. For example, from the colors red, yellow, blue, orange and green, red, yellow and blue are arranged.
3. Explain that when the order of the arrangement does not matter, the arrangements are called combinations. Yellow, red and blue are the same combination as red, yellow and blue. Have the child arrange and record all the combinations of three of the five selected cards. There are ten combinations.
4. Have the child read Rule Card 4.
5. The child may compute combinations following the rule or put the materials away.

Procedure for finding circular permutations

1. Invite the child to bring the materials to the table, placing the sets of cards in front and the recording paper and pencil on the dominant side.
2. Have the child select four cards from a set, arrange them in a circular configuration and record the arrangement.
3. Explain that any arrangement are rotated to match another arrangement is not distinct or a different arrangement. Have the child record all the circular permutations of the four items. There are six permutations.
4. Have the child read Rule Card 5.
5. The child may compute permutations following the rule or put the materials away.

Procedure for finding permutations in which some items are not distinct

1. Invite the child to bring the materials to the table, placing the sets of cards in front and the recording paper and pencil on the dominant side.
2. Have the child select four pictures from one set with two of the pictures alike.
3. Invite the child to arrange and record the possible permutations of the four cards. There are twelve permutations.
4. Ask the child to read Rule Card 6.
5. The child may compute permutations following the rule or put the materials away.

Control of error:

Answers for problems
Definition cards
Rule cards

Variations:

Invite the child to experiment by using the sets of cards to find permutations with more items.
Invite the child to read the permutation information booklet.
Invite child to find applications for using permutations and list these in a booklet.
Invite the child to match permutation rules to operations.

Vocabulary:

n factorial permutation combination distinct combinatorial analysis

Algebra

Purposes

To develop an understanding of the use of variables
To relate variables to operations
To become familiar with the concept of functions
To acquire the ability to solve algebraic equations
To understand commutative, associative and distributive laws
To become acquainted with the concepts underlying set theory, Venn diagrams, Cartesian coordinates Preliminary

Exercises

Work with the binomial and trinomial cubes and powers of two
material Experience with decanomial square, binomial square,
trinomial square Practice in all arithmetic operations
Exercises with ratio and proportion
Experience in multiplying and dividing fractions
Work with missing number problems

Materials

Definition cards or booklets
Problem cards or booklets for missing signs and missing numbers Recording paper for problems with missing numbers and with missing signs Paper for comparison
Binomial square materials and control chart
Trinomial square materials and control chart
Decanomial square materials and control chart
Binomial cube materials and control chart
Trinomial cube materials and control chart
Power of two materials and control chart
Cubing material with one cube and twenty-seven squares each for one to nine

Procedure for Binomial Square

1. Invite a child to bring the binomial square materials and control charts to a table, then to place the unlabeled control chart to the right of the container of binomial square materials with the labeled control chart and container of labels at the left.

2. Ask the child to place the separate squares and rectangles in a row in front of the unlabeled chart, then move them to the appropriate locations on the unlabeled chart

3. Have the child place labels on the unlabeled chart and arrange the components of the equation in order at the bottom by referring to the control chart.

4. Tell the child that the goal is to be able to sequence the equation $(a+ b)^2 = a^2+ 2ab+ b^2$ without referring to the control, then to be able to write the equation without referral to any materials.

Procedure for Trinomial Square

1. Invite a child to bring the trinomial square materials and control charts to a table, then to place the unlabeled control chart to the right of the container of trinomial square materials with the labeled control chart and container of labels at the left.
2. Ask the child to place the separate squares and rectangles in a row in front of the unlabeled chart, then move them to the appropriate locations on the unlabeled chart
3. Have the child place labels on the unlabeled chart and arrange the components of the equation in order at the bottom by referring to the control chart.
4. Tell the child that the goal is to be able to sequence the equation

$(a+b+c)^2 = a^2+2ab+2ac+b^2+2bc+c^2$ without referring to the control, then to be able to write the equation without referral to any materials.

Procedure for Binomial Cube

1. Invite a child to bring the binomial cube materials and control charts to a table, then to place the unlabeled control chart to the right of the container of binomial cube materials with the labeled control chart and container of labels at the left.
2. Ask the child to place the components of the binomial cube on the unlabeled chart
3. Have the child place labels on the unlabeled chart and arrange the components of the equation in order at the bottom by referring to the control chart.
4. Tell the child that the goal is to be able to sequence the equation

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ without referring to the control, then to be able to write the equation without referral to any materials.

Procedure for Trinomial Cube

1. Invite a child to bring the trinomial cube materials and control charts to a table, then to place the unlabeled control chart to the right of the container of trinomial cube materials with the labeled control chart and container of labels at the left.
2. Ask the child to place the components of the trinomial cube on the unlabeled chart
3. Have the child place labels on the unlabeled chart and arrange the components of the equation in order at the bottom by referring to the control chart.
4. Tell the child that the goal is to be able to sequence the equation

$(a+b+c)^3 = a^3+3a^2b+3ab^2+b^3+3a^2c+6abc+3b^2c+3ac^2+3bc^2+c^3$ without referring to the control, then to be able to write the equation without referral to any materials.

Procedure for Power of Two

1. Invite a child to bring the power of two materials and control charts to a table, then to place the unlabeled control chart to the right of the container of other materials with the labeled control chart and container of labels at the left.
2. Ask the child to place the components of the power of two on the unlabeled chart.
3. Have the child place labels on the unlabeled chart.
4. Tell the child that the goal is to be able to sequence the values of the power of two without referring to the control, then to be able to write the values by calculating the powers of two.

Procedure for Finding Missing Numbers

1. Invite a child to bring paper for problems with missing numbers, pencil and problem cards or booklet to a table, placing the problems on the non-dominant side.
2. Tell the child to copy a problem on the paper but to draw a square instead of the x in the problem to indicate the location of a missing number.
3. Have the child determine the operation necessary for finding the missing number. For example, in the problem $9 \div x = 3$, the child copies it as $9 \div [\ ] = 3$, then determines that the missing number is obtained by dividing the answer three into nine and writes three in the box.
4. Invite the child to continue with other problems or to put the materials away.

Procedure for Finding Missing Signs

1. Invite a child to bring paper for problems with missing signs, pencil and problem cards or booklet to a table, placing the problems on the non-dominant side.
2. Tell the child to copy a problem on the paper and explain that the triangle indicates the missing sign and is to be copied as part of the problem.
3. Have the child determine the operation necessary for finding the missing number. For example, in the problem 8 112 = 4, the child copies it exactly, then determines that the missing sign is obtained by dividing eight by two to get four and writes the division sign in the triangle.
4. Invite the child to continue with other problems or to put the materials away.

Procedure for Recording Equalities and Inequalities

1. Invite a child to bring paper for problems, pencil and problem cards or booklet to a table, placing the problems on the non-dominant side.
2. Tell the child to copy a problem on the paper and explain that the triangle indicates the missing sign and is to be copied as part of the problem.
3. Have the child determine the sign to be written in the triangle.
4. Invite the child to continue with other problems or to put the materials away.

Procedure for Decanomial Square Material

1. Invite the child to bring the decanomial square materials, paper and pencil to the table.
2. Ask the child to select two numbers from one to ten and to remove the squares for those numbers from the box, for example eight and two.
3. Show the child how to arrange the squares in a diagonal in the same pattern as the binomial square.
4. Have the child find rectangles in the materials to complete the construction of the binomial square.
5. Once the square has been constructed, have the child arrange the parts according to the pattern for the binomial square.
6. Invite the child to multiply and record the products for each component of the square, then add the products and record the sum. In this example, $8^2 + 2(8 \times 2) + 2^2 = 100$.
7. Have the child add the two numbers originally chosen and find the square of that sum by multiplying. In this example $(8 + 2)^2 = 100$.
   Note: The child may also use this procedure for constructing trinomial squares.
8. Invite the child to continue with other problems or to put the materials away.

Procedure for Building the Decanomial Square

1. Invite the child to bring the decanomial square materials, recording paper and pencil to the table.
2. Beginning with the square of ten at the upper left of the table, have the child place the squares of each number in a diagonal.
3. The remaining rectangles are placed adjacent to the squares for the same number in descending size.
4. The child computes the amount for each square and rectangle and records it in the appropriate area of the recording paper .
5. Invite the child to continue with other problems or to put the materials away.

Procedure for Cubing Material

1. Invite the child to bring paper and pencil to the work space for cubing.
2. Ask the child to remove the lid from the cubing material and choose two cubes from those representing the cubes of the numbers from one to nine, for example the cubes of two and eight.
3. Hold the cubes in the correct positions for the construction of the binomial cube.
   Note: One cube will be diagonally in front of and higher than the other.
4. Ask the child to select squares from the cubing materials to complete construction of the binomial cube.
5. Once the cube has been constructed, have the child arrange the parts according to the pattern for the binomial cube.
6. Invite the child to multiply and record the products for each component of the cube, then add the products and record the sum. In this example, $8^3 + 3(8^2 \times 2) + 3(8 \times 2^2) + 2^3 = 1000$.
7. Have the child add the two numbers represented by the original cubes chosen and find the cube of that sum by multiplying. In this example $(8 + 2)^3 = 1000$.
8. Invite the child to continue with other problems or to put the materials away.

Algebra with Materials

Purposes

To develop an understanding of the use of variables To relate variables to operations
To acquire the ability to solve algebraic equations
To acquire the ability to factor algebraic equations

Preliminary Exercises

Work with the binomial and trinomial squares
Experience with the binomial and trinomial cube and the powers of two material Practice in all arithmetic operations
Practice with negative numbers
Experience with the decanomial square material and the cubing material

Materials

45 unit squares in green with gray on the reverse to represent negative units 30 blue bars to represent x with gray on the reverse to represent negative x

9 red squares to represent x^2 with gray on the reverse to represent negative x^2
Problem cards or booklets
Recording paper

Preliminary Procedure

1. Invite the child to bring the materials to the table.
2. Select some units and have the child arrange them in a column, then count them. Repeat with several different amounts of units.
3. Demonstrate that the units represent an area with two dimensions, over and up. The over distance is one.
4. Select some units and have the child arrange them in a rectangle.
5. Ask the child to determine the over and up dimensions of the rectangle. Continue if interest is shown.
6. Place an x bar in front of the child and explain that the over distance is one, but the up distance cannot be counted since there are no divisions on the bar. Tell the child that an undefinable amount is called "x."
7. Put an x square in front of the child. Explain that the dimensions of the square cannot be counted. Have the child compare the sides of the square to the x bar. Tell the child that the over distance is x and the up distance is x. This is written "x^2" to indicate that the square is x distance in two dimensions.
8. Continue with a three period lesson to establish that the child can recognize the x and x square.
9. Have the child return the materials to the shelf.

Procedure for Addition

1. Invite the child to bring the materials to the table.
2. Select some x squares, x bars and units. Have the child count them. Indicate the notation of he amounts with a plus sign between each type, for example, $8x^2 + 5x +$ 3. Remove the materials.
3. Write an equation and have the child select the appropriate amounts, for example, $4x^2 + 2x + 7$. Wrlte a second equation under the first and have the child select the appropriate amounts, for example $3x^2 + 4x + 1$.
4. Explain that in adding algebraic equations, the amounts are combined and counted.
5. Ask the child to combine the units, x bars and x squares and record the total.
6. Invite the child to continue by using a problem booklet for addition or return the materials to the shelf.

Algebraic Factoring

1. Invite the child to bring the materials to the table.
2. Have the child select the appropriate amount of materials to match a factoring problem from a card or booklet, for example $x^2 + 5x + 4$.
3. Tell the child that for factoring the materials are into a rectangle in order to determine the over and up dimensions.
4. Have the child put the x^2 on the left of the work area with all the x bars to its right, then determine if the remaining units make a complete rectangle. In this case, the materials do not make a rectangle.
5. Tell the child to move one x bar to the top of the x square and determine if the remaining units can be arranged into a rectangle. The child should continue to move one x bar up at a time until all the materials are arranged in a rectangle. In this case there will be one x bar above the x square, four x bars to the right of the x square and four units in the upper right corner.
6. Have the child find the over and up dimensions of the rectangle, in this case, over $x + 4$ and up $x + 1$, or $(x + 4)(x + 1)$. Note: The over dimension is always written first. 7. Invite the child to continue with additional problems or put the materials away.

Procedure for subtraction

1. Have the child bring the materials to the table.
2. Explain that the gray side of the algebra materials indicates an amount to be removed (negative).
3. Have the child record a problem from a subtraction booklet and put out the amount for the minuend, for example, $2x^2 + 5x + 9$.
4. Tell the child to place materials for the subtrahend, gray side up, below the minuend, for example, $x^2 + 3x + 2$. The gray materials are then placed on top of corresponding colored materials. The difference is material which is uncovered. This is moved to the bottom of the work space and recorded. In this case, $x^2 + 2x + 7$ is the difference.
   Note: If there is a greater number of negative gray materials in the subtra-hend, the difference will be negative, for example, $3x^2 + 2x + 7$ minus $2x^2 + ax + 4$ will equal $x^2 - 6x + 3$.
5. The child may continue to do additional problems or put the materials away.

Procedure for Algebraic Multiplication of a Binomial and a Monomial

1. Invite the child bring the materials to the table.

2. Have the child record a multiplication problem from a problem booklet or card, for example, x + 3 times 4 or (x + 3)(4). The multiplicand and the multiplier represent the dimensions of the rectangle to be constructed. In this example an x bar and three units are placed in a row along the bottom of the work area. This represents x + 3 times 1. Three additional rows of x + 3 are be placed above the first row to complete the rectangle.

3. Ask the child to confirm the dimensions of the rectangle before counting the amounts to determine the product. In this example the product is 4x + 12.

4. Invite the child to continue to do additional problems or put the materials away.

Procedure for Algebraic Multiplication of a Binomial and a Binomial

1. Invite the child bring the materials to the table.

2. Have the child record a multiplication problem from a problem booklet or card, for example, x + 2 times x + 4 or (x + 2)(x + 4). The multiplicand and the multiplier represent the dimensions of the rectangle to be constructed. In this example an x bar and two units are placed in a row along the bottom of the work area to represent the over dimension or x + 2.

3. Explain that the up dimension is x + 4. The unit at the right is part of the up dimension, so three more units are placed above it with an x above them. Show the child that three more x bars are placed horizontally above the x bar along the bottom, and three more units are placed above the unit to the left of the corner unit. An x bar will be placed vertically to the left of the x bar representing the up dimension. An x square completes the rectangle.
4. Ask the child to confirm the dimensions of the rectangle before counting the amounts to determine the product. In this example the product is $x^2 + 6x + 8$.
5. Invite the child to continue to do additional problems or put the materials away.

Procedure for Algebraic Multiplication of Binomials including Negatives

1. Invite the child bring the materials to the table.

2. Have the child record a multiplication problem from a problem booklet or card, for example, x - 2 times x + 4 or (x - 2)(x + 4). The multiplicand and the multiplier represent the dimensions of the rectangle to be constructed. In this example an x bar with two negative units on top of it is placed horizontally along the bottom of the work area to represent the over dimension or x - 2.

3. Explain that the up dimension is x + 4. The amount in the work areais part of the up dimension, so three more rows of x - 2 are placed above it to represent x - 2 times 4. An x square with 2 negative x bars is placed above this amount to represent x - 2 times x.

4. Ask the child to confirm the dimensions of the rectangle before counting the amounts to determine the product. In this example the product is $x^2 + 4x - 2x - 8$ or $x^2 + 2x - 8$.
5. Invite the child to continue to do additional problems or put the materials away.

Procedure for Algebraic Division

1. Invite the child to bring the materials to the table.

2. Have the child record a problem from a booklet or card and select the materials to represent the dividend.

   For the example $2x^2 + 11x + 15 \div (x + 3)$, the child will select $2x^2 + 11x + 15$.

3. Explain that in division the materials are rearranged in a rectangle with the divisor as the over dimension, in this example $x + 3$. Beginning with the largest piece, have the child place materials so that the dimension along the top edge is the divisor. In this example the child will use an x square and three x bars to build a rectangle which is $x + 3$ over and x down. Since there is still an x square, the child continues to build the rectangle down using the x square and three additional x bars for a down dimension of 2x. Remaining are 5 x bars and 15 units to be arranged in the rectangle. The x bars are placed horizontally under the bottom x square with the units arranged under the vertical x bars. In this example the down dimension is $2x + 5$ which represents the quotient.

4. Invite the child to continue to do additional problems or put the materials away.

Procedure for Algebraic Division with Negatives

1. Invite the child to bring the materials to the table.

2. Have the child record a problem from a booklet or card and select the materials to represent the dividend. For the example $4x^2 + 2x - 6 \div (2x - 2)$, the child will select $4x^2 + 2x - 6$.

3. Explain that in division the materials are rearranged in a rectangle with the divisor as the over dimension, in this example $2x - 2$. Beginning with the largest piece, have the child place materials so that the dimension along the top edge is the divisor. In this example the child will begin with two x squares as part of the over dimension. The remainder of the over dimension will be a negative which is formed with negative x bars.

4. Tell the child that zero is added to the dividend in the form negative and positive x bars. In this example 4 negative and 4 positive x bars are added, then two negative x bars are placed on top of the right-most x square. Remaining are two x squares to build into the rectangle, which are placed below the previously placed x squares with the remaining two negative x bars on top of the right-most square. The child continues to build the rectangle down using the remaining six positive x bars placed horizontally below the x squares with the six negative units placed on top of the three right-most x bars.

5. Have the child check the dimensions of the rectangle before recording the down dimension as the quotient. In this example the over dimension along the top and the bottom of the rectangle is $2x - 2$ and the down dimension along the left and the right of the rectangle is $2x+3$.

6. Invite the child to continue to do additional problems or put the materials away.

Trigonometry

Preliminary Exercises

Work with parts of the circle
Practice with decimal fractions
Exercises involving ratio and
proportion Experience with Cartesian
coordinates

Materials

Labeled and unlabeled trigonometry charts with container of separate
labels Container of Trigonometry definition cards ?
Container of labeled and unlabeled formula cards with separate labels
Information booklet
Container of cards or booklets with Sine problems
Container of cards or booklets with Cosine problems
Trigonometry board
Note: See directions for preparation in red pages.
Perpendicular guide
Pencil and recording paper

Procedure for charts

1. Invite a child to bring labeled and unlabeled trigonometry charts with container of labels to a table.
2. Have the child place the unlabeled chart to the right of the labeled chart.
3. Ask the child to read the labeled chart and then place the labels appropriately on the unlabeled chart.
4. Invite the child to attempt to place labels without referring to the labeled control chart except to check the work once completed, or to put the work away.

Control of error

Labeled control chart

Variation

Invite the child to read the information booklet.

Procedure for Using Formula Cards

1. Invite a child to bring the container of formula cards to a table.
2. Have the child match the labeled and unlabeled cards and place the labels appropriately.
3. Invite the child to place the labels without referring to the labeled control cards except to check the work once completed, or to put the work

away. Control of Error

Labeled control formula cards

Procedure for Solving Sine Problems on the Trigonometry Board

1. Invite a child to bring the Trigonometry Board, perpendicular guide ,sine problems, paper and pencil to a table.
2. Have the child select a problem, copy it, then set the movable radius according to the angle indicated in the problem. For example, in the problem, Find the sine of 30°, the child adjusts the movable radius to form an angle of 30°.
3. Ask the child to move the perpendicular attached to the circumference end of the radius to intersect the x axis of the circle, using the perpendicular guide.
4. Tell the child that the sine of the angle is the length of the perpendicular from the x axis to the circumference of the circle. Have the child determine the length and record it. In this example, the sine of a 30 degree angle is 0.05.
5. Invite the child to continue with other sine problems or to put the work away.

Control of Error
Answers on reverse of problem cards or in back of problem booklet

Procedure for Solving Cosine Problems on the Trigonometry Board

1. Invite a child to bring the Trigonometry Board, perpendicular guide ,cosine problems, paper and pencil to a table.
2. Have the child select a problem, copy it, then set the movable radius according to the angle indicated in the problem. For example, in the problem, Find the cosine of 30°, the child adjusts the movable radius to form an angle of 30°.
3. Ask the child to move the perpendicular attached to the circumference end of the radius to intersect the x axis of the circle, using the perpendicular guide.
4. Tell the child that the cosine of the angle is the distance along the x axis from the center of the circle to the intersection of the perpendicular with the x axis. Have the child determine the length and record it. In this example, the cosine of a 30 degree angle is .87.
5. Invite the child to continue with other cosine problems or to put the work away.

Control of Error
Answers on reverse of problem cards or in back of problem booklet

Directions for Preparation of Trigonometry Board

1. Using 24 by 24 inch graph paper with a 1/5 inch grid, draw a quarter circle whose circumference lies 2 inches within the boundary of the paper with the center at the lower right.
2. Construct the x axis horizontally and the y axis vertically.
3. In the upper right quadrant of the circle mark along the circumference of the circle the number of degrees, starting with 0 degrees at the intersection of the x axis with the circumference and proceeding counterclockwise to 90 degrees at the intersection of the y axis with the circumference.
4. Measure the radius of the circle and cut rigid transparent plastic that length plus one half inch, and one half inch wide.
5. Measure another strip of transparent plastic one half inch wide and about 1 inch longer than the radius. This will be the perpendicular.
6. Attach the perpendicular to one end of the radius with a small brad so that it can be moved.
7. Attach the radius (with its perpendicular) to the center of the circle where the x and y axis intersect, using a small brad so that it can be moved. The brad holding the perpendicular to the radius must be exactly at the point where the radius intersects the circumference.
8. Using rigid transparent plastic, make a perpendicular guide 1 inch wide with a vertical line carefully constructed from a base line.